ENTRI

Particle properties of waves
Part 5
Photoelectric effect problems & Compton Effect

1. A radiation of frequency $5.2\times10^{14}S^{-1}$ produces photoelectric effect and eject with K.E $1.3\times10^{-20}J$. Calculate the threshold frequency of the metal.

Solution :

$\nu = 5.2\times10^{14}S^{-1}$

$K = 1.3\times10^{-20}J$

$\nu_0 = ?$

$h\nu = h\nu_0 + K$

$\nu = \nu_0 + \frac{K}{h}$

$\nu_0 = \nu - \frac{K}{h}$

$= 5.2\times10^{14} - \frac{1.3\times10^{-20}}{6.626\times10^{-34}}$

$\nu_0 = 5\times10^{14}\text{Hz}$

2. **A photon of wavelength 4×10^{-7}m strikes on metal surface, the work function of a metal being 2.13 eV. Calculate**

(a) The energy of photon
(b) The kinetic energy of emission
(c) The velocity of photoelectron

Solution :

Given $\lambda = 4\times10^{-7}$

W = 2.13 eV

$E = h\nu = \frac{hc}{\lambda} = \frac{1.26\times10^{-6}}{4\times10^{-7}} = 3.1$ eV

$h\nu_0 = W + K$

$K = h\nu_0 - W = 3.1 - 2.13 = 0.97\text{eV}$

$K = \frac{1}{2}mv^2$

$V = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2\times0.97\times1.6\times10^{-19}}{9.1\times10^{-31}}} = 5.8\times10^{5}$ m/s

3. (a) What are the energy and momentum of a photon of red light of wavelength 650 nm? (b) What is the wavelength of a photon of energy 2.40 eV?

Solution :

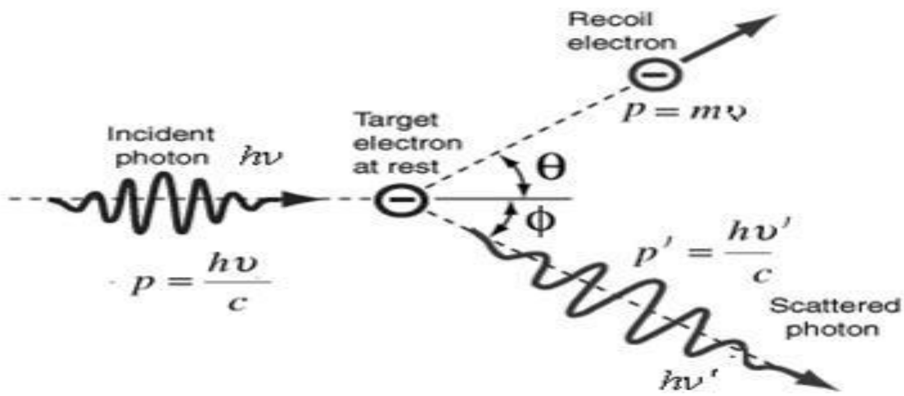
(a) $E = \frac{hc}{\lambda} = \frac{1240\ eV\ nm}{650\ nm} = 1.91$ eV

$$P = \frac{h}{\lambda} = \frac{1}{c}\frac{hc}{\lambda} = \frac{1}{c}\frac{1240\ eV.nm}{650\ nm} = 1.91\ \text{eV/c}$$

(b) $$\lambda = \frac{hc}{E} = \frac{1240\ eV\ nm}{2.40\ eV} = 517\ \text{nm}$$

Compton Effect

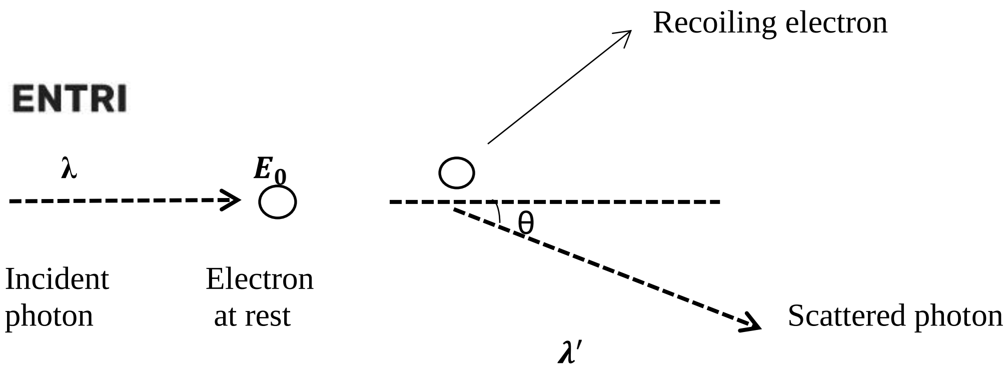
- In his experiment, Compton provided the most conclusive conformation of the particle aspect of radiation.
- By scattering X-rays off free electrons, he found that the wavelength of scattered radiation is higher than the wavelength of the incident radiation.
- This can be explained only by assuming that the X- ray photons behave like particles.



Before collision **After collision**

Compton scattering of a photon (of energy hν and momentum p) off a free stationary electron. After collision, the photon is scattered with an angle θ with energy hν'

- The wavelength of the scattered X-ray increases by an amount Δλ , called the wavelength shift, and that Δλ depends not on the intensity of the incident radiation, but only on the scattering angle.
- Compton succeeded in explaining this only after treating the incident radiation as a stream of particles (photons) colliding elastically with individual electrons.
- This scattering process can be illustrated by the elastic scattering of a photon from a free electron, the laws of elastic collisions can be applied here; conservation of energy and momentum



$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta) = \lambda_c\,(1-\cos\theta)$$

$$\lambda_c = \frac{h}{m_e c} = 2.426\times10^{-12}\ \text{m}$$

$$\Delta\lambda = \lambda_c(1-\cos\theta)$$

$$\text{Or } \lambda' - \lambda = \lambda_c(1-\cos\theta)$$

$$\lambda' = \lambda + \lambda_c(1-\cos\theta)$$

Cases :

(I) $\theta = 90^0$

$\cos\theta = \cos 90^0 = 0$

$\Delta\lambda = \lambda_c = 2.426$ pm

Or $\lambda' = \lambda + 2.426$ pm

(ii) $\theta = 180^0$

$\cos\theta = \cos 180^0 = -1$

$\Delta\lambda = \lambda_c(1 - -1) = 2\lambda_c$

Or $\lambda' = \lambda + 2\times 2.426$ pm

Or $\lambda' = \lambda + 4.852$ pm

Maximum wavelength shift

Problems

1. X-rays of wavelength 10.0 pm are scattered from a target.

(a) Find the wavelength of the X-rays scattered through 45^0.

(b) Find the maximum wavelength present in the scattered X-rays.

(c) Find the maximum kinetic energy of the recoil electrons.

Solution :

Given, λ= 10 pm

$\theta = 45^0$

$\Delta \lambda = \lambda_c(1- \cos\theta)$

$\lambda' = \lambda + \lambda_c(1- \cos\theta)$

$= 10 \text{ pm} + 2.426 \text{ pm} (1- \text{Cos } 45^0)$

$= 10 \text{ pm} + 2.426 (1- 0.7)$

$\lambda' = 10.7 \text{ pm}$

(b) $\lambda' = \lambda + 2\lambda_c$

$= 10 \text{ pm} + 2\times 2.426$

$\lambda' = 14.852 \text{ pm}$

(c) $K = h\nu - h\nu'$

$$K_{max} = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

$$= 6.626\times10^{-36}\times3\times10^{8}\left(\frac{1}{10} - \frac{1}{14.9}\right)\times 10^{12}$$

$$K_{max} = 6.6\times10^{-15}\text{J} = \frac{6.6\times10^{-15}}{1.6\times10^{-19}} = 41\times10^{3}\text{eV} = 41 \text{ KeV}$$