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Mathematics
Paper-I

1- **Real Analysis :**

Functions of Several variables: Eucledian spaces, continuous functions, derivatives in an open subset of R^n, Chainrule, Partial derivatives, interchange of order of differentiation, Derivatives of higher orders, Taylors's theorem, Inverse function theorem, implicit function theorem.

Lebesgue outer measure, Measurable sets, Measurable functions, Borel and Lobesgue measurability.

Integration of non-negative functions, Integration of series, Riemann and Lebesgue integrals.

Functions of Bounded variation.

Measure and outer measure, Extension of a measure, Uniqueness of extension, Completion of a measure, Measure spaces, Integration with respect to a measure. The Lp-spaces, Convex functions, Jensen's inequality, Holder and Minkowski inequalities, Completeness of Lp, Convergence in measure.

2- **Complex Analysis:**

Cauchy-Goursat theorem, Poisson integral formula, Cauchy's integral formula for derivatives, Cauchy's inequality, Liouville's theorem, Morera's theorem, Taylor's and Laurent's theorems, Maximum modulus principle, Schwarz lemma, Meromorphic functions, Inverse function theorem.

Residues, Cauchy's residue theorem, Evaluation of integrals.

Weierstrass factorization theorem, Gamma function, Riemann zeta function, Mittag-Leffler's theorem, Riemann mapping theorem.

Analytic continuation, Uniqueness of analytic continuation along a curve, Power Series method of analytic continuation, Natural boundary, Schwarz reflection principle. Harnack's inequality and theorem.

Canonical products, Jensen's formula, Hadamard's three circles theorem, Order of an entire function, Exponent of convergence, Borel's theorem, Hadamard's factorization theorem.

3- **Topology :**

Completeness of metric spaces, Cantor's Intersection theorem, Dense sets, Baire's category theorem, Separable spaces, Continuous functions, Uniform continuity, Isometry and homeomorphism, Compactness, Sequential compactness, Totally bounded spaces, Finite intersection property.

Definition and examples of topological spaces, Neighbourhoods, Closed sets, Limit point and derived sets, Closure, interior, exterior and boundary of a set, Dense and nowhere dense sets, Bases and sub-bases, Subspaces and relative topology, Metric topology and equivalent metrics.

Characterization of topology in terms of Kuratowaski closure operator and fundamental system of neighbourhoods.

Continuous maps and homeomorphisms.

First and second countable spaces, Lindelof's theorem, Separable spaces.

Separation axioms, T_o, T_1, T_2, T_3 Tychonoff ($T_{3/2}$) and T_4 spaces Urysohn's lemma, Tietze extension theorem.

Compactness, Continuous functions and compact sets.

Connected spaces, connectedness on the real line, components, Locally connected spaces.

Tychonoff product topology, projection maps, separation axioms and product spaces, connectedness and product spaces, compactness and product spaces, countability and product spaces.

4. Rigid Dynamics :

Moments and product of interia, momental ellipsoid, Equimomental systems, Principal axes.

D'Alembert's principle, General equations of motion of a rigid body, Motion of the centre of inertia and motion relative to the centre of interia.

Motion about a fixed axis, Compound pendulum,

Motion of rigid body in two dimensions under finite and impulsive forces.

Conservation of momentum and energy, Lagrange's equation, Euler's equations of motion, Hamilton's principle, Hamilton's equations of motion.

5.Calculus of Variations :

Variational problems with fixed boundaries – Euler's equation for functionals containing first order derivative and one independent variable, functionals dependent on higher order derivatives, functionals dependent on more than one independent variable, variational problems in parametric form, Invariance of Euler's equation under coordinate transformation.

Variational problems with moving boundaries : Functionals dependent on one and two functions, one sided variations.

Sufficient conditions for an extremum : Jacobian and Legendre conditions, Second variation, Variational principle of least action.

6.Discrete Mathematics:

Formal Logic – Statements, symbolic Representation and Tautologies, Quantifiers, Predicates and Validity, Propositional Logic.

Semigroups and Monoids – Definitions and Examples, Homomorphisms of Semigroups and Monoids, congruence relation and Quotient Semigroup, Subsemigroup and sub-monoids, direct products, basic homomorphism theorem.

Lattices – Lattices as partially ordered sets, their properties, Lattices as Algebraic system, Sub-lattices, direct product and Homomorphisms, complete, complemented and Distributive Lattices.

Boolean Algebras – Boolean Algebras as Lattices, Various Boolean Identities, switching algebra example, Sub-algebras, Direct products and Homomorphisms, Join-irreducible elements, Atoms and Minterms, Boolean forms and their equivalence, Minterm Boolean forms, sum of products, Canonical Forms, Minimization of Boolean Functions, Applications of Boolean Algebra to switching theory, the Karnaugh Map Method.

Graph Theory – Definition of Graphs, Paths, Circuits, Cycles and Subgraphs, Induced Subgraphs, Degree of a vertex, Connectivity, Planar Graphs, trees, Euler's formula for connected planar graphs, complete and complete Bipartite Graphs, Kuratowski's theorem and its use, Spanning trees, matrix representation of graphs, Euler's theorem on the existence of Eulerian Paths and circuits, directed graphs, Indegree and Outdegree of vertex, Weighted undirected Graphs, directed trees, search Trees, Tree Traversals.

7.Mathematical Statistics:

Moments, method of least squares and curve fitting, moments of bivariate distribution, correlation coefficient and regression, partial and multiple correlation for three variables.

Probability : Axiomatic definition of probability, Independent events, Baye's theorem, discrete and continuous random variables.

Probability distribution functions : Binomial, Poisson's and normal probability distributions.

Chi square distribution, t-, f- and z- distributions.

Mathematics
Paper-II

1. Abstract Algebra :

Groups-Normal and subnormal series, composition series, Jordan-Holder theorem, Solvable groups, Nilpotent groups, p-Sylow subgroups, Cauchy's theorem, Conjugacy relation, Class equation, Direct product, Sylow theorems, Structure theorem for finite abelian groups.

Rings- Integral domain, Imbedding theorem, prime and maximal ideals, Quotient rings, Euclidean rings, Polynomial rings, Gaussian rings, unique factorization theorem.

Modules- Modules and submodules, quotient module, Isomorphism theorem, cyclic modules, simple modules, semi-simple modules, Sehuler's Lemma, Free modules.

Field Theory- Extension fields, algebraic and transcendental extensions separable and inseparable extensions, normal extensions, perfect fields finite fields, Primitive elements, Algebraically closed fields, Automorphisms of extensions, Galois extensions, fundamental theorem of Galois theory solution of polynomial equations by radicals.

Canonical Forms- Similarity of linear transformations, Invariant subspaces reduction to triangular forms, Nilpotent transformations, Index of nilpotency, Invariants of a nilpotent transformation, The primary decomposition theorem, Jordan blocks and Jordan forms.

2. Fluid Dynamics :

Kinematics- Lagrangian and Eulerian method, Equation of continuity, Boundary surface, Stream lines, Path lines and streak lines, Velocity potential, Vortex line, Rotational and irrotational motion.

Equation of motion- Euler's and Lagrange's equation of motion, Bernoulli's equations, Equation of impulsive action.

Motion in two dimensions- Sources and Sinks, stream or current function, Complex potential, Doublets, Images Milne-Thomson circle theorem, Theorem of Blasius, Flow and circulation, Kelvin's circulation theorem, Performance of irrotational motion, Kinetic energy of infinite liquid.

Motion of cylinders- General equation of cylinder, Kinetic energy, Motion of circular and elliptic cylinders, Liquid streaming past a fixed circular/elliptic cylinder, Kinetic energy of elliptic cylinder.

Motion of Spheres- Motion of a sphere through liquid at rest of infinity, liquid streaming past a fixed sphere, Equation of motion of a sphere, Stoke's stream function.

3. Functional Analysis :

Normed linear spaces, Banach spaces, Continuous linear transformations, spaces of continuous linear transformations from a linear space to a Branch space, Continuous linear functionals.

Hahn-Banach theorem for real linear spaces, complex linear spaces, and normed linear spaces, Natural imbedding of N in N **, open mapping theorem, closed graph theorem, conjugate of an operator, Banach-Steinhaus theorem, Uniform boundedness theorem and some of its consequences.

Conjugate spaces, Weak of weak* topologies on conjugate space, Simple applications to reflexive separable spaces and to the calculus of variations.

Hilbert spaces, Schwarz's inequality, Orthogonal complement of a subspace, Orthonormal sets, Bessel's inequality, Continuous linear functionals, Riesz representation theorem, Reflexivity of Hilbert spaces, adjoint of an operator, self-adjoint operators, normal and unitary operators, Projections.

Finite dimensional spectral theory: Determinant and the spectrum of an operator, the spectral theorem.

4. **Integral Equations :**

Definition and classification of integral equations.

Fredholm integral equation of the second kind with separable kernel, reduction to a system of algebraic equations. Method of successive appoximations, interative scheme for Fredholm integral equation of second kind, condition of uniform convergence and uniqueness of series solution, resolvent kernel, Eigen values and eigen functions, Classical Fredholm theory.

Conversion of differential equations into integral equations, initial value problems, application of inerative scheme to Volterra integral equation of second kind, method of successive approximations for Volterra integral equations.

Integral transform methods :

Fourier transform, Laplace transform, convolution integral, application to Volterra integral equations with convolution type kernels.

Symmetric kernels:

Complex Hilbert space, Orthonormal system of functions, Fundamental properties of eigen values and eigen functions for symmetric kernels, Hilbert-Schmidt theorem and its immediate consequences, solution of integral equations with symmetric kernels.

5. **Partial Differential Equations :**

Nonlinear partial differential equations of the first order, Cauchy's method of characteristics, Compatible systems of first order equations, Charpit's method, Special types of first order equations, solutions satisfying given conditions, Jacobi's method.

Partial differential equations of the Second order with variable coefficients, Canonical forms, Separation of variables, Nonlinear equations of the second order-Monge's method, Laplace equation, wave equation and diffusion equation.

6. **Analytical Dynamics :**

Generalized coordinates, Holonomic and non-holonomic systems, Scleronomic and rheonomic systems, generalized potential, Lagrange's equations, uniqueness of solution, Energy equation for conservative fields.

Hamilton's canonical equations, Cyclic coordinates, Routh's equations, Poisson Brackets, Poisson Identity, Jacobi-Poisson theorem. Shortest distance, Minimum surface of revolution, Brachistochrone problem, Isoperimetic Problems.

Hamilton's principle, Principle of least action, Poincare Cartan integral invariant, Hamilton-Jacobi equation, Lagrange's brackets. Invariance of Lagrange's brackets and poisson brackets under canonical transformations.

7. **Operations Research :**

Operations research and its scope, Necessity of operations research in industry.

Linear Programming : Simplex method, Big-M Mehod, Duality and sensitivity analysis.

Transportation and Assignment problems.

Game Theory:

Two-Person zero-sum games, Games with mixed stategies, Graphical solution, solution by linear programming.

Sequencing Models:

Assumptions for sequencing problem, Processing n jobs through two machines, Processing n jobs through three machines, Processing two jobs through m machines.

Network Analysis :

Construction of a network diagram. PERT and CPM, Time estimates in PERT, Project Crashing.