MENSURATION

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Mensuration is the branch of mathematics that studies the measurement of geometric figures and their parameters like length, volume, surface area, lateral surface area, etc. It discusses all the essential equations and properties of various geometric shapes and figures.

PERIMETER

Perimeter is the distance covered along the boundary forming a closed figure when you go round the figure once. It is the continuous line along the edge of the closed vessel. It is represented by P and measures are taken in unit of length.

UNIT OF LENGTH

10 mm = 1 cm

100 cm = 1 m

1000 m = 1 km

AREA

The amount of surface enclosed by a closed figure is called area. It is represented by the letter A and expressed in square units.

UNIT OF AREA

1 cm^2 = 100 mm^2

1 m^2 = 10000 cm^2

1 hectare (ha) = 100 × 100 = 10,000 m^2

VOLUME

Volume is the amount of three-dimensional space enclosed by a closed surface. Volume is denoted by V and SI unit of volume is cubic meter.

UNIT OF VOLUME

1 cm^3= 1000 mm^3

1 m^3= 1000000 cm^3

1 L = 1000 cm^3 and 1 m^3= 1000 L

POLYGONS

- Polygons are the two-dimensional closed shapes that are formed by joining three or more straight line segments with each other.

- A polygon with a minimum of three sides is known as triangle and it is also called 3-gon. An n-sided polygon is called n-gon.
- The sum of all the interior angles of a simple n-gon is, (n-2)180°
- The measure of exterior angle of any polygon is 360°

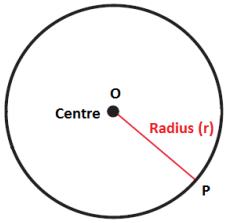
Here, we are going through some special polygons

Name of polygon	Area	Perimeter
Triangle	$\frac{1}{2} \times$ base $\times$ $height$	Sum of all sides
Square	side^2	4 × side
Rectangle	Length ×breadth	2 × (length+breadth)
Parallelogram	Base × height	2 × (sum of pair of adjacent sides)
Trapezium	$\frac{1}{2}$ ×(sum of parallel side)	Sum of all sides
Rhombus	$\frac{1}{2}$ ×(product of diagonals)	4 × side
Pentagon	$\frac{1}{4}\sqrt{5(5+2\sqrt{5})}\ \text{side}^2$	Sum of all five sides
Hexagon	$\frac{3\sqrt{3}}{2}\text{side}^2$	Sum of all six sides

LENGTH OF ARCS

CIRCLE

- The collection of all the points in a plane, which are at a fixed distance from a fixed point in the plane is called circle.
- The circle formula in the plane is $(x-h)^2+(y-k)^2 = r^2$
- The fixed point is called centre of the circle.
- The fixed distance is called radius of the circle.



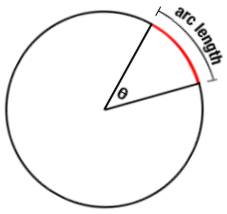
- Area of circle = πr^2
- Perimeter of circle = $2\pi r$

CHORD

- The chord of a circle is defined as the line segment joining any two points on the circumference of the circle.
- Diameter is the longest chord and all diameters have the same length, which is equal to two times the radius.

ARC AND ARC LENGTH

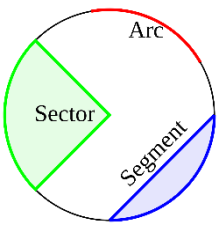
- The arc of a circle is defined as the part of segment of the circumference of a circle.
- The arc length is defined as the interspace between two points along a section of curve.



- Arc length = $2\pi r \times (\frac{\theta}{360})$ if $\theta$is in degrees
- Arc length = rθ if θ is in radian

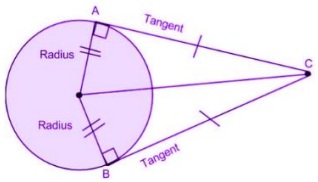
SEGMENT AND SECTOR

- The region between a chord and either of its arcs is called a segment of the circular region or simply segment of the circle.
- The region between an arc and the two radii, joining the centre to the end points of the arcs is called a sector.
- Area of the sector of angle θ = $2\pi r \times (\frac{\theta}{360})$



TANGENTS

- A line that touches the circle at a single point is known as the tangent to a circle.
- The point where tangent meets the circle is known as point of tangency.
- The tangent is perpendicular to the radius of the circle, with which it intersects.
- In a circle, the angles between the radii through two points and the angle tangents at these points are supplementary.
- From a point outside a circle, two tangents can be drawn and these are of same length.



CIRCUMCIRCLE AND INCIRCLE OF POLYGONS

CIRCUMCIRCLE

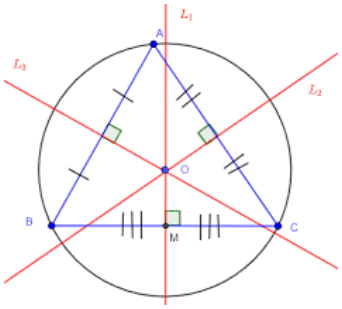
- Circumscribed circle or circumcircle of a polygon is a circle that passes through all the vertices of the polygon.
- The centre of the circumcircle is called the circumcentre. Which is the intersection point of perpendicular bisectors of each side of the polygon.
- The radius is called the circumradius.
- A polygon that does have a circumcircle is called a cyclic polygon. ie, a polygon whose vertices are on the same circle.
- Every triangle, every rectangle and every regular polygon are cyclic polygons.

CIRCUMCIRCLE OF TRIANGLES

Circumcircle of a triangle is circle that passes through all three vertices of the triangle.

CIRCUMCENTRE

Circumcentre of any triangle is intersection point of perpendicular bisectors of sides of the triangle.



- For an acute triangle (ie, a triangle with all angles less than 90°) the circumcentre always lies inside the triangle.
- For right angled triangle, the circumcentre always lies in the midpoint of hypotenuse.
- For an obtuse angle triangle (ie, tringle with one angle bigger than right angle) circumcentre always lies outside the triangle

CIRCUMRADIUS

- The diameter of a circumcircle is,

$$\boldsymbol{d = \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$$

- Radius

$$R = \frac{abc}{4 \times \ area\ of\ \Delta}$$

- Circumradius of an equilateral triangle

$$R = \frac{a}{\sqrt{3}}$$

Where a, b and c are three sides and A, B and C are angles of triangle.

CIRCUMCIRCLE OF SQUARE

- The circumcircle of a square is the circle that passes through all four vertices.
- The centre of the circumcircle is the point where the two diagonals of the square meet.
- Circumradius of a square is,

$$R = \frac{a}{\sqrt{2}}$$

- Perimeter of circumcircle is,

$$P = \sqrt{2}\pi a$$

- Area of circumcircle is,

$$A = \frac{1}{2}\pi a^2$$

Where a is the side of the square.

CIRCUMCIRCLE OF RECTANGLE

- The circumcircle of a rectangle is a circle that passes through all four vertices of the rectangle.
- The circumcentre of a rectangle is the point where the two diagonals of the rectangles meet.
- Circumradius of the rectangle is,

$$R = \frac{\sqrt{a^2 + b^2}}{2}$$

- Perimeter of circumcircle of a triangle is,

$$P = \pi\sqrt{a^2 + b^2}$$

- Area of circumcircle is,

$$A = \frac{\pi}{4}(a^2 + b^2)$$

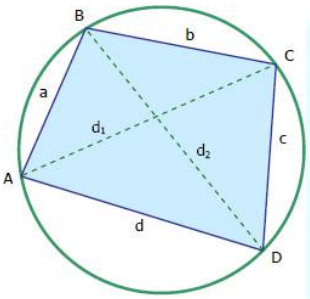
Where a and b are two sides of rectangle.

CYCLIC QUADRILATERAL AND ITS PROPERTIES

A cyclic quadrilateral is a quadrilateral whose vertices all lie on a single circle.

PROPERTIES

- In a cyclic quadrilateral the sum of a pair of opposite angles is 180°(supplementary)



Cyclic Quadrilateral

- For the given cyclic quadrilateral, $d_1 \times d_2 = ac + bd$

- Area of cyclic quadrilateral, $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$

  Where $s = \frac{a+b+c+d}{2}$

PARAMESHVARA'S CIRCUMRADIUS FORMULA

Let ABCD be a cyclic quadrilateral with sides a, b, c, and d respectively. Then its circumradius R will be,

$$R = \frac{1}{4}\sqrt{\frac{(ab+cd)(ac+bd)(ad+bc)}{(s-a)(s-b)(s-c)(s-d)}}$$

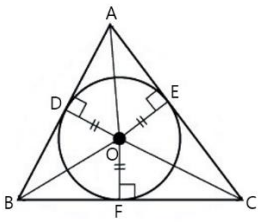
$$R = \sqrt{\frac{(ab+cd)(ac+bd)(ad+bc)}{16\Delta^2}}$$

INCIRCLE OF POLYGONS

- The incircle of a polygon is the circle that will fit inside the polygon and touch each side in just one place.
- The centre of the incircle is called the incentre and can be constructed by drawing the intersection of angle bisectors.
- Its radius is called the inradius.
- Every triangle and regular polygon have a unique incircle.
- A quadrilateral that does have an incircle is called tangential quadrilateral.
- Incircle is the largest circle that can be inscribed in the polygon.
- All the sides of the triangle are tangents of the circle.

INCIRCLES OF TRIANGLE

The incircle or inscribed circle of a triangle is the largest circle contained in the triangle, it touches the three sides.



- The radius of an incircle of a right triangle is,

$$r = \frac{ab}{a+b+c}$$

Where, a, b and c are sides of triangle.

- The radius of an incircle of equilateral triangle is

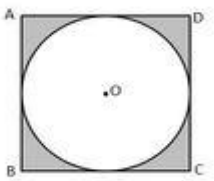
$$r = \frac{\sqrt{3}a}{6} = \frac{a}{2\sqrt{3}}$$

where a is the side of the triangle.

- The radius of the incircle of an equilateral triangle is half the radius of its circumcircle.

INCIRCLE OF A SQUARE

The incircle or inscribed circle of a square is the largest circle contained in the square, it touches all the four sides.



- Incentre and circumcentre of a square is the same point.

SOLIDS

Solids are three dimensional shapes because they have three dimensions such as length, breadth and height. The study of the properties, volume and surface area of three-dimensional shapes is called solid geometry. Now, we have to go through some special solid shapes.

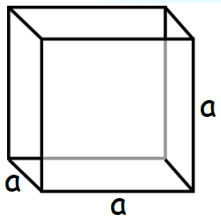
CUBE

Cube is a solid shape which has 6 square faces, 8 vertices and 12 edges. It is also said to be a regular hexahedron.

- Surface area of a cube having length of an edge a is

$$surface\ area = 6a^2$$

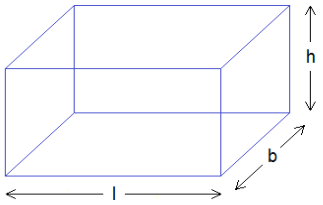
- Volume $= a^3$



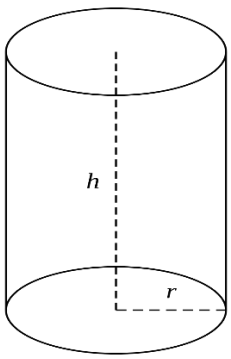
CUBOID

A cuboid is a three dimensional solid that has 6 rectangular faces, 8 vertices and 12 edges.

- Surface area $= 2(lb + lh + bh)$
- Volume $= lbh$



CYLINDER

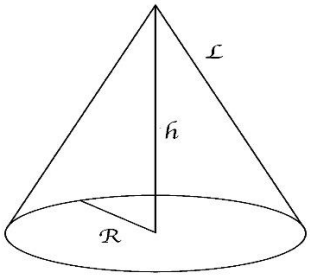


Cylinder is a solid shape which has two circular bases joined by a curved surface. The line segment joining the centre of two circular bases is axis of the cylinder. A cylinder has 3 faces and 2 edges.

- Curved surface area $= 2\pi rh$
- Surface area $= 2\pi(r + h)$
- Volume $\pi r^2 h$

CONE

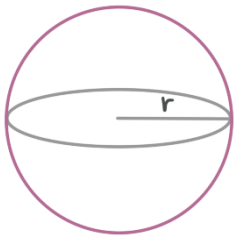
A cone is solid shape formed by using a set of line segments or the line which connects a common point called the apex or vertex, to all the points of a circular base which does not contain the apex. Cone has 2 faces, one edge and one vertex.



- Curved surface area $= \pi rl$
- Surface area $= \pi r(r + l)$
- Volume $= \frac{1}{3}\pi r^2 h$

SPHERE

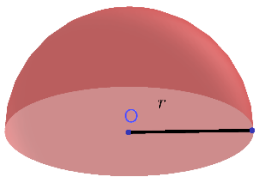
Sphere is a three-dimensional object that is round in shape. It has only one curved surface.



- Surface area $= 4\pi r^2$
- Volume $= \frac{4}{3}\pi r^3$

HEMISPHERE

Hemisphere is exactly half of the sphere.



- Curved surface area $= 2\pi r^2$
- Total surface area $= 3\pi r^2$
- Volume $= \frac{2}{3}\pi r^3$

NUMBER THEORY

Number theory is a branch of pure mathematics devoted primarily to the study of the integers and natural numbers.

WELL ORDERING PRINCIPLE

The well ordering principle states that every non-empty set of positive integers contains a least element.

DIVISION ALGORITHM

The division algorithm states that for any integer a and any positive integer b and r such that $a = bq + r$, where r $0 \leq r < b$. We call a the divident, b the devisor, q the quotient and r the remainder.

DIVISIBILITY

- Let a and b be two integers such that a≠0. Then the following statements are equivalent.
  - a devides b
  - a is a devisor of b
  - a is a factor of b
  - b is a multiple of a
  - b is divisible by a
- A positive integer a is a proper devisor of b if a \ b and |a|<|b|.
- An integer p is a prime if its positive devisors are 1 and p itself.
- There are infinitely many primes

FUNDAMENTAL THEOREM OF ARITHMETIC

This theorem says that primes can be regarded as the building blocks of all integers with respect to multiplication.ie, for any given integer n≥2, there exist primes $p_1 \leq p_2 \leq p_3 \leq \ldots \leq p_s$ such that $n = p_1 p_2 p_3 \ldots p_s$. Furthermore, this factorization is unique.

GREATEST COMMON DEVISOR

The GCD of two non-zero integers a and b is the greatest positive integer d such that d is a devisor of both a and b.ie, there are integers e and f such that, a= de and b= df and d is the largest such integer. The GCD of a and b is represented as gcd(a, b).

LEAST COMMON MULTIPLE

The LCM of the integers a and b, denoted by lcm(a, b) is the smallest positive common multiple of both a and b.

CONTINUED FRACTIONS

Continued fraction is an expression obtained through an iterative process of representing a number as the sum of its integer part and the reciprocal of another number, then writing this other number as the sum of its integer part and another reciprocal and so on.

$$x = b_0 + \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2 + \cfrac{a_3}{b_3 + \cfrac{a_4}{b_4 + \ddots}}}}$$

When b_i= 1 for all the expression then it is called a simple continued fraction.

FINITE CONTINUED FRACTION

In a finite continued fraction the iteration is terminated after finitely many steps by using an integer instead of another continued fraction.

INFINITE CONTINUED FRACTION

If the expression contains infinitely many terms, then it is called infinite continued fraction.

PERIODIC CONTINUED FRACTION

When the terms eventually repeat from some point onwards, the expression is called as periodic continued fraction.

CALCULATION OF CONTINUED FRACTION

To calculate a continued fraction representation of a number r, write down the integer part of r. Subtract this integer part from r. If the difference is 0, then stop. Otherwise find the reciprocal of the difference and repeat. The procedure will halt if and only if r is rational.

Let us consider an example,

- $\frac{13}{9}$
- Here the integer part = 1
- Subtract 1 from $\frac{13}{9}$ then the difference is $\frac{4}{9}$ which is non-zero
- Hence take its reciprocal $\frac{9}{4}$ and repeat.
- So, the continued fraction of $\frac{13}{9}$ will be,

$$\frac{13}{9} = 1 + \frac{4}{9} = 1 + \cfrac{1}{\cfrac{9}{4}}$$

$$= 1 + \frac{1}{2 + \frac{1}{4}}$$

BOOLEAN ALGEBRA

Boolean algebra is the branch of algebra, in which the values of the variables are truth values true and false usually denoted as 1 and 0 respectively. The basic operations of Boolean algebra are

- Conjunction
- Disjunction
- Negation

These Boolean operations are expressed with the corresponding binary operations AND and OR and the unary operator NOT, collectively referred as Boolean operators.

CONJUNCTION

A conjunction is a statement formed by adding two statements with the connector AND. The symbol for conjunction is ′ ∧ ′ which can be read as and. When two statements p and q are joined in a statement, the conjunction will be expressed symbolically as p ∧ q. The conjunction statement will only be true if both the combining statements are true. Otherwise, false.

TRUTH TABLE FOR CONJUNCTION

P	Q	P∧Q
T	T	T
T	F	F
F	T	F
F	F	F

DISJUNCTION

Disjunction is a compound statement formed by combining two statements using the word OR. It is represented using the symbol ′ ∨ ′ and can be read as or. If p and q are connected using OR it can be represented as p ∨ q.

TRUTH TABLE FOR DISJUNCTION

P	Q	P∨Q
T	T	T
T	F	T
F	T	T
F	F	F

NEGATION

A statement that is constructed by interchanging the truth value of the statement is called negation of that statement. It is done by using the word NOT and is represented by ′ ∼ ′.

TRUTH TABLE FOR NEGATION

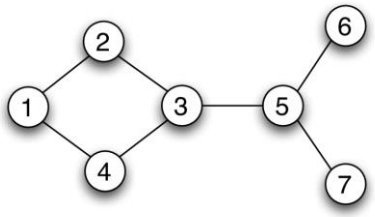
P	∼P
T	F
F	T

GRAPH THEORY

Graph theory is the study of graphs which are mathematical structures used to model pairwise relation between objects. Graph is an ordered pair (V, E) where V is the set of vertices and E is the set of edges.ie, graph is a diagram of points and lines connected to the points.

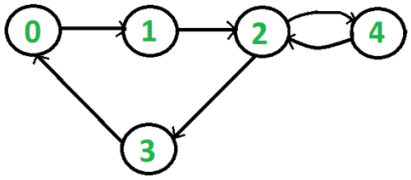
UNDIRECTED GRAPH

- An undirected graph is a set of nodes and links between the nodes.
- Each node is called vertex and each link is called edge. Each edge connects two vertices.
- The order of two connected vertices is not important.
- Both the sets might be empty. Which is called an empty graph.



DIRECTED GRAPH

- A directed graph is a finite set of vertices together with a finite set of edges.
- Each edge is associated with two vertices, called its source and target vertices.
- We say that the edge connects its source to its target.
- The order of the two connected vertices is important.



TERMINOLOGIES

- **Loop:** An edge that connects a vertex to itself.
- **Path:** A sequence of vertices, $p_0, p_1, p_2 \ldots p_m$ such that each adjacent pair of vertices $p_i \; and \; p_{i+1}$ are connected by an edge.
- **Cycle:** A simple path with no repeated vertices or edges other than the starting and ending vertices. A cycle in directed graph is called a directed cycle.
- **Simple graph:** The graphs that have no loops and no multiple edges.