Particle properties of waves
Part -6
Problems on Compton Effect

1. Calculate the minimum energy required by a photon to transfer half of its energy to an electron at rest.

Solution :

Here we have to find the minimum energy that means the frequency is also minimum, so the wavelength will be maximum. Maximum wavelength will occur when $\theta = 180^0$

$$E = h\nu$$

$$\lambda' - \lambda = \lambda_c(1-\cos\theta)$$

$$\lambda' - \lambda = 2\lambda_c \longrightarrow \quad (1)$$

According to the question $E_{initial} = h\nu$

$$E_{final} = h\nu' = \frac{h\nu}{2}$$

$$\frac{hC}{\lambda'} = \frac{hC}{2\lambda}$$

$\lambda' = 2\lambda$, Substituting this value in (1)

$$2\lambda - \lambda = 2\lambda_c$$

$$\lambda = 2\lambda_c = 2\times 2.426 \text{ pm}$$

$$E_{min} = \frac{hc}{\lambda} = \frac{1.26\times10^{-6}}{2\times 2.426} = 25\times10^4 \text{ev} = 0.25 \text{ MeV}$$

2. **The kinetic energy of the recoil electron if a photon of wavelength 2.5Å scattered at 90^0 is -----------**

(a) 40 eV
(b) 47.8 eV
(c) 88 eV
(d)24.6 eV

Solution : (b) 47.8 ev

Given

λ= 2.5Å = 250×10^{-17}m

$\theta = 90^0$

K = ?

λ' - λ = λ_c

λ'= λ + λ_c

λ' = 250 pm + 2.626 pm= 252.4pm

K = hν - hν'

$$k = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \left(\frac{\lambda' - \lambda}{\lambda\lambda'}\right)$$

$$= 1.26\times10^{-6}eV\left(\frac{2.426pm}{250pm\times252.4\,pm}\right)$$

K = 48 eV ≈ 47.8 ev

3. Calculate the maximum percentage change in wavelength of 10Å incident photons in Compton scattering.

Solution :

Given λ = 10Å = 10×10^{-10}m

$\frac{\Delta\lambda}{\lambda}\times100\%$ = ?

Δλ = $2\lambda_C$

For λ to be maximum θ= 180^0

$$\frac{\Delta\lambda}{\lambda}\times100\% = \frac{2\times2.426\times10^{-12}}{10\times10^{-10}}\times 100\,\% = 0.484\,\%$$

4. A beam of X-rays of wavelength 0.2 nm is incident on a free electron and gets scattered in a direction with respect to the direction of the incident radiation resulting in maximum wavelength shift. Find the percentage energy loss of the incident radiation

(a) 4.8%

(b) 6.2%

(c) 2.4%

(d) 1.2%

Solution : (c) 2.4%

Given **λ**= 0.2 nm = 200 pm

λ' - λ = maximum = $2\lambda_C$ (θ = 180^0)

$\frac{\Delta E}{E}\times 100\ \% = ?$

$\lambda' = \lambda + 2\lambda_C$

= (200 + 4.852) pm = 204.8 pm

$$\frac{\Delta E}{E}\times 100\ \% = \frac{h\nu - h\nu'}{h\nu} = \frac{\frac{C}{\lambda} - \frac{C}{\lambda'}}{\frac{C}{\lambda}}$$

$$= \left(\frac{\lambda' - \lambda}{\lambda\lambda'}\right)\times\lambda\times 100\%$$

$$= \left(\frac{\lambda' - \lambda}{\lambda'}\right)\times 100\%$$

$$\frac{\Delta E}{E}\times 100\ \% = \frac{2\times 2.426\ pm}{204.8 pm}\times 100\% = 2.4\%$$

5. **X-rays of 4 Å wavelength falls on an electron cloud and gets scattered. Determine the maximum change in**

i) Kinetic energy of e-

ii) Velocity of e-

Solution :

Given λ = 4Å = 400 pm

i) Maximum change in kinetic energy occur only when $\theta = 180^0$

$\lambda - \lambda' = 2\lambda_c = 4.852$ pm ≈ 5 pm

$\lambda' = 400 + 4.852 = 404.852$ pm ≈ 405 pm

$$K = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

$$= hc\left(\frac{\lambda' - \lambda}{\lambda\lambda'}\right) = \frac{1.24\times 10^{-6}\times 5}{400pm\times 405pm}$$

K = 37 eV

ii) $K = \frac{1}{2} mv^2$

$$V = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2\times 37\times 1.6\times 10^{-19}}{9.1\times 10^{-31}}} = 3.6\times 10^{6} m/s$$