Quantum mechanics
Particle properties of waves
Part:1

Black body Radiation

- When heated, a solid object glows and emits thermal radiation.
- As the temperature increases, the object becomes red, then yellow, then white.
- The thermal radiation emitted by glowing solid objects consist of a continuous distribution of frequencies ranging from infrared to ultraviolet.
- The radiation emitted by gases has a discrete distribution spectrum.
- When radiation falls on an object, some of it might be absorbed and some reflected.

Blackbody

- An idealized " blackbody " is a material object that absorbs all the radiation falling on it.
- Appears as black under the reflection when illuminated from outside.
- When an object is heated, it radiates electromagnetic energy as a result of thermal agitation of electrons in its surface.
- A blackbody is perfect absorber as well as a perfect emitter of radiation.

Practical Blackbody

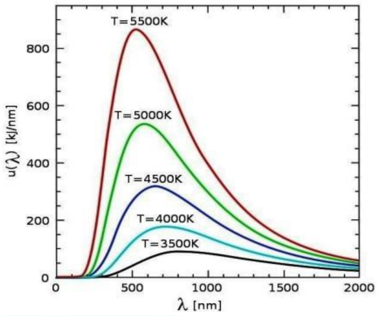
- A Hollow cavity whose internal walls perfectly reflect electromagnetic radiation with a very small hole on its surface.
- Radiation that enters through the hole will be trapped inside the cavity and gets completely absorbed after successive reflection on the inner surface of the cavity.
- Hole thus absorbs radiation like a black body.
- When this cavity is heated to a temperature T, radiation leaves the hole, and is blackbody radiation.
- The hole behaves as a perfect emitter; as the temperature increases, the hole will eventually begin to glow.

- To understand the radiation inside the cavity, one needs simply to analyze the spectral distribution of the radiation coming out of the hole.

Blackbody Spectrum

- The radiation emitted has a well defined , continuous energy distribution.
- To each frequency, there corresponds an energy density which depends neither on the chemical composition of the object nor its shape, but only on the temperature of the cavity's walls.

Spectral energy density (ν) of a blackbody



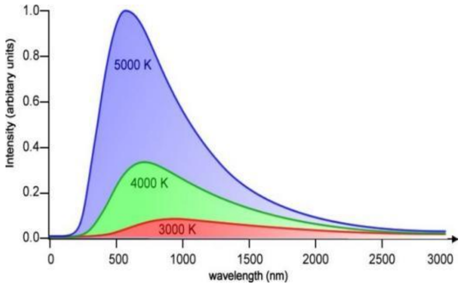
- The energy density shows a pronounced maximum at a given frequency, which increases with temperature.
- That is, the peak of the radiation spectrum occurs at a frequency that is proportional to the temperature. [Reason behind the change in color.]

Wien's Displacement Law

- For a given temperature, there is a maximum intensity. The wavelength corresponding to this is called λmax
- $\lambda_{max} \propto \frac{1}{T}$

  λ_{max} T = Constant

  λ_{max} T = 2.898 $\times 10^{-3}$ mK



Stefan-Boltzmann Law

According to Stefan-Boltzmann law, the amount of radiation emitted per unit time from an area of the black body is proportional to the fourth power of temperature.

$I = \frac{P}{A} = a\,\sigma\,T^4$ I = Intensity

P = Power

A = Area

a= emissivity (1 for perfect blackbody)

σ = Stefan- Boltzmann constant

$\sigma = 5.67\times10^{-8} \mathrm{w}/m^2K^4$

Wien's Formula for Energy Density

$u(\nu,T) = A\nu^3 e^{-\beta\nu/T}$

- A and β: Empirically defined parameters to fit the data.
- Fits only high frequency data.

Rayleigh's Energy Density Distribution

- He considered the radiation to consist of standing waves having a temperature T with nodes at the metallic surface.
- Standing waves are equivalent to harmonic oscillators, for they result from the harmonic oscillators of a large number of electrical charges, electrons, that are present in the walls of the cavity.
- Total energy of the radiation in the cavity can be obtained by multiplying the average energy of the oscillators by the number of standing waves of radiation.

$$u(\nu,T) = N(\nu)\,(E) = \frac{8\Pi\nu^2}{C^2}\,(E)$$

Rayleigh's Energy Density Distribution

- According to the equipartition theorem of classical thermodynamics, all oscillators in the cavity have the same mean energy, irrespective of their frequencies

$$\langle E \rangle = \frac{\int_0^\infty E\, e^{-E/KT}\, dE}{\int_0^\infty e^{-E/kT}\, dE} = k\,T$$

$$u(\nu,T) = N(\nu)\,\langle E \rangle = \frac{8\Pi\nu^2}{C^2}\,\langle E \rangle$$

$$u(\nu,T) = N(\nu)\,\langle E \rangle = \frac{8\Pi\nu^2}{C^2}\,k\,T$$

Ultraviolet Catastrophe

- Except for low frequencies, this law is in complete disagreement with experimental data.
- Energy density diverges for high value of frequency, whereas experimentally it must be finite.
- If we integrate over all frequencies, integral diverges. This implies that the cavity contains an infinite amount of energy.

$$u(\nu,T) = \frac{8\Pi\nu^2}{C^2}\,k\,T$$

Planck's Energy Density Distribution

$$u(\nu,T) = N(\nu)\,\langle E \rangle = \frac{8\Pi\nu^2}{C^2}\,\langle E \rangle \text{ ------------ (A)}$$

- In contrast to Rayleigh's assumption that a standing wave can exchange any amount (continuum) of energy with matter, Planck considered that the energy exchange between radiation and must be discrete.

$$E = nh\nu \qquad n = 0, 1, 2, 3, \ldots..$$

h = Planck's constant= 6.626×10^{-34} j s

- ν represents the frequency of the oscillating charge in the cavity's walls as well as the frequency of the radiation from the walls.

- **Integration to summation**

$$\langle E \rangle = \frac{\sum_{n=0}^{\infty} nh\nu e^{-nh\nu/kT}}{\sum_{n=0}^{\infty} e^{-h\nu/kT}} = \frac{h\nu}{e^{h\nu/kT}-1}$$

$$\boxed{u(\nu,T) = \frac{8\Pi\nu^2}{C^2}\,\frac{h\nu}{e^{h\nu/kT}-1}}$$

This is known as Planck's distribution

Since C = ν λ

$$u(\lambda,T) = \frac{8\Pi hc}{\lambda^3}\frac{1}{e^{hc/\lambda kT}-1}$$

Comparison of different formulations