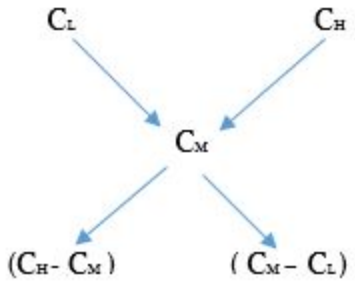
# ALLIGATION AND MIXTURES

Mixture is a specific type of ratio and proportion. One or two questions of this chapter are asked regularly for any competitive exams. Questions are of limited types hence marks can be ensured with very less efforts.

## ALLIGATION

### Rule of alligation.

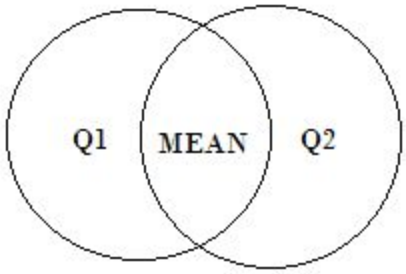
Let $C_L$ be the cost of a lower price product A and $C_H$ be the cost of higher price product B. Let $C_M$ be the mean cost after mixing the two products. Then the ratio for which the quantities of products A and B should be mixed to get the desired cost as $C_M$ is defined as,




Quantity of A : Quantity of B = $(C_H - C_M) : (C_M - C_L)$

Always remember that $C_L < C_M < C_H$ and they must have the same unit. Alligation must be done only on a single object i.e. parameters relating to a single object must be considered for alligation.

### Venn diagram interpretation of Alligation




Q1: Price of Quantity 1

Q2: Price of Quantity 2

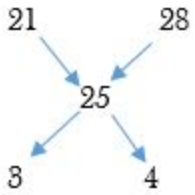
MEAN: Mean price

- Example Questions.

## Case 1: Basic Alligation

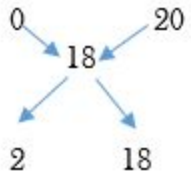
1. In what ratio must wheat at Rs. 21 per kg be mixed with wheat at Rs. 28 per kg, so that the mixture worth Rs. 25 per kg?

Here the mean price is Rs. 25
Alligating,




Required ratio = 3:4

2. In what ratio must water and wine at Rs.20 per liter be mixed to produce a mixture worth Rs.18 per liter?
Here the cost price of water = Rs. 0 per liter
Alligating,

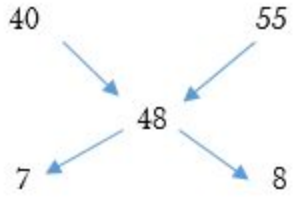



Required ratio = 2: 18
= 1: 9

## Case 2: To find unknown quantity

3. How many kilograms of salt costing 40 paise/kg must be mixed with 16 kg of salt consisting 55 paise/kg so that 25% may be gained by selling the mixture at 60 paise/kg?

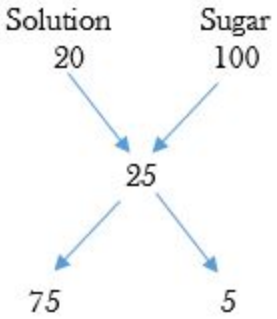
Here the amount 60 paise/kg is not the mean price instead it is the selling price of the salt. This amount is actually 25% more than the mean price (i.e. 1/4 more). So in order to find the mean price 1/5 th of the selling price must be subtracted from the selling price.
Therefore mean price = 60 – (60/5)
= 48 paise/kg
Then we can alligate,




Here the ratio of mixing salt at 40 paise/kg : salt at 55 paise/kg = 7:8
In the question it is given that quantity of salt at 55 paise/kg = 16 kg
Which means that 8 is representing 16 therefore 7 will represent 14 kg.

## Case 3: Addition of pure quantity

4. 150 gm. of sugar solution has 20% sugar in it. How much sugar should be added to make it 25 % in the solution?
Percentage of sugar content in pure sugar = 100 %
Then we can alligate,



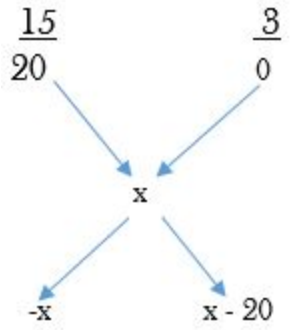

Therefore required ratio = 75:5 = 15:1
From the question quantity of sugar solution = 150 gm.
Which means that 15 is representing 150 therefore quantity of sugar = 150/15 * 1 = 10 gm.

## Case 4: Percentage of a quantity in a solution after mixing.

5. 15 liters of a mixture contains 20% alcohol and the rest water. If 3 liters of water be mixed in it, the percentage of alcohol in the new mixture will be?

In these kind of questions we have to focus on one of the contents of the mixture i.e. either alcohol or water.
In the question we are asked to find the percentage of alcohol so it is good to focus only on alcohol part.

Percentage of alcohol in pure water = 0 %
Then we can alligate,

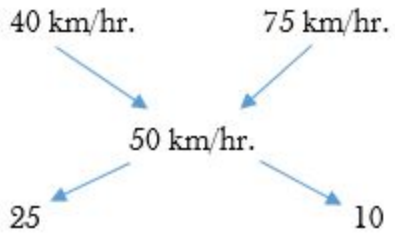



Ratio of mixing = -x :( x-20) = 15:3
= x :( 20-x) = 5:1
Percentage of alcohol after mixing = x = 16.66%

## Case 5: Alligation in Time and Distance.

Alligation has many applications. One of which is in problems of Time and Distance.
If a man travels a part of journey at ‘x’ km/hr. and the rest at ‘y’ km/hr. then the time for which he travelled at ‘x’ km/hr. or ‘y’ km/hr. can be obtained alligation.

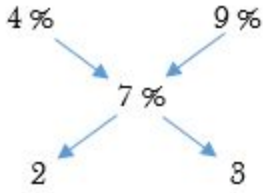
Where Time 1 is the time for which he travelled at ‘x’ km/hr. and Time 2 is the time for which he travelled at ‘y’ km/hr.
6. A man travels a distance of 200 km in 4 hours, partly by bus at 40 km/hr. and rest by train at 75 km/hr. Find the distance covered in each part?
Average speed = 200 / 4 = 50 km/hr.
Alligating,



Ratio of Time division = 25:10 = 5:2
Therefore Distance travelled at 40 km/hr. = (5/7) * 4 * 40 = 114 2/7 kms
Distance travelled at 75 km/hr. = (2/7) * 4 * 75 = 85 5/7 kms

## Case 6: Alligation in Profit and Loss.

7. A trader has 25 kg of rice, part of which he sells at 4 % profit and the rest at 9 % profit. He gains 7 % on the total. Find the quantity sold at 9 % profit?
Here 7 % is the mean profit.




Quantity of rice sold at 9 % profit = (3/5)*25 = 15 kg.

## REPLACEMENT.

Replacement is one of the most important subtopics in Mixtures.

8. A jar contains a mixture of two liquids of two A and B in the ratio 3:1. When 15 liters of the mixture is taken out and 9 liters of liquid B is poured into the jar, the ratio becomes 3:4. How many liters of liquid A was contained by the can initially?

Let initial quantity of A and B be 3x and 1x respectively.
Quantity of A in 15 liters of mixture = 15*(3/4)
Quantity of B in 15 liters of mixture = 15*(1/4)

Then we can write,
(3x -15*(3/4)) / (x-15*(1/4)) + 9 = 3/4
x = 6.75
Therefore, quantity of A initially = 3 * 6.75 = 20.25 liters.

### REPLACEMENT EQUATION.

This is generally used in case of repeated replacement.
Pure quantity remains = $x\ (1 - y/x)^n$
x = initial pure quantity.
y = replaced quantity.
n= Number of operations.

9. A container contains 60 kg of milk. From this container 6 kg of milk was taken out and replaced water. This process was repeated a further two times. The amount of milk left in the container is?

Pure quantity remains = $x\,(1 - y/x)^n$

$= 60(\,1 - 6/60)^3$

$= 43.74$ kg.

## Shortcut /OR method

After the first replacement, the quantity of milk will become 54 kg and quantity of water will be 6 kg.

Ratio of milk to water = 54:6 = 9:1

After the second replacement, Quantity of milk = 60 * (9/10) -6 * (9/10) = 48.6 kg

After the third replacement, Quantity of milk remaining = 48.6 - 6 * (48.6/60) = 43.74 kg

# MIXING OF TWO MIXTURES.

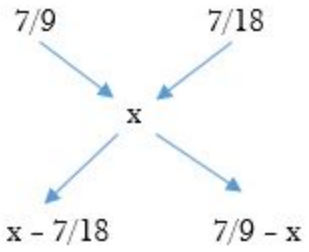
10. A and B are two alloys of gold and copper prepared by mixing metal in the ratio 7:2 and 7:11 respectively. If equal quantities of the alloys are melted to form a third alloy C, the ratio of gold and copper in C will be?

In this type of questions alligation can be applied,

Fraction of Gold in A = 7/9

Fraction of Gold in B = 7/18

Equal quantities of A and B are taken therefore the ratio will be 1:1.




$x - 7/18 \,/\, 7/9 - x = 1/1$

$x = 7/12$

Fraction of Gold in C = 7/12

Fraction of Copper in C= 12-7/12 = 5/ 12

Ratio of Gold and Copper = 7:5

## MIXING OF THREE MIXTURES.

11. Three utensils contain equal quantity of mixtures of milk and water in the ratio 6:1, 5:2 and 3:1 respectively. If all the solutions are mixed together, the ratio of milk and water in the final mixture?

In utensil 1 the fraction of milk is 6/7

Similarly in utensil 2 fraction of milk is 5/7and in utensil 3 fraction of milk is ¾

Like this we can see that the fraction of water in the 3 utensils are 1/7 , 2/7 and ¼

In the resultant solution the fraction of milk will be the sum of the fractions of milk in the 3 utensils since equal quantities are taken from each utensil.Similarly the fraction of water will be the sum of the fractions of water in three utensil.Therefore,

The ratio of milk and water in the resultant solution = 6/7 + 5/7 + ¾ : 1/7 + 2/7 + ¼

= 65:19