# Postulates of Quantum mechanics
# Part 3
# Operators

## Postualte 2 : Operators

- To every observable physical quantity there corresponds a Hermitian operator or Metrix. The operators in quantum mechanics are derived from the Poisson bracket of the corresponding pair of classical variables according to the rule.

$$[\boldsymbol{Q}, \boldsymbol{R}] = i\hbar \{q,r\}$$

Where Q and R are the operators selected for the dynamical variables q and r and { q,r } is the Poisson bracket of q and r.

| Observable K | Classical mechanics | Quantum mechanics |
|---|---|---|
| 1. Momentum Components | $P_x , P_y , P_z$ | $\widehat{P_x} = i\hbar\frac{\partial}{\partial x}$, $\widehat{P_y} = i\hbar\frac{\partial}{\partial y}$, $\widehat{P_z} = i\hbar\frac{\partial}{\partial z}$ |
| 2. Momentum | $\widehat{P}$ | $\widehat{p} = -i\hbar\Delta$ |
| 3. Energy I | E | $\widehat{E} = i\hbar\frac{\partial}{\partial t}$ |

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## Kinetic energy operator

We have KE = $\frac{P^2}{2m}$

$$\widehat{KE} = \frac{\widehat{P}^2}{2m}$$

Also Momentum $\vec{P} = P_x\mathbf{i} + P_y\mathbf{j} + P_z\mathbf{k}$

$$P^2 = P_x{}^2 + P_y{}^2 + P_z{}^2$$

$$P_x{}^2 = -\hbar^2 \frac{\partial^2}{\partial x^2}$$

$$P_y{}^2 = -\hbar^2 \frac{\partial^2}{\partial y^2}$$

ENTRI

$$P_z^{2} = -\hbar^2 \frac{\partial^2}{\partial z^2}$$

$$\widehat{KE} = \frac{P_x^{2} + P_y^{2} + P_z^{2}}{2m}$$

$$\widehat{KE} = \frac{-\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right] = \frac{-\hbar^2}{2m}\Delta^2$$

## Hamiltonian operator

$$\widehat{H} = \frac{\widehat{P^2}}{2m} + V$$

$$\widehat{H} = \frac{-\hbar^2}{2m}\Delta^2 + V$$

## Angular Momentum

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\hat{L} = \hat{r} \times \hat{p}$$

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

$$\vec{p} = \widehat{p_x}\,\hat{\imath} + \widehat{p_y}\,\hat{\jmath} + \widehat{p_z}\,\hat{k}$$

$$\vec{L} = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{bmatrix}$$

$$\vec{L} = \hat{\imath}\,(yp_z - zp_y) - \hat{\jmath}\,(xp_z - zp_x) + \hat{k}\,(xp_y - yp_x)$$

$$\vec{L} = \hat{\imath}\,(yp_z - zp_y) + \hat{\jmath}\,(zp_x - xp_z) + \hat{k}\,(xp_y - yp_x)$$

Since $\hat{L} = L_x\,\hat{\imath} + L_y\,\hat{\jmath} + L_z\hat{k}$ we can equate the corresponding components.

$$\widehat{L_x} = \hat{y}\,\widehat{P_z} - \hat{z}\,\widehat{p_y} = -i\hbar\left[\hat{y}\frac{\partial}{\partial z} - \hat{z}\frac{\partial}{\partial y}\right]$$

$$\widehat{L_y} = \hat{z}\widehat{p_x} - \hat{x}\,\widehat{p_z} = -i\hbar\left[\hat{z}\frac{\partial}{\partial z} - \hat{x}\frac{\partial}{\partial x}\right]$$

$$\widehat{L_z} = \hat{x}\,\widehat{p_y} - \hat{y}\,\widehat{p_x} = -i\hbar\left[\hat{x}\frac{\partial}{\partial y} - \hat{y}\frac{\partial}{\partial x}\right]$$

## Quantum Commutator

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

1. $[\,x, \frac{d}{dx}\,] = ?$

$$[x, \frac{d}{dx}]\Psi = [x\frac{d}{dx} - \frac{d}{dx}x]\Psi$$

$$= x\frac{d\Psi}{dx} - \frac{d(x\Psi)}{dx}$$

$$= x\frac{d\Psi}{dx} - (x\frac{d\Psi}{dx} + \Psi \times 1)$$

$$= x\frac{d\Psi}{dx} - x\frac{d\Psi}{dx} - \Psi$$

$$[x, \frac{d}{dx}]\Psi = -\Psi$$

But $\hat{A}x = \lambda x$

$$[x, \frac{d}{dx}] = -1$$

$$[\frac{d}{dx}, x] = 1$$

$$[\widehat{A, B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

$$[\hat{B}, \hat{A}] = \hat{B}\hat{A} - \hat{A}\hat{B}$$

$$= -[\hat{A}\hat{B} - \hat{B}\hat{A}]$$

$$[\hat{B}, \hat{A}] = -[\widehat{A, B}]$$