

## The Nucleus

### Introduction

Most of the physical and chemical properties of matter which we are familiar with are a result of the number and configuration of atomic electrons.

That's why we have spent most of the course to date concentrating topics related to atomic electrons.

Nevertheless, atomic nuclei are vitally important for a number of reasons, including:

- The number of electrons an atom can have depends on how many protons the nuclei has. Thus, the nucleus plays a large, if indirect, role in determining atomic structure.
- Most of the energies liberated in everyday processes involve nuclear reactions.

The first couple of sections of this chapter describe several problems the nucleus presents us with.

For example, consider this problem.

Take two protons at their approximate separation in the nucleus. Calculate the repulsive Coulomb energy between them.

$$V = \frac{e^2}{4\pi\epsilon_0 r} = \frac{(1.6 \times 10^{-19})^2}{(9 \times 10^9)(10^{-15})} \approx 1.4 \times 10^{-6} \text{ eV} = 1.4 \text{ MeV}.$$

That's a large repulsive potential.

Now imagine the repulsive Coulomb energy for several dozen protons packed tightly into a nucleus.

We have a major problem here. How can a nucleus stick together?

A logical guess is that the nucleus contains electrons, which reduce the Coulomb repulsion.

## 11.1 Nuclear Composition

It is quite logical to assume that nuclei contain electrons. For example, a good guess might be that half of an atom's electrons are contained within the nucleus, and reduce the electrostatic repulsive forces between protons.

Besides the Coulomb repulsion mentioned above, facts which suggest the nucleus might contain electrons are nuclide masses, which are nearly multiples of the hydrogen mass (which contains an electron).

In addition, some nuclei undergo beta decay, in which an electron is spontaneously emitted from the nucleus.

But other experiments demonstrate that the nucleus cannot contain electrons. Here are some reasons why.

Reason 1 -- nuclear size.

The Heisenberg uncertainty principle places a lower bound on the energies of particles confined to a nucleus.

Take a typical nucleus of radius  $5 \times 10^{-15}$  m. Suppose an electron exists inside the nucleus. In the example on page 112, we estimated the minimum momentum such an electron must have. The estimated momentum corresponds to a kinetic energy of *at least* 20 MeV.

Experimentally, electrons emitted during nuclear decay are found to have only 2 or 3 MeV of energy. This isn't nearly enough to correspond to an electron escaping from a nucleus.

On the other hand, protons, with their much larger masses, would only need to have a few tenths of an MeV of energy to be confined to a nucleus. This is within the realm of possibility.

Reason 2 -- nuclear spin.

Electrons and protons both have spins of  $1/2$ . A deuteron (an isotope of hydrogen) is known to have a mass roughly equal to two protons.

If the deuteron nucleus contains two protons and one electron (whose mass is small enough to not worry about here), then the deuterium should have a nuclear spin of  $\pm 1/2$  or  $\pm 3/2$  ( $\pm 1/2 \pm 1/2 \pm 1/2$ ).

The deuterium nuclear spin is measured to be 1. Its nucleus cannot contain an electron.

Reason 3 -- nuclear magnetic moments.

Electrons have magnetic moments about 6 times larger than protons.

If nuclei contain electrons, their magnetic moments should be comparable to electron magnetic moments.

Observed nuclear magnetic moments are comparable to proton magnetic moments.  
Nuclei cannot contain electrons.

Reason 4 -- electron-nuclear interactions.

The energies binding nuclear particles together are observed to be very large, on the order of 8 MeV per particle.

Remember that atomic electronic binding energies are of the order eV to a few keV.

Why, then, can some atomic electrons "escape" from being bound inside the nucleus? In other words, if you allow any electrons to be bound inside the nucleus, you really must require all of them to.

Of course, it is obvious to us that nuclei don't contain electrons.

But that's mainly because we've been taught that way for so long.

If we were starting from scratch 60 years ago, we would probably try to "put" electrons inside nuclei.

This section also contains a number of facts pertaining to nuclei.

Atomic masses.

The most abundant type of carbon atom is defined to have a mass of exactly 12 u, where u is one atomic mass unit:

$$1 \text{ u} = 1.6604 \times 10^{-27} \text{ kg} = 931.48 \text{ MeV}.$$

Atomic masses always refer to neutral atoms. In other words, atomic masses *include* the masses of all of the electrons in the neutral atom.

Isotopes.

Not all atoms of an element have the same mass. *Isotopes* are atoms of the same element having different masses.

Nuclides.

A *nuclide* is simply any particular nuclear species. Hydrogen and deuterium are isotopes. They are also nuclides. Carbon-12 is a nuclide, but it is not an isotope of hydrogen.

Since nuclei contain neutrons, let's consider the neutron for a while.

Neutrons were observed in 1930 and "hypothesized" in 1932.

Here are some of the details about the observations. Figure 11.1 illustrates these observations, but not particularly well.

Beryllium bombarded with alpha particles emits radiation which easily penetrates matter, and therefore must not be charged particles.

It is logical to guess that the radiation is gamma rays (photons).

The radiation is able to knock protons out of paraffin (wax, which contains lots of hydrogen; i.e., protons).

The protons come out with energies up to 5.7 MeV (big energies). The gamma ray energy needed to produce such energetic protons is about 55 MeV. Gamma rays of this much energy were not observed experimentally. About 1/5th of the 5.7 MeV energy is the most that can be produced in the reaction which creates the unknown radiation.

Chadwick in 1932 proposed that the unknown radiation could be neutral particles having about the mass of protons.

Charge neutrality is necessary for the radiation to easily penetrate matter.

Because a collision between particles of equal mass can transfer all of the kinetic energy from the projectile to the target, the neutrons needed to have only 5.7 MeV of energy, which was a much more reasonable value.

Some properties of neutrons:

- mass=1.00867u, just a little more than the proton
- charge=0
- spin=1/2

- neutrons are unstable outside of nuclei, and decay into a proton, an electron, and an antineutrino; the neutron lifetime is about 15 minutes outside of the nucleus
- neutrons produce attractive forces which help hold nuclei together.

Now we can fully describe the nucleus.

The number of protons in a nucleus (and electrons in the atom, if the atom is not ionized) is represented by  $Z$ .

$N$  is the number of neutrons in the nucleus.

The atomic mass number  $A$  is given by  $A=Z+N$ .

Neutrons and protons are called *nucleons*, so  $A$  is the number of nucleons in a nucleus.

We identify nuclides by writing  ${}^A_ZX$ . For example, the most abundant isotope of iron has 26 protons and electrons, and a mass number of 56, so we write  ${}_{26}^{56}\text{Fe}$ .

Because most physical and chemical properties are determined by the number and arrangement of atomic electrons, isotopes of an element are very similar.

As Beiser states, all isotopes of chlorine make good bleach and are poisonous.

Some properties, such as density and freezing points, are different for different isotopes of the same element, but the differences are usually so slight that isotopes are difficult to separate.

## 11.2 Some Nuclear Properties

Scattering is the technique by which nuclear sizes are usually measured.

Highly energetic electrons can interact with nuclear charges and give us information about nuclear charge distribution.

Neutrons interact with the nuclear potential, and provide information about the distribution of nuclear matter.

Experimentally, using neutrons of energy 20 MeV or more, or electrons of 1 GeV ( $10^9$  eV) or greater, it is found that the volume of a nucleus is proportional to the number of nucleons (neutrons and protons) it contains.

Since the mass number  $A$  is proportional to volume and volume is proportional to the  $R^3$ , where  $R$  is the nuclear radius, it follows that  $R$  is proportional to  $A^{1/3}$ . We usually write

$$R = R_0 A^{1/3},$$

where  $R_0$  is a constant approximately equal to  $1.2 \times 10^{-15}$  m.

The nucleus does not have a sharp boundary, so the "constant"  $R_0$  is only approximate; also, nuclear matter and nuclear charge do not seem to be identically distributed.

The unit of length  $10^{-15}$  m is called a femtometer, abbreviated fm, and also often called a fermi, so  $R = 1.2A^{1/3}$  in units of fm.

Example: the radius of the  ${}^{107}_{47}\text{Ag}$  nucleus is  $R = 1.2(107)^{1/3} \approx 5.7$  fm.

Nuclear shapes.

If a nucleus is not spherically symmetric, it will produce an electric field that will perturb atomic electronic energy levels.

Such an effect is, in fact, observed, but it is small -- "hyperfine." The departures from spherical symmetry are small.

*Skip the subsection on nuclear spin and magnetic moment.*

### 11.3 Stable Nuclei

We can begin to understand why certain nuclei are stable and others unstable by realizing that nucleons have spins of  $\frac{1}{2}$  and obey the Pauli exclusion principle.

Nucleons, like electrons and their electronic energy levels, occupy discrete nuclear energy levels.

Minimum energy configurations (i.e., nucleons in the lowest possible energy levels) give the most stable nuclei.

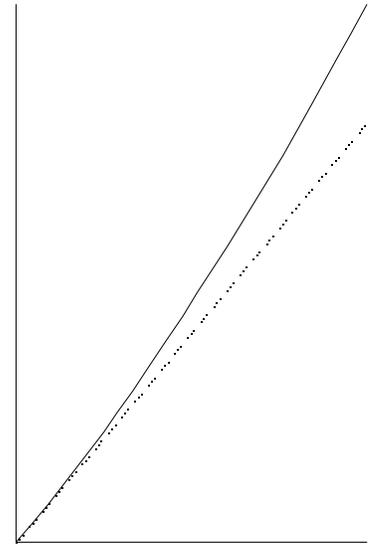
A plot of N versus Z for the stable nuclides looks like this:

Sorry, the graph didn't translate well between word processing programs. The vertical axis should be labeled "Neutron Number, N."

Neutrons produce attractive forces within nuclei, and help hold the protons together.

For small numbers of protons, about an equal number of neutrons is enough to provide stability, hence  $N \approx Z$  for small Z.

As the number of protons gets larger, an excess of neutrons is needed to overcome the proton-proton repulsion.



Proton Number, Z

The stability of nuclei follows a definite pattern.

The majority of stable nuclei have both even Z and even N ("even-even" nuclides).

Most of the rest have either even Z and odd N ("even-odd") or odd Z and even N ("oddeven").

Very few stable nuclei have both Z and N odd.

The reasons for this pattern are the Pauli exclusion principle and the existence of nuclear energy levels.

Each nuclear energy level can contain two nucleons of opposite spin.

The neutrons and protons occupy separate sets of energy levels.

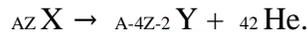
When both Z and N are even, the energy levels are filled. The nucleus doesn't "want" to gain or lose nucleons by participating in nuclear reactions. The nucleus is stable.

When both Z and N are odd, the nucleus is much more likely to "want" to participate in nuclear reactions or nuclear decay, because it has unfilled nuclear energy levels.

Example:  $^{12}_6\text{C}$  has all of its neutrons and protons in filled energy levels. It is very stable.  $^{12}_5\text{B}$  has one more neutron, which is in a higher energy level, and is therefore unstable and decays by beta decay (which we will talk about in a minute) into  $^{12}_6\text{C}$ .

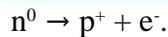
Attractive nuclear forces are limited in range and primarily operate between nearest neighbors ("saturation"), so there is a nuclear size beyond which neutrons are unable to overcome the proton-proton repulsion.

The heaviest stable nuclide is  $^{209}_{83}\text{Bi}$ . Heavier ones decay into lighter nuclides through alpha decay (the emission of a  $^4_2\text{He}$  nucleus).

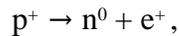


X is called the *parent* nucleus and Y is called the *daughter* nucleus.

It may be that a nucleus produced by alpha decay has too many neutrons to be stable. In this case, it may decay via beta decay:



If the nucleus has too few neutrons, it may decay via positron emission



or by electron capture



Note that Z decreases by 1 as a result of positron emission or electron capture, but *increases* by 1 as a result of beta decay.

## 11.4 Binding Energy

The binding energy that holds nuclei together shows up as "missing" mass.

For example, deuterium is an isotope of hydrogen which contains a neutron, a proton, and an orbiting electron.

If we add the mass of a neutron to the mass of an atom of hydrogen, we get  $1.0087 \text{ u} + 1.0078$

$\text{u} = 2.0165 \text{ u}$ .

The actual deuterium mass is  $2.0141 \text{ u}$ , or  $0.0024 \text{ u}$  less.

Since 1 u of mass has an energy equivalent of 931 MeV, the missing mass is equal to  $931 \times 0.0024 \text{ MeV} = 2.2 \text{ MeV}$ .

The fact that this mass deficit is the binding energy is demonstrated by experiments which show that it takes 2.2 MeV of energy to split a deuterium into a neutron and a proton.

Nuclear binding energies range from 2.2 MeV for deuterium to 1640 MeV for bismuth-209 (also written  $^{209}_{83}\text{Bi}$ ).

These binding energies are enormous; millions of times greater than even the energies given off in highly energetic chemical reactions.

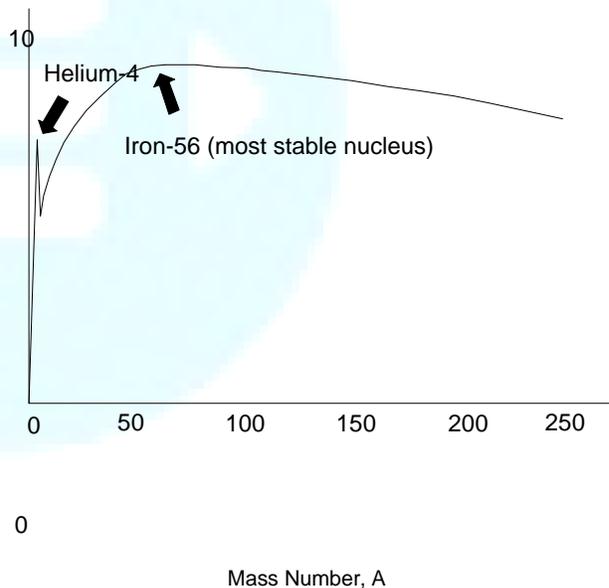
We usually talk in terms of binding energy per nucleon, which is  $2.2/2 = 1.1 \text{ MeV}$  per nucleon for deuterium, or  $1640/209 = 7.8 \text{ MeV}$  per nucleon for bismuth-209.

The figure below shows a plot of binding energy per nucleon as a function of nucleon number. Again, the graph didn't make the transition well between word processors, and the vertical axis should be labeled "Binding Energy per Nucleon, MeV."

This figure can be used to demonstrate how fission and fusion are feasible.

There is one potentially confusing aspect to this figure. The energy is **binding energy**, so it is actually a **negative energy**.

Elements with higher binding energies per nucleon are more stable.



Here's how the figure shows fission is possible:

Take an element with a large A. If we could split it into two smaller nuclei, with A's closer to iron, the two nuclei would have **more** binding energy per nucleon.

But remember, binding energies are negative. If the resulting nuclei have more negative energy than the starting element, some **positive** energy must have been released in splitting the starting element.

The positive energy is the energy released in the reaction.

Similarly, the figure shows how fusion is possible.

If we begin with two nuclei significantly lighter than iron-56 (with the exception of helium-4), and somehow make them fuse, the resulting nucleus will have more binding energy per nucleon.

Again, the more negative binding energy of the product nucleus means that positive energy had to be released to maintain energy balance.

Here's a fission example:

Split uranium-235 into two lighter nuclei; the difference in binding energy between uranium-238 and the lighter nuclei is about 0.8 MeV per nucleon.

Multiply this by the total number of nucleons involved, 235, and you get 188 MeV released in the fission. This is a huge amount of energy.

Here's a fusion example:

Two deuterium-2 nuclei can be fused to form helium-4.

There is a huge binding energy per nucleon difference between deuterium and helium.

The end result is a release of about 5.5 MeV per nucleon, or 23 MeV total, again, a huge energy difference, especially considering the very small number of nucleons involved.

There's another interesting thing about figure 11.9, which I'll just suggest.

It turns out that to make nuclei heavier than iron, you need enormous energies to force particles to fuse. That's because they have *less* binding energy per nucleon than nuclei near iron.

Such energies are available only in supernova.

For example, gold can only be created in a supernova.

## 11.5 Liquid-Drop Model--*skip this section*

This is a relatively long section which presents a semi-empirical formula for binding energy. We won't cover it this semester for quiz or exam purposes, but I will briefly discuss it, and you can read the book or my notes.

"Semi-empirical" means there are fudge factors in it which help us match up theory with experiment.

I won't do this section in great detail, but you ought to read it.

The attractive forces between nucleons are very strong but very short-ranged. In fact, nucleons essentially interact only with their nearest neighbors.

Nuclei do not act as if they are solids, because nucleons move around too much to be like solids.

A picture of the nucleus as a drop of liquid, with nucleons free to move around inside it, works very well.

Using the model of the nucleus as a liquid drop, we attempt to calculate nuclear binding energies.

There are five contributions to the binding energy:

- The volume energy is proportional to the number of nucleons.
- The surface energy is the reduction in the binding energy due to nuclear surface effects.
- The Coulomb energy is the repulsion between protons.
- The asymmetry energy is a reduction in binding energy which occurs when there are more neutrons than protons, and vice versa.
- The pairing energy is somewhat of a "fudge factor" which accounts for stability of even-even and even-odd nuclei, and the reduction of stability for odd-odd nuclei.

We now consider these five contributions in a little more detail.

The volume energy.

We suppose that each pair of nucleons shares a bond energy  $U$ .

Close-packed nucleons will have 12 nearest neighbors, each sharing the bond energy  $U$ .

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Each interior nucleon has a total binding energy of  $(12)(2)U$ , with the factor of 2 entering because each  $U$  is shared between two nucleons.

The volume energy is just  $E_V = 6AU = a_1 A$ .

The surface energy.

Some of the nucleons are on the surface of the nucleus, and don't have 12 near neighbors.

The number of surface nucleons is proportional to the surface area.

The surface area is proportional to  $R^2$ , but  $R$  is proportional to  $A^{1/3}$ , so the surface energy is proportional to  $A^{2/3}$ .

Thus,  $E_S = -a_2 A^{2/3}$ . The - sign is because this is a reduction in the binding energy.

The Coulomb energy.

Protons repel each other. The Coulomb repulsion is proportional to  $1/R$ , or  $1/A^{1/3}$ .

In a nucleus with  $Z$  protons, there are a total of  $Z(Z-1)/2$  pairs of protons.

(If you don't believe that, try it with small numbers, like  $Z=2, 3, 4$ .)

The Coulomb energy is thus  $E_C = -a_3(Z)(Z-1)/A^{1/3}$ . Again, it is a reduction in the binding energy, so it is negative.

The other two terms are a little more obscure, and deviate from the liquid drop model. In fact, they take into consideration the fact that there are discrete nuclear energy levels.

The asymmetry energy.

When there is an excess of neutrons (or protons), the "extra" nucleons have to go into higher energy levels than the "original" nucleons.

This results in a decrease in the binding energy.

Beiser shows that this can be written in the form  $E_a = -a_4(A-2Z)^2/A$ .

The pairing energy.

Even-even nuclei are the most stable. Even-odd and odd-even are stable, but less so. Odd-odd nuclei are the least stable.

The pairing energy accounts for this relative increase/decrease in the nuclear binding energy.

The pairing energy is  $E_p = (\pm, 0)a_5/A^{3/4}$ .

The nuclear binding energy is the sum of all of these terms:

$$E_b = E_V + E_S + E_C + E_a + E_p, \text{ or}$$

$$E_b = a_1A - a_2A^{2/3} - a_3(Z)(Z-1)/A^{1/3} - a_4(A-2Z)^2/A + (\pm, 0)a_5/A^{3/4}.$$

### 11.6 Shell Model--skip this section

This is an *alternative* to the liquid drop model.

Both the liquid drop and shell models explain some facts about the nucleus.

In the shell model:

Just as atoms having filled electron shells are more stable, so are nuclei, except that the shells are different.

The most stable are:

atomic electrons	2	10	18	36	54	86	
neutrons/protons	2	8	20	28	50	82	126

The numbers 2, 8, etc., for which nuclei are most stable, are called "magic numbers."

I'll skip the additional details of the shell model, except to say that it predicts, among other things, stability of even-even nuclei, and nuclear angular momentum.

The "Collective Model" by A. Bohr and B. Mottelson combines liquid drop and shell models into a single theory which is the current state of the art.

### 11.7 Meson Theory of Nuclear Forces

Nucleons exchange mesons to produce an attractive force.



See Figure 11.18 for an idea how throwing/taking a basketball can produce repulsive/attractive forces.

A pion (pi meson) is a moderately massive (about 250 times the mass of the electron, depending on the exact type of pi meson).

It is a very short lived particle which is the primary contributor to attractive nuclear forces.

