1. The maximum value of $|z|$ when z satisfies the condition $|z + \frac{1}{z}| = 4$ is

   A) $2 - \sqrt{5}$ B) $2 + \sqrt{5}$ C) $4 - \sqrt{5}$ D) $4 + \sqrt{5}$

2. If $1, \omega_1, \omega_2, \ldots \omega_9$ are the 10^{th} roots of unity, then $(1 + \omega_1)(1 + \omega_2) \cdots (1 + \omega_9)$ is

   A) 0 B) 1 C) -1 D) 9

3. If x is a real number, then $(x - 1)^2 + (x - 2)^2 + \cdots + (x - 100)^2$ is least when x is

   A) 50 B) 100 C) 101 D) $\frac{101}{2}$

4. The sum $100C_0 + 101C_1 + 102C_2 + \cdots + 150C_{50}$ is

   A) $200C_{100}$ B) $201C_{50}$ C) $201C_{100}$ D) $151C_{50}$

5. If $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{pmatrix}$ then A^{101} is

   A) I B) $A - I$ C) A D) $(a + b)(A - I)$

6. The value of the determinant $\begin{vmatrix} 1 & \log_5 10 & \log_5 15 \\ \log_{10} 5 & 1 & \log_{10} 15 \\ \log_{15} 5 & \log_{15} 10 & 1 \end{vmatrix}$ is

   A) 0
   B) 1
   C) $\log_5 150 + \log_{10} 75 + \log_{15} 50$
   D) $\log_5 25 + \log_{10} 20 + \log_{15} 15$

7. For what value of λ will the equation $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represent a pair of straight lines

   A) 4 B) 2 C) -2 D) 3

8. The equation of a tangent to the circle $x^2 + y^2 - 2x - 6y - 12 = 0$ is

   A) $\sqrt{3}(x - 2) + (y - 3) = 0$
   B) $\sqrt{3}(x - 2) + (y - 3) = 5$
   C) $\sqrt{3}(x - 2) + (y - 3) = 10$
   D) $(x - 2) + \sqrt{3}(y - 3) = 5$

9. The director circle of the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ is

A) $x^2 + y^2 = 16$ B) $x^2 + y^2 = 9$
C) $x^2 + y^2 = 7$ D) $x^2 + y^2 = 25$

10. The angle between the planes $2x - y + z = 6$ and $x + y + 2z = 3$ is

A) π B) $\dfrac{\pi}{2}$ C) $\dfrac{\pi}{3}$ D) $\dfrac{\pi}{6}$

11. The equation of the perpendicular bisector of the straight line joining the points $(2, 3)$ and $(1, 2)$ is

A) $x - y + 4 = 0$ B) $x - y - 2 = 0$
C) $x + y - 4 = 0$ D) $x + y - 2 = 0$

12. The spheres $x^2+y^2+z^2 = 25$ and $x^2+y^2+z^2-24x-40y-18z+225 = 0$

A) touch internally
B) touch externally
C) do not touch each other
D) intersect each other

13. $\cos 2x + a \sin x = 2a - 7$ possesses a solution for

A) all a B) $a > 6$ C) $a < 2$ D) $a \in [2, 6]$

14. The lowest degree of the polynomial with real coefficients having roots $2, -3, 2 + i, 1 + i$ is

A) 2 B) 4 C) 6 D) 8

15. Let $f(x) = 6x + 5$. If f_n denotes the function $f \circ f \circ \cdots \circ f$ n times then $f_{15}(5)$ is

A) $6^{15} - 1$ B) $6^{15} + 1$ C) $6^{16} - 1$ D) $5(6^{15} + 1)$

16. If $f(x) = 2^x + 2^{x+1} + \cdots + 2^{x+9}$ then $f'(2)$ is

A) $1023 \log_e 16$ B) $1023 \log_e 8$ C) $1023 \log_e 4$ D) $1023 \log_e 2$

17. If $f(x) = \min\{x, x^2\}$ for every real value of x, then which one of the following is not true

A) f is continuous for all x
B) f is differentiable for all x
C) $f'(x) = 1$ for all $x > 1$
D) one of the above statement is wrong

18. If $\int_0^{\frac{\pi}{2}} \cos^n x dx = A$, then the value of $n \int_{\frac{\pi}{2}}^{0} \sin^n x dx$ is

A) $-A$ B) A C) nA D) $-nA$

19. If $\int\limits_0^x f(t)\,dt = x + \int\limits_x^1 tf(t)\,dt$ then the value of $f(1)$ is

A) $\frac{1}{2}$ B) $-\frac{1}{2}$ C) 1 D) -1

20. The general solution of the equation $(e^{-x} + \sin y)dx + \cos y dy = 0$ is

A) $x + e^{-x}\cos y + C = 0$
B) $x - e^{-x}\sin y + C = 0$
C) $x + e^{x}\sin y + C = 0$
D) $x - e^{x}\sin y + C = 0$

21. $\lim\limits_{n\to\infty} \{\sqrt{n^2+n} - n\}$ is

A) 0 B) 1 C) $\frac{1}{2}$ D) ∞

22. $\lim\limits_{n\to\infty} (n^{\frac{1}{n}} - 1)^n$ is

A) 1 B) 0 C) e D) ∞

23. Which of the following series is divergent

A) $\sum\limits_{n=1}^{\infty} \frac{(-1)^n}{n}$ B) $\sum\limits_{n=1}^{\infty} \frac{1}{n\log(n+1)}$
C) $\sum\limits_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{n}$ D) $\sum\limits_{n=1}^{\infty} \frac{1}{n^2}$

24. Which of the following sequence is convergent for all x in $[0,1]$, but is not uniformly convergent on $[0,1]$?

A) $\{\frac{\sin nx}{\sqrt{n}}\}$ B) $\{\sin nx\}$ C) $\{x^n(1+x)^{-n}\}$ D) $\{x^n\}$

25. If $A = \lim_{x\to 0} x\sin\frac{1}{x}$ and $B = \lim_{x\to\infty} x\sin\frac{1}{x}$, then

A) $A = B = 0$ B) $A = 0$ and $B = \infty$
C) $A = 0$ and $B = 1$ D) $A = 1$ and $B = \infty$

26. Let $[x]$ denote the greatest integer not exceeding x, then the value of the Riemann Stielgies integral $\int\limits_0^2 x^2 d[x]$ is equal to

A) 1 B) 3 C) 5 D) 0

27. Let the function f be defined on $\mathbb{R}$ by

$$f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ x, & \text{Otherwise} \end{cases}$$

Let μ be the Lebesgue measure on $[0,1]$, then the Lebesgue integral $\int\limits_0^1 f d\mu$ has the value

A) 1 B) 0 C) $\frac{1}{2}$ D) 2

28. Let $f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ -1, & \text{if } x \text{ is irrational} \end{cases}$
Then which of the following function is Riemann integrable on $[0,1]$

A) f B) $|f|$ C) f^+ D) f^-

29. If the radius of convergence of the power series $\sum\limits_{n=0}^{\infty} a_n z^n$ is R, then the radius of convergence of the power series $\sum\limits_{n=0}^{\infty} n^2 a_n z^n$ is

A) R B) $2R$ C) $\frac{R}{2}$ D) R^2

30. Which of the following power series represent the principal branch of $\log(1+z)$?

A) $z - \frac{z^2}{2} + \frac{z^3}{3} - \cdots$ B) $z + \frac{z^2}{2} + \frac{z^3}{3} + \cdots$

C) $1 + z + \frac{z^2}{2} + \cdots$ D) $1 - z + \frac{z^2}{2} - \cdots$

31. Let γ be the path defined by $\gamma(t) = e^{4\pi i t}$, $0 \leq t \leq 1$. Then the value of the integral $\int_\gamma \frac{dz}{z}$ is

A) $2\pi i$ B) $4\pi i$ C) 0 D) $-2\pi i$

32. The singularity of the function $\frac{1-\cos z}{z^2}$ at $z = 0$ is

A) a simple pole B) a pole of order 2
C) a removable singularity D) an essential singularity

33. Let γ be a positively oriented unit circle, then $\int_\gamma \frac{\sin z}{z^2} dz$ has the value

A) $2\pi i$ B) 0 C) $-2\pi i$ D) $4\pi i$

34. At $z = 0$, the function $f(z) = \frac{1}{z} + \frac{1}{z^2} + e^{\frac{1}{z}}$ has

A) an essential singularity B) a simple pole
C) a pole of order 2 D) a removable singularity

35. The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{n^2 z^{2n}}{2^n}$ is

A) $\frac{1}{\sqrt{2}}$ B) 2 C) $\sqrt{2}$ D) $\frac{1}{2}$

36. Which of the following subsets of the complex plane is simply connected?

A) $\{z : |z| > 1\}$
B) $\{z : |z-1| \leq 2\} \cup \{z : |z+1| \leq 2\}$
C) $\{z : 0 < |z| < 1\}$
D) $\{z : |z-1| > 1\}$

37. Let T be the Mobius transformation defined by $T(z) = \dfrac{z+i}{iz+1}$. Then T maps the real axis $\{z : \text{Im } z = 0\}$ onto

A) the imaginary axis $\{z : \text{Re } z = 0\}$
B) the unit circle $\{z : |z| = 1\}$
C) the line $\{z : \text{Re } z = 1\}$
D) the circle $\{z : |z - i| = 1\}$

38. Let $f(z) = \sin \dfrac{\pi}{z}$, $z \in \mathbb{C}$, $z \neq 0$. Then which of the following statements is incorrect.

A) $f(z)$ has infinite number of zeros in $\mathbb{C}$
B) $z = 0$ is an essential singularity of f
C) $\lim\limits_{|z|\to\infty} f(z) = 0$
D) $f(z)$ is bounded in the annulus $\{z : 0 < |z| < 1\}$

39. The residue at $z = 1$ of the function $\dfrac{1}{(z-1)(z-3)^2}$ is

A) 2 B) 0 C) $\dfrac{1}{4}$ D) 4

40. The coefficient of $\dfrac{1}{z}$ in the Laurent series expansion of $f(z) = \dfrac{1}{z(z-1)}$ in the region $1 < |z| < \infty$ is

A) 1 B) 0 C) -1 D) 2

41. Which of the following permutations is even

A) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \end{pmatrix}$ B) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 4 & 2 & 1 \end{pmatrix}$
C) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 5 & 3 \end{pmatrix}$ D) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 5 & 4 \end{pmatrix}$

42. If $a + bi$ with $a, b \in \mathbb{Z}$ is a unit in the ring $\mathbb{Z}[i]$ of Gaussian integers, then which of the following is true

A) $a = 1$ B) $a = -1$ C) $b = 1$ D) $ab = 0$

43. Which of the following groups is cyclic

A) $\mathbb{Z}_6 \oplus \mathbb{Z}_8$ B) $\mathbb{Z}_3 \oplus \mathbb{Z}_{16}$
C) $\mathbb{Z}_4 \oplus \mathbb{Z}_{12}$ D) $\mathbb{Z}_2 \oplus \mathbb{Z}_{24}$

44. The order of the element $(2, 2)$ in the group $\mathbb{Z}_4 \oplus \mathbb{Z}_6$ is

A) 2 B) 4 C) 6 D) 8

45. For which of the following numbers all groups of that order are abelian

A) 6 B) 8 C) 12 D) 25

46. Which of the following pair of groups are isomorphic

A) $\mathbb{Z}_{24}$ and $\mathbb{Z}_8 \oplus \mathbb{Z}_3$
B) $\mathbb{Z}_{25}$ and $\mathbb{Z}_5 \oplus \mathbb{Z}_5$
C) $\mathbb{Z}_4$ and $\mathbb{Z}_2 \oplus \mathbb{Z}_2$
D) $\mathbb{Z}_{20}$ and $\mathbb{Z}_2 \oplus \mathbb{Z}_{10}$

47. Which of the following maps is a homomorphism on the ring $\mathbb{Z} \times \mathbb{Z}$

A) $\phi(x, y) = (2x, 2y)$
B) $\phi(x, y) = (x + y, 0)$
C) $\phi(x, y) = (2x, 3y)$
D) $\phi(x, y) = (y, x)$

48. Which of the following is a unit in the ring $\mathbb{Z}(\sqrt{2}) = \{a+b\sqrt{2} : a, b \in \mathbb{Z}\}$

A) $3 + 2\sqrt{2}$ B) $2 + 3\sqrt{2}$ C) $2 + \sqrt{2}$ D) $1 + 2\sqrt{2}$

49. Which of the following equations has a solution in $\mathbb{Z}_{18}$

A) $3x = 5$ B) $4x = 3$ C) $5x = 4$ D) $6x = 7$

50. Which of the following polynomials is not irreducible in $\mathbb{Z}_3[x]$

A) $x^2 + 1$
B) $x^2 + x + 2$
C) $x^3 + x^2 + 2$
D) $x^3 + x + 1$

51. Which of the following is an ideal in the ring $F[x]$ of all polynomials over a field F

A) set of all polynomials in $F[x]$ of degree> 1
B) set of all polynomials in $F[x]$ of degree≤ 1
C) set of all polynomials in $F[x]$ without constant term
D) set of all polynomials $f(x) \in F[x]$ such that $f(0) \neq 0$

52. The degree of the field extension $[\mathbb{Q}(\sqrt{2} + \sqrt{3}), \mathbb{Q}]$ is

A) 1 B) 2 C) 3 D) 4

53. Which of the following statement is not true about an algebraically closed field K

A) Every non constant polynomial in $K[x]$ has a zero in K
B) Every polynomial in $K[x]$ of degree n has a factorization into n linear factors in $K[x]$
C) Irreducible polynomials in $K[x]$ have degree≤ 1
D) Every extension of K is an algebraic extension

54. Let $K = \mathbb{Q}(\alpha)$ where α is the real cube root of 2, then the order of the automorphism group Aut $(K, \mathbb{Q})$ is

A) 1 B) 2 C) 4 D) 6

55. Let σ be an automorphism in Aut $(\mathbb{Q}(\sqrt{2}, \sqrt{3}) : \mathbb{Q})$. Then which of the following can not hold

A) $\sigma(\sqrt{2}) = -\sqrt{2}$ B) $\sigma(\sqrt{2}) = \sqrt{3}$
C) $\sigma(\sqrt{2} + \sqrt{3}) = \sqrt{2} - \sqrt{3}$ D) $\sigma(\sqrt{2} + \sqrt{3}) = -\sqrt{2} + \sqrt{3}$

56. In the vector space $\mathbb{R}^3$ over $\mathbb{R}$, W is the subspace given by $W = \{(x_1, x_2, x_3) : x_1 + x_2 + x_3 = 0\}$. Then dim W is

A) 0 B) 1 C) 2 D) 3

57. Which of the following is a linearly independent set in $\mathbb{R}^2$

A) $\{(1, -1), (-2, 2)\}$ B) $\{(1, -1), (3, -1)\}$
C) $\{(1, 2), (2, 4)\}$ D) $\{(3, 1), (-3, -1)\}$

58. Which of the following is an eigen vector of the matrix $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

A) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ B) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ C) $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ D) $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$

59. Which of the following matrix is diagonalizable

A) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ C) $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ D) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

60. Let T from $\mathbb{R}^2$ to $\mathbb{R}^3$ be defined by $T(x, y) = (x + y, x + y, 0)$. Then rank T is

A) 0 B) 1 C) 2 D) 3

61. With usual metric in $\mathbb{R}$ which of the following subspaces of $\mathbb{R}$ is complete

A) the rationals in $\mathbb{R}$
B) the irrationals in $\mathbb{R}$
C) the closed interval $[0, 1]$
D) the open interval $(0, 1)$

62. With usual topology on the spaces concerned which of the following spaces is not connected?

A) $\{z \in \mathbb{C} : |z| < 1\}$
B) $\{x \in \mathbb{R} : |x| < 1\}$
C) $\{z \in \mathbb{C} : |z| > 1\}$
D) $\{x \in \mathbb{R} : |x| > 1\}$

63. Which of the following is not a property of $\mathbb{R}$ (with usual topology)

A) second countability
B) compactness
C) separability
D) local compactness

64. Which among the following topologies on $\mathbb{R}$ is an example of a topology not induced by a pseudo metric?

A) usual topology
B) discrete topology
C) indiscrete topology
D) cofinite topology

65. Which of the following functions $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is not a metric

A) $d(x, y) = |x - y|$
B) $d(x, y) = 2|x - y|$
C) $d(x, y) = \dfrac{|x - y|}{1 + |x - y|}$
D) $d(x, y) = |x - y|^2$

66. Let X be a topological space and let A, B be subsets of X. Then it is not always true that

A) $\bar{\bar{A}} = \bar{A}$
B) $\overline{(A \cup B)} = \bar{A} \cup \bar{B}$
C) $\overline{(A \cap B)} = \bar{A} \cap \bar{B}$
D) $\bar{X} = X$

67. With the usual topology, which of the following subspaces of $\mathbb{R}$ is not homeomorphic to $(0, 1)$?

A) $\{x | x > 0\}$
B) $[0, 1]$
C) $\mathbb{R}$
D) $(-1, 1)$

68. Let X be a metric space. Three of the following properties of X are equivalent to each other, pick the odd one out

A) X is compact
B) X is sequentially compact
C) X has the Bolzano-Weierstrass property
D) X is totally bounded

69. Let $\mathbb{R}$ be the space of real numbers with usual topology. Which of the following subspaces of $\mathbb{R}$ is compact?

A) $(0, 1)$
B) $[0, 1] \cup [2, 3]$
C) $[0, 1)$
D) set of all rationals in $\mathbb{R}$

70. Let (X, τ) be the Sierpinski topology with $X = \{a, b\}$, $\tau = \{\phi, \{a\}, X\}$. Then X is not a

A) compact space
B) connected space
C) T_0 space
D) T_1 space

71. Let X be the normed linear space of square summable real sequences with $\|\ \|_2$ and Y be the subspace generated by the elements $(1, 0, 0, \ldots)$ and $(0, 1, 0, \ldots)$. If $U = \{x \in X : \|x\|_2 < 1\}$ Then

A) $Y + U$ is open in X
B) $Y + U$ is closed in X
C) $Y + U$ is neither open nor closed in X
D) $Y + U$ is not bounded in X

72. Let X be the complex normed linear space of summable sequences of complex numbers with norm $\|\ \|_1$ and $Y = \{x \in X : \|x\|_1 \leq 1\}$ then

A) Y is compact and convex
B) Y is compact but not convex
C) Y is neither compact nor convex
D) Y is convex but not compact

73. Let $X = C_{00}$, the space of all real sequences which have only finitely many nonzero members, and f be the linear functional on X defined by $f(x(1), x(2), \ldots) = x(1) + x(2) + \cdots$ for $x = (x(1), x(2), \ldots) \in X$. Then f is continuous

A) with respect to $\|\ \|_1$ and $\|\ \|_2$ but not with respect to $\|\ \|_\infty$
B) with respect to $\|\ \|_1$ and $\|\ \|_\infty$ but not with respect to $\|\ \|_2$
C) with respect to $\|\ \|_2$ and $\|\ \|_\infty$ but not with respect to $\|\ \|_1$
D) with respect to $\|\ \|_1$, $\|\ \|_2$ and $\|\ \|_\infty$

74. Let $X = C_{00}$ with $|| \;||_\infty$ and $F : X \to l^\infty$ be a bounded linear map. Then there is a bounded linear map $G : C_0 \to l^\infty$ such that

A) G is unique, $G/C_{00} = F$ and $||F|| < ||G||$
B) G is unique, $G/C_{00} = F$ and $||F|| = ||G||$
C) $G/C_{00} = F$ and $||F|| = ||G||$ but G is not necessarily unique
D) G is unique, $R(G) = R(F)$ and $||F|| < ||G||$

75. Let X be a normed linear space and Y be a subspace of X with basis $\{y_1, y_2, \ldots, y_n\}$. Let $x'_1, x'_2, \ldots, x'_n$ be linear functionals with

$$x'_i(y_j) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

If $Z = \{x : x'_j(x) = 0, \text{ for } j = 1, 2, \ldots, n\}$ then which one of the following is not correct?

A) $Y \cap Z = \{0\}$
B) $Y + Z = X$
C) Z is open
D) Z is closed

76. If H is the Hilbert space of square summable sequences of complex numbers and if $x = (x(1), x(2), \ldots) \in H$ has the property that $2 \sum\limits_{i=1, i\neq j}^{\infty} |x(i)|^2 + |x(j) - 1|^2 + |x(j) + 1|^2 = 18$ then $||x||$ is equal to

A) 1
B) 2
C) $2\sqrt{2}$
D) 4

77. Let H be the complex Hilbert space of square summable sequences of complex numbers and $T : H \to H$ be defined $T(x(1), x(2), \ldots) = (0, x(1), x(2), \ldots)$ for $x = (x(1), x(2), \ldots) \in H$. Then which one of the following is not correct?

A) T is bounded
B) $||T|| = 1$
C) T is one-one but not onto
D) T is one-one and onto

78. Let M be a closed subspace of a complex Hilbert space H. Let P and Q be orthogonal projections of H onto M and $M^\perp$ respectively. Then the set of all values of $\alpha,\ \beta$ such that $\alpha P + \beta Q$ is selfadjoint is

A) ϕ
B) $\{1\}$
C) the set of all real numbers
D) set of all complex numbers

79. Let H be the real Hilbert space $L^2([0, 2\pi])$ and f be a linear functional on H defined by $f(x) = \int_0^{2\pi} x \sin 2x dx$. Then $||f||$ is

A) 1 B) π C) 2π D) $\sqrt{\pi}$

80. Let X_1 and X_2 be closed subspaces of a Hilbert space H and let P_1 and P_2 be orthogonal projections on X_1 and X_2 respectively. If $< x, y >= 0$ for all $x \in X_1$, $y \in X_2$ then which one of the following is not correct?

A) $X_1 + X_2$ is a closed subspace of H
B) $P_1 - P_2$ is an orthogonal projection
C) $(P_1 - P_2)^2$ is an orthogonal projection
D) $P_1 + P_2$ is an orthogonal projection
