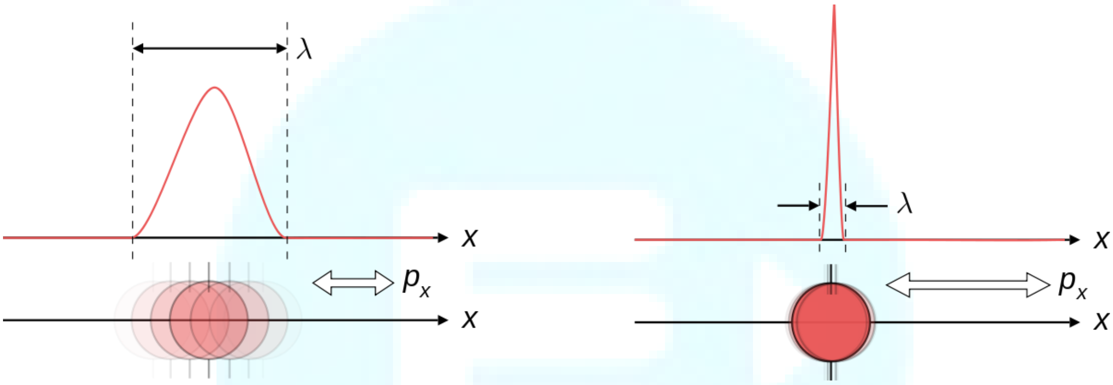
Wave properties of particles
Part 5
Heisenberg's uncertainty Principle

Uncertainty Principle

- We cannot know the future because we cannot know the present.
- To regard a moving particle as a wave implies that there are fundamental limits to the accuracy with which we can measure such " Particle " properties as position and momentum.



- It is impossible to know both the exact position and exact momentum of an object at the same time.

$$\Delta x \ \Delta p \geq \frac{\hbar}{2}$$

$$\hbar = \frac{h}{2\pi} = 1.054 \times 10^{-34} \text{J.s}$$

Uncertainties in energy and momentum

$$\Delta E \ \Delta t \geq \frac{\hbar}{2}$$

This law help us to know;

- The non - existence of free electrons inside Nucleus.
- Ground state energy of systems like Hydrogen atom, Harmonic oscillator etc.
- Width of spectral lines.

Problems

1. Find the uncertainty in the measurement of position of :

a) A neutron moving with a velocity 5×10^6 m/s

b) A 50kg man moving with a velocity 2 ms^{-1}

Solution :

a) Given V = 5×10^6 m/s

$\Delta x = ?$

$$\Delta x \, \Delta p \geq \frac{\hbar}{2} \quad \text{------} \quad (1)$$

$$P = mv$$

$$\Delta p = m \, \Delta V$$

$$\Delta x \, m \, \Delta V = \frac{\hbar}{2}$$

$$\Delta x = \frac{\hbar}{2mV}$$

$$= \frac{1.054\times 10^{-34}}{2\times 1.6\times 10^{-27}\times 5\times 10^{6}}$$

$$\Delta x = 0.65\times 10^{-16} = 6.5\times 10^{-15}\text{m} = 6.5 \text{ fm}$$

b) V = 2 ms^{-1}

$$\Delta x = \frac{\hbar}{2mV}$$

$$= \frac{1.054\times 10^{-34}}{2\times 50\times 2}$$

$$= 0.5\times 10^{-36}\text{m}$$

2. A ball of mass 145g is moving at a speed 42.5 m/s. Assume that its speed can be measured to a precision of 1%. With what precision we can measure its position?

Solution :

Given m = 145 g = 145×10^{-3} Kg

V = 42.5 m/s

$$\Delta V = \frac{1}{100}\times 42.5 = 0.425$$

$\Delta x = ?$

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x \, m \, \Delta V = \frac{\hbar}{2}$$

$$\Delta x = \frac{\hbar}{2m\Delta V}$$

$$\Delta x = \frac{1.05\times10^{-36}}{2\times145\times10^{-3}\times0.625}$$

$$\Delta x = 8.5\times10^{-36} m$$

3. A measurement establishes the position of a proton with accuracy of $\pm1\times10^{-11}$ m. Find the uncertainty in the proton's position 1.00 s later. Assume v << c

Solution :

Given $\Delta x_{t=0} = 10^{-11} m$

$$\Delta x_{t=1} = ?$$

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

$$\Delta p = \frac{\hbar}{2\Delta x}$$

$$m\Delta V = = \frac{\hbar}{2\Delta x}$$

$$\Delta v = = \frac{\hbar}{2m\Delta x}$$

$$\Delta x_{t=1} = \Delta V \, t$$

$$= \frac{\hbar}{2m\Delta x}$$

$$\Delta x_{t=1} = \frac{1.05\times10^{-36}}{2\times1.6\times10^{-27}\times10^{-11}}$$

$$\Delta x_{t=1} = 3.15\times10^{3} m$$

4. A hydrogen atom is 5.3×10^{-11}m in radius. Use the uncertainty principle to estimate the minimum energy an electron can have in this atom.

Solution :

Given Δ x = 5.3×10^{-11}

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

From the above relation we can find that $\Delta x \propto \frac{1}{\Delta p}$

$$\Delta x_{max} \times \Delta p_{min} = \frac{\hbar}{2}$$

$$\Delta x_{min} = 5.3\times10^{-11}$$

$$\Delta p_{min} = \frac{\hbar}{2\Delta x}$$

$$= \frac{1.05\times10^{-36}}{2\times5.3\times10^{-11}}$$

$$\Delta p_{min} = 9.9\times10^{-25} \text{kg m} s^{-1}$$

$$K_{min} = \frac{{P_{min}}^2}{2m} = \frac{(9.9\times10^{-25})^2}{2\times9.1\times10^{-31}}$$

$$= \frac{5.6\times10^{-19} J}{1.6\times10^{-19}}$$

$$K_{min} = 3.4 \text{ eV}$$