Postulates Quantum mechanics
Part 2
Wave function,Operators& Quantum commutators

Probability

Given a normalized and otherwise acceptable wave function Ψ, the probability that the particle it describes will be found in a certain region is simply the integral of the probability density $|\Psi|^2$ over the region. Thus for a particle restricted to motion in the x direction, the probability of finding between x_1 and x_2 is given by

$$P_{x_1 x_2} = \int_{x_1}^{x_2} |\Psi|^2$$

Schrödinger's Equation

- Schrödinger's equation is a wave equation in the variable Ψ
- It is the fundamental equation of quantum mechanics in the same sense that the second law of motion is the fundamental equation of Newtonian mechanics.
- It is the equation of motion in QM.
- Schrödinger's equation cannot be derived from other basic principle of physics. It is a basic principle itself.

General Wave Equation is given by ;

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{V^2} \frac{\partial^2 y}{\partial t^2}$$

It's General solution is $\mathbf{y = A}\, e^{i(kx-\omega t)}$

In terms of wave function, the equation for a freely moving particle along x-direction with energy E and momentum P is

$$\Psi = A\, e^{-i/\hbar (Et-Px)}$$

Derivation of Schrödinger Equation

Wave equation for a free particle

$$\Psi = A\, e^{-i/\hbar (Et-Px)}$$

Diff w. r.to x

$$\frac{\partial \Psi}{\partial x} = A\, e^{-i/\hbar(Et-Px)} \times \frac{-i}{\hbar} \times -p$$

$$= \frac{iP}{\hbar} A\, e^{-i/\hbar(Et-Px)}$$

$$\frac{\partial^2 \Psi}{\partial x^2} = \left(\frac{i\,P}{\hbar}\right)^2 A\, e^{-i/\hbar(Et-Px)}$$

$$= \left(\frac{i\,P}{\hbar}\right)^2 \Psi = \left(\frac{-\,P}{\hbar}\right)^2 \Psi$$

$$P^2\Psi = -\hbar^2 \frac{\partial^2 \Psi}{\partial x^2}$$

Diff w . r. to t

$$\frac{\partial \Psi}{\partial t} = A\, e^{-i/\hbar(Et-Px)} \times \frac{-i}{\hbar} \times E$$

$$= \frac{-i\,E\,\Psi}{\hbar}$$

$$E\,\Psi = \frac{-\hbar}{i} \frac{\partial \Psi}{\partial t} \times \frac{i}{i}$$

$$E\,\Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

Total energy E = K + V

$$= \frac{P^2}{2m} + V$$

Multiplying L.H.S and R.H.S with Ψ

$$E\Psi = \frac{P^2\Psi}{2m} + V\Psi$$

$$i\hbar \frac{\partial \Psi}{\partial t} = -\hbar^2 \frac{\partial^2 \Psi}{\partial x^2} + v\Psi$$

The above equation is called Schrödinger's equation of motion.

$$\Psi = A\, e^{-i/\hbar(Et-Px)}$$

$$\frac{\partial \Psi}{\partial x} = \frac{-i}{\hbar} \times -P\,\Psi$$

$$\frac{\partial \Psi}{\partial x} = \frac{i}{\hbar} P\Psi$$

$$P\Psi = \frac{\hbar}{i} \frac{\partial \Psi}{\partial x} \times \frac{i}{i}$$

$$P\ \Psi = -\ i\hbar \frac{\partial \Psi}{\partial x}$$

The momentum operator

$$\hat{P}\Psi = -\ i\hbar \frac{\partial \Psi}{\partial x}$$

$$\boxed{\hat{P} = -\ i\hbar \frac{\partial}{\partial x}}$$

Also energy operator

$$\boxed{\hat{E} = i\hbar \frac{\partial}{\partial t}}$$