Wave properties of Particles
Part -3
de-Broglie Waves

1. In a television picture tube electrons are accelerated through 20000 volts. What would be the velocity and de Broglie wavelength of one of these electrons ?

Solution : If a particle have a charge q and is accelerated to a potential V then the kinetic energy will be

$K = qV$ Then $\lambda = \frac{h}{\sqrt{2mk}}$

Also $\frac{1}{2} m v^2 = qV$

$$v^2 = \frac{2qV}{m}$$

$$V = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 2 \times 10^4}{9.1 \times 10^{-31}}} = 8 \times 10^7 \text{m/s}$$

$$\lambda = \frac{h}{\sqrt{2mk}} = \frac{h}{\sqrt{2mqV}} = 8.6 \times 10^{-11} \text{m}$$

Matter Waves

- In water waves, the quantity that varies periodically is the height of the water surface. In sound waves, it is pressure. In light waves, electric and magnetic field vary. What is it that varies in the case of matter waves ?
- The quantity whose variation make up matter waves is called the wave function (Ψ)
- The value of the wave function associated with a moving body at the particular point x,y,z in space at the time t is related to the likelihood of finding the body there at the time.
- The wave function itself, however, has no direct physical significance. Ψ cannot be interpreted in terms of an experiment.

- The probability that something be in a certain place at a given time must lie between 0 (the object is not definitely not there.) and 1 (the object is definitely there).
- An intermediate probability, say 0.2, means that there is a 20% chance of finding the object. But the amplitude of a wave can be negative as well as positive, and a negative probability, say -0.2, is meaningless. Hence Ψ by itself cannot be an observable quantity.
- This objection does not apply to $|\Psi|^2$, square of the absolute value of the wave function, which is known as probability density :
- The probability of experimentally finding the body describe by the wave function at the point x,y,z at the time t is perpendicular to the value of $|\Psi|^2$there at t.
- A large value of $|\Psi|^2$ means the strong possibility of the body's presence, while a small value of $|\Psi|^2$ means the slight possibility of its presence.
- As long as $|\Psi|^2$ is not actually 0 somewhere, however , there is a definite chance, however small, of detecting it there.

➢ How fast do de broglie waves travel ? since we associate a de Broglie wave with a moving body?

Since we associate a de Broglie wave with a moving body, we expect that this wave has the same velocity as that of the body.

If we call the de Broglie wave velocity v_p we can apply the usual formula

$$\boldsymbol{v_p} = \boldsymbol{\nu\lambda}$$

To find v_p

The wavelength λ is simply the de Broglie wavelength $\boldsymbol{\lambda} = \frac{\boldsymbol{h}}{\boldsymbol{mV}}$

To find the frequency, we equate the quantum expression **E = hν** with the relativistic formula for total energy $\mathbf{E} = \mathbf{m}\boldsymbol{C}^2$ to obtain

$$\mathbf{h\nu} = \mathbf{m}\boldsymbol{C}^2$$

$$\boldsymbol{\nu} = \frac{\mathbf{m}\boldsymbol{C}^2}{\boldsymbol{h}}$$

$$\text{Then } V_p = \frac{\mathbf{m}\boldsymbol{C}^2}{\boldsymbol{h}} \times \frac{\boldsymbol{h}}{\boldsymbol{mV}}$$

$$\boldsymbol{V_p} = \frac{\boldsymbol{C}^2}{\boldsymbol{V}}$$

Because the particle velocity V must be less than the velocity of light C, the de Broglie waves are always travel faster than light !

In order to understand this unexpected result, we must look into the distinction between phase velocity and group velocity.

Phase and group velocities

- The wave representation of a moving body corresponds to a wave packet, or a wave group.
- The velocity of individual waves that combine together to form the wave group is called Phase Velocity.
- The velocity of the wave group is called Group velocity.
- If a particle of mass m travels with a velocity V and there must be a matter wave associated with the particle.If we consider an individual wave or a phase wave its velocity will be phase velocity V_p and V_g is the group velocity.

$$V_g = V$$

$$V_p = \frac{c^2}{V}$$