Wave Properties of Particles
Part 2
de- Broglie Waves

Problems

1. What is the wavelength of a particle of mass 2 mg with velocity 0.5c ?

Solution : Given

m= 2 mg = 2×10^{-6}kg

V = 0.5c

$$\lambda = \frac{h}{m_0\gamma v}$$

$$= \frac{h}{m_0 v}\sqrt{1-\frac{v^2}{c^2}}$$

$$= \frac{6.626\times10^{-34}}{2\times10^{-6}\times\frac{C}{2}}\sqrt{1-\frac{c^2}{4c^2}}$$

$$\lambda = 1.9\times 10^{-36}$$

2. What is the de-Broglie wavelength of 1 MeV proton ?

Solution : Given

K = 1 MeV

Rest mass energy of proton , $E_0 = \frac{mc^2}{e}$

$$E_0 = \frac{1.6\times10^{-27}(3\times10^{8})^2}{1.6\times10^{-19}} = 939.4 \text{ Mev}$$

Since k << E_0 , it is non-relativistic case

$$\lambda = \frac{h}{\sqrt{2mk}} = \frac{6.626\times10^{-34}}{\sqrt{2\times1.6\times10^{-27}\times10^{6}\times1.6\times10^{-19}}}$$

$$\lambda = 2.96\times 10^{-14}\text{m}$$

3. What is the de- Broglie wavelength of 1 MeV Electron ?

Solution : Given

K = 1 MeV

Rest mass energy of electron $E_0 = \frac{mc^2}{e} = 0.511$ MeV

Since k >> E_0, it is relativistic

$$\lambda = \frac{hc}{\sqrt{k^2 + 2kE_0}}$$

$$= \frac{1.26\times10^{-6} eV\ m}{\sqrt{1\ MeV^2 + 2\times MeV\times 0.5MeV}}$$

$= 0.87$pm

4. Find the kinetic energy of a proton whose de Broglie wavelength is 1 fm

Solution ; Given

λ = 1 fm

If Pc > E_0 , it is relativistic

$$\lambda = \frac{h}{p}$$

$$Pc = \frac{hc}{\lambda}$$

As E_0 =939.6 MeV pc must be in eV

$$Pc = \frac{1.26\times10^{-6} eV\ m}{10^{-15} m} = 1.6\times 10^9 eV = 1.26\ GeV$$

E_0= 0.939 GeV

Clearly If Pc > E_0 , it is relativistic

Also E = K + E_0

$$E = \sqrt{p^2C^2 + {E_0}^2}$$

$$E = \sqrt{(1.26^2)(GeV)^2 + (0.939^2)(GeV^2)}$$

$= \sqrt{1.53 + 0.9}$GeV = $\sqrt{2.5}$ GeV = 1.55 GeV

Then Kinetic energy , K= E - E_0

K = 1.55 - 0.939 = 0.617 GeV = 617 MeV