# Numbers

In Decimal number system, there are ten symbols namely 0,1,2,3,4,5,6,7,8 and 9 called digits. A number is denoted by group of these digits called as numerals.

# Face Value

Face value of a digit in a numeral is value of the digit itself. For example in 321, face value of 1 is 1, face value of 2 is 2 and face value of 3 is 3.

# Place Value

Place value of a digit in a numeral is value of the digit multiplied by $10^n$ where n starts from 0. For example in 321:

- Place value of $1 = 1 \times 10^0 = 1 \times 1 = 1$
- Place value of $2 = 2 \times 10^1 = 2 \times 10 = 20$
- Place value of $3 = 3 \times 10^2 = 3 \times 100 = 300$

# Types of Numbers

1. **Natural Numbers** - $n > 0$ where n is counting number; [1,2,3...]
2. **Whole Numbers** - $n \geq 0$ where n is counting number; [0,1,2,3...].

   0 is the only whole number which is not a natural number.

   Every natural number is a whole number.
3. **Integers** - $n \geq 0$ or $n \leq 0$ where n is counting number;...,-3,-2,-1,0,1,2,3... are integers.
   - **Positive Integers** - $n > 0$; [1,2,3...]
   - **Negative Integers** - $n < 0$; [-1,-2,-3...]
   - **Non-Positive Integers** - $n \leq 0$; [0,-1,-2,-3...]
   - **Non-Negative Integers** - $n \geq 0$; [0,1,2,3...]

   0 is neither positive nor negative integer.
4. **Even Numbers** - $n / 2 = 0$ where n is counting number; [0,2,4,...]
5. **Odd Numbers** - $n / 2 \neq 0$ where n is counting number; [1,3,5,...]
6. **Prime Numbers** - Numbers which is divisible by themselves only apart from 1.

# Decimal Fractions

Fractions having denominators in power of 10 are called decimal fractions.

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1/10 = .1, 2/10 = .2, ....
1/100 = .01, 2/100 = .02, ...
1/1000 = .001, 2/1000 = .002, ...
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# Converting a decimal number into a fraction

In the denominator part, place 1 under decimal point and suffix with as many zeroes as is the total number of digits after decimal point. Remove the decimal point and reduce the fraction to its lowest term.

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.56 = 56/100 = 14/25
.0024 = 24/10000 = 3/1250
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Suffixing zeroes to the right of a decimal fraction does not change its value. Thus 0.6 = 0.60 = 0.600 etc.

If numerator and denominator contains same number of decimal places, we can remove decimal signs from each number.

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2.71/3.41 = 271/341
14.4/15.6 = 144/156 = 12/13
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# Factors and Multiples

If a number P divides another number Q exactly, we say that P is a factor of Q i.e. Q is a multiple of P.

# H.C.F

The H.C.F of two or more than two numbers is the greatest number that divides each of them exactly.

# L.C.M

The least number which is exactly divisible by each one of the given numbers is called their L.C.M

# Product of two numbers

Product of their H.C.F and L.C.M

# Co-primes

Two numbers are co-primes if their H.C.F is 1.

# H.C.F and L.C.M of fractions

1. H.C.F = $\frac{\text{H.C.F of Numerators}}{\text{L.C.M of Denominators}}$
2. L.C.M = $\frac{\text{L.C.M of Numerators}}{\text{H.C.F of Denominators}}$

# Percentage

Percent means many hundredths.Example: z% is z percent which means z hundredths. It will be written as:

$z\% = \frac{z}{100}$

$\frac{p}{q}$ as percent: $(\frac{p}{q} \times 100)\%$

# Commodity

If the price of a commodity increases by R%, then the reduction in consumption so as not to increase the expenditure is:

$[\frac{R}{(100 + R)} \times 100]\%$

If the price of a commodity decreases by R%, then the increase in consumption so as not to decrease the expenditure is:

$[\frac{R}{(100 - R)} \times 100]\%$

# Population

The population of a city is P and let it increases at the rate of R% per annum:

Population after t years: $P(1 + \frac{R}{100})^t$

Population t years ago: $\frac{P}{(1 + \frac{R}{100})^t}$

# Depreciation

Let V be the present value of machine. Suppose it depreciates at the rate of R% per annum:

Machine's value after t years:$P(1 - \frac{R}{100})^t$

Machine's value t years ago: $\frac{P}{(1 - \frac{R}{100})^t}$

- **If P is R% more than Q, then Q is less than P by how many percent?**

  $[\frac{R}{(100 + R)} \times 100]\%$

- **If P is R% more than Q, then Q is more than P by how many percent?**

  $[\frac{R}{(100 - R)} \times 100]\%$

## Ratio

Ratio represents the proportion in which a number contains another number. A ratio is represented by a/b or a:b. The duplication or division of every term of a proportion by the same non zero number does not influence the proportion.Subsequently, 3:5 is the same as 6:10 or 9:15 or 12:20 and so on.

## Extent

The fairness of two proportions is called extent. As 2:3 = 6:9, we compose, 2:3::6:9 and we can say that 2,3,6,9 are in extent . Here 2 and 9 are called extremes while 3 and 6 are called implies.

In an extent, result of extremes=product of means.

## Fourth Relative

if a:b::c:d, then d is known as the fourth relative to a,b and c.

## Third Relative

if a:b::c:d, then c is known as the third relative to a and b.

## Mean Relative

Mean relative is the middle of a and b.

# Profit and loss

1. **Cost Price, (c.p.)** = The price, at which an article is purchased, is called its cost price.
2. **Selling price (s.p)** = The price, at which an article is sold, is called its selling price.
3. **Profit or Gain** = (S.P) - (C.P)
4. **Loss** = (C.P) - (S.P)
5. Gain or Loss is always reckoned on C.P.

## Formulae

1. Gain% = (Gain*100) / C.P
2. Loss% = (Loss*100) / C.P
3. S.P = (100+ Gain %) /100 * (C.P)
4. S.P = (100-Loss %) / 100 * (C.P)
5. C.P = 100 / (100 + Gain %)* (S.P)
6. C.P = 100 / (100-Loss %)* (S.P)

## Important cases

1. If an article is sold at a profit of say, 20%, then SP = 120% of CP.
2. If an article is sold at a loss of say, 20%, then SP = 80% of CP.
3. When a person sells two similar items, one at a gain of say x% and the other at a loss of say x%. then the seller always incurs a loss given by:

$$\text{Loss\%} = (x/10)^2$$

4. If a seller sells his goods at cost price but uses false weights, then

$$\text{Gain\%} = [\text{Error}/(\text{True value} - \text{Error}) * 100]\%$$