Particle properties of waves
Part-2
Problems on Blackbody Radiation

1. The wavelength of radiation emitted by a body depends upon

(a) The nature of its surface

(b) The area of its surface

(c) The temperature of its surface

(d) All of the above factors

Solution : (c) The temperature of its surface

According to Wien's displacement law $\lambda_{max} \propto \frac{1}{T}$

So the wavelength of radiation emitted by a body only depends upon the temperature of its surface.

2. If a blackbody is heated at high temperature, it seems to be

(a) Blue

(b) white

(c) Red

(d) Black

Solution : (b) White

When heated, depending upon the temperature of the cavity, the wavelength of radiation emitted by the blackbody changes. At first the body seems to be red then to yellow and at high temperatures the blackbody appears to be white.

3.If a black wire of platinum is heated, then its colour first appear red, then yellow and finally white. It can be understood on the basis of

(a) Wien's displacement law

(b) Prevost theory of heat exchange

(c) Newton's law of cooling

(d) None of the above

Solution : (a) Wien's displacement law

Since Wien's displacement law defines the relation between wavelength and temperature.

4.Calculate the Sun's temperature assuming that it can be modified as a perfect blackbody. Also calculate the wavelength of maximum emission. Average solar power = 3.85×10^{26}W, radius of sun = 6.96×10^{8}m

Given P = 3.85×10^{26}W

r = 6.96×10^{8}m

T = ?

According to Stefen's Boltzmann law

$$\frac{P}{A} = a\,\sigma\,T^4 \rightarrow (1)$$

For a perfect blackbody emissivity a = 1

$\sigma = 5.67\times10^{-8} w/m^2K^4$

From (1) $T^4 = \frac{P}{A\sigma}$

$$T = \sqrt[4]{\frac{P}{4\pi r^2\sigma}}$$

$$= \sqrt[4]{(\frac{3.85\times10^{26}}{4\pi\times5.67\times10^{-8}\times(6.96\times10^{8})^2}}$$

T $\approx$ 5559 k

Wavelength corresponding to maximum emission

λ_{max}T= Constant= 2.898×10^{-3}

$$\lambda_{max} = \frac{2.898\times10^{-3}}{5779\times10^{3}} = 501 \text{ nm}$$

5.Estimate the surface temperature of a star if the radiation it emits has a maximum intensity at a wavelength of 446 nm. What is the intensity radiated by the star ?

Here λ_{max} = 446 nm

λ_{max} T = 2.898×10^{-3}

$$T = \frac{2.898\times10^{-3}}{446\times10^{-9}}$$

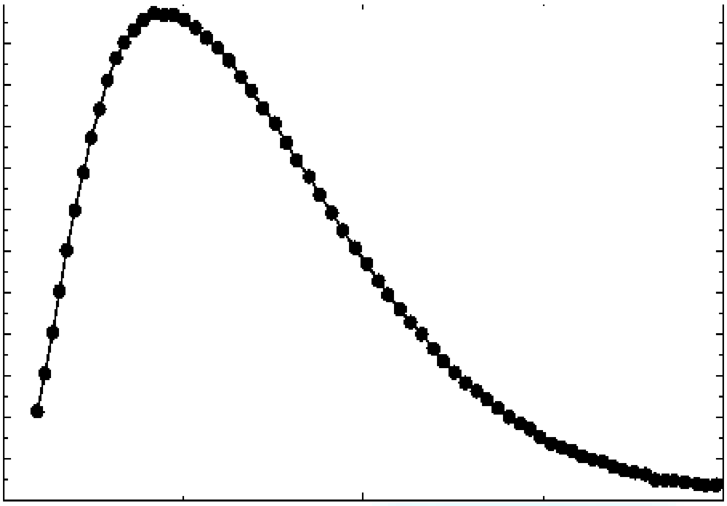
T = 6500 k

Intensity, I= $a\,\sigma\,T^4$ (a=1)

I = $5.67\times10^{-8}\times6500$

= $101.2 \times 10^{6} \frac{w}{m^2}$

6. Calculate the frequency corresponding to the highest energy density from the curve given:



E T= 1000K

Frequency

By Wien's displacement law

λ_{max} T = 2.898×10^{-3} → (A)

To find frequency we can use the relation between frequency and wavelength

C = ν λ

$\lambda = \frac{C}{\nu}$

Substituting the value of λ in equation (A)

$\frac{C}{\nu}$ T = 2.898×10^{-3}

T= 1000 k

$$\nu = \frac{CT}{2.898 \times 10^{-3}} = \frac{3 \times 10^{8}}{2.898 \times 10^{-3}} \times 1000$$

$\nu \approx 1.03 \times 10^{14}$ Hz

7.A maximum wavelength of λ= 2µm is emitted at 1600K. If the temperature is increased to 2000K, then the wavelength of the spectrum is —

We know, at a particular temperature λ_{max} T = constant

λ_1 = 2µm

T_1 = 1600K

T_2 = 2000K

λ_2 = ?

$\lambda_1 T_1$= Constant — (1)

$\lambda_2 T_2$ = Constant — (2)

From (1) and (2)

$\lambda_1 T_1 = \lambda_2 T_2$

$$\lambda_2 = \frac{2\mu m \times 1600}{2000} = 1.6\ \mu m$$

8.The rate of radiation of a black body at 0 °c is EJ/sec. The rate of radiation of this black body at 273°C will be .

(a) 16E

(b) 8E

(c) 4E

(d) E

Solution: (a) 16E

By Stefan- Boltzmann law

$$E \propto T^4$$

T_1= 0°C=273K

T_2 = 273°C= 546K

E_1= E

E_2 = ?

$$\frac{E_1}{E_2} = \frac{{T_1}^4}{{T_2}^4}$$

$$\frac{E_2}{E_1} = \frac{546^4}{273^4}$$

E_2= 16E