ENTRI

Pauli's Exclusion Principle & Identical Particles

No two electrons in an atom can exist in the same quantum state. Each electron in an atom must have a different set of quantum numbers n , l, m_l , m_s. Pauli came into the conclusion from the study of atomic spectra, hence the principle is empirical.

The principle governs the electronic configuration of atom having more than one electron.

If in an element the spin of two electrons are in the same direction so that the total spin is 1 then no transitions are observed in the element (except hydrogen) from ground state or vice versa.

If the spins are in opposite direction so that the total spin is 0 the transitions are observed from the ground state and vice versa.

In the missing state the quantum numbers of both electrons would be n = 1 , l = 0 , m_l = 0, m_s = 1/2.

In the state that exist one of the electrons has m_s= 1/2 and other m_s= -1/2 .

Thus every missing atomic state involves two or more atomic electrons with identical quantum numbers.

Consider that one of the particle is in quantum state **a** and the other is in state **b**. Since the particle are identical the probability $|\Psi|^2$ of the system remains unaffected if positions are exchanged, with one in state a replacing one in state **b**, and vice versa.

$$|\Psi|^2(1,2) = |\Psi|^2(2,1)$$

Let the wave function $|\Psi|^2(2,1)$, represents the exchanged particles, it may be expressed as

$$|\Psi|(2,1) = |\Psi|(1,2) \quad \text{or} \quad |\Psi|(2,1) = -|\Psi|(1,2)$$

Wave functions unchanged by exchange of particles are said to be symmetric, and those reversing sign upon exchange are said to be antisymmetric.

If a particle 1 is in state a and particle 2 is in state b, the wave function of the system may be expressed as

$$\Psi_I = \Psi_a(1)\Psi_b(2)$$

If a particle 2 is in state a and particle 1 is in state b the wave function is

$$\Psi_{II} = \Psi_a(2)\Psi_b(1)$$

The two particles are identical.

Whether Ψ_I describes the system or Ψ_{II}, we cannot know.

The probability that Ψ_I describes the system at any moment id the same as the probability that Ψ_{II} does it.

It may be assumed that the system spends half the time in configuration having wave function Ψ_I and the other half in the configuration with wave function Ψ_{II}.

Thus, a linear combination of Ψ_I and Ψ_{II} will describe the system appropriately. There are two such combinations possible.

The symmetric one

$$\Psi_s = \frac{1}{\sqrt{2}} [\ \Psi_a(1)\Psi_b(2) + \Psi_a(2)\Psi_b(1)\ ]$$

The antisymmetric one

$$\Psi_A = \frac{1}{\sqrt{2}} [\ \Psi_a(1)\Psi_b(2) - \Psi_a(2)\,\Psi_b(1)\ ]$$

$\sqrt{2}$ factor is required to normalize Ψ_s and Ψ_A .

Two cases

Behaviour of particles in a system whose wave functions are symmetric: both the particles 1 and 2 can simultaneously exist in the same state, with a = b.

Behaviour of particles in a system whose wave functions are antisymmetric, with a= b $\Psi_A = 0$; the two particles cannot be in the same quantum state.

Compare above two statements with Pauli's exclusion principle that no two electrons in an atom can be in the same quantum state.

Conclusion

Systems of electrons described by wave functions that reverse sign upon the exchange of any pair of them obey Pauli,s Exclusion principle.

The results of various experiments show that all particles which have spin of 1/2 have wave functions that are antisymmetric to exchange of any pair of them.

Such particle like proton, neutrons, and electrons obey the exchange principle when they are in the same system; I.e when they move in a common force field each member of the system must be in a different quantum state.

Particles of spin 1/2 are often referred as Fermi particles or fermions.

Particles whose spin are 0 or an integer have wave functions that are symmetric on exchange of any pair of them. These particles do not obey the exchange principle. Particles of 0 or integral spins are often referred to as Bose particles or bossons.