Electrodynamics
Vector Analysis Part 1

Vector

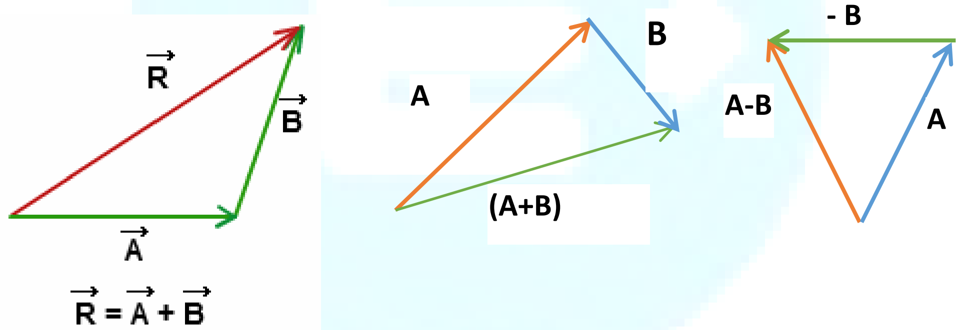
- Tensors of rank 1
- They have direction as well as magnitude.
- Examples : Displacement, Velocity, Acceleration, Force, Momentum

Vector Operations

- Four vector operations : Addition and three kinds of multiplication

1. **Addition of two vectors** : Place the tail of B at the head of A; the sum, A+B, is the vector from the tail of A to the head of B.

- Addition is commutative: **A+B = B+A**
- To subtract a vector, add its opposite: **A - B = A+(-B)**



2. **Multiplication by a Scalar**:

- Multiplication of a vector by a positive scalar **a** multiplies the magnitude but leaves the direction unchanged.
- If **a** is negative, the direction is reversed.
- Scalar multiplication is distributive**: a(A+B) = aA + aB**

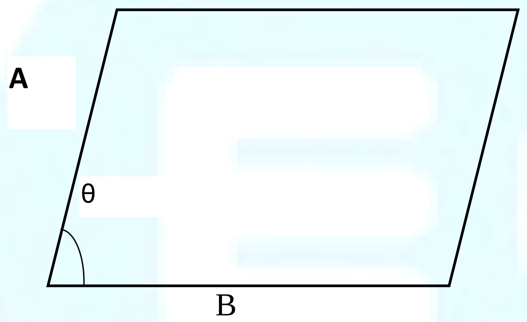
3. **Dot product of two vectors**

- If **A** and **B** are two vectors then **A.B = AB cos θ ;** Where θ is the angle they form when placed tail to tail.
- Note that **A.B** is itself scalar (so the name scalar product).
- The dot product is commutative : **A.B = B.A**
- The dot product is distributive, **A.(B+C) = A.B + A.C**

- If the two vectors are parallel, then **A.B = AB**
- In particular, for any vector A**, A.A =** A^2
- If A and B are perpendicular, then **A.B = 0**

4. **Cross product of two vectors**

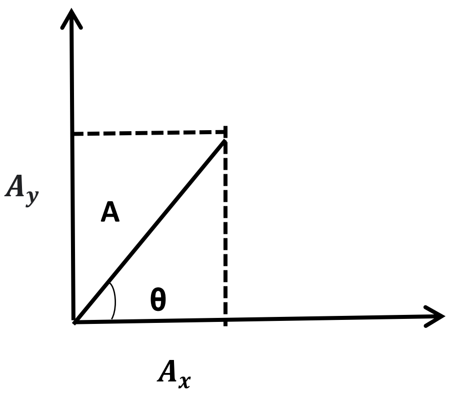
- The cross product of two vectors is defined by **A×B= AB Sinθ n**
- Right - hand rule : let your fingers point in the direction of the first vector and curl around (via the smaller angle) towards the second; then your thumb indicates the direction of n
- Note that A×B is itself a vector (hence the name vector product).
- Cross product is distributive, **A×(B+C) = (A×B) + (A×C)** **n̂ n̂**
- But not commutative; **(B×A) = - (A×B)**
- Geometrically**, [A×B]** is the area of the parallelogram generated by **A** and **B**.



- If two vectors are parallel, their cross product is Zero. In particular, **A×A = 0** for any vector **A**

Vector Resolution

- The splitting of a single vector into two or more vectors in different directions which together produce a similar effect as is produced by a single vector itself. The vectors formed after splitting are called components vectors.



$A = A_x \hat{x} + A_y \hat{Y}$
$A_x = A \cos \theta$
$A_y = A \sin \theta$

Vector Algebra : Component Form

$$\mathbf{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

- The numbers A_x, A_y and A_z are the components of **A**
- We can now reformulate each of the four vector operations as a rule for manipulating components ;

$$\mathbf{A}+\mathbf{B} = (A_x \hat{x} + A_y \hat{y} + A_z \mathrm{Z}) + (B_x \hat{x} + B_y \hat{y} + B_z \hat{z})$$

$$= (A_x + B_x)\hat{x} + (A_y + B_y)\hat{y} + (A_z + B_z)\hat{z}$$

- **Rule (i) : To add vectors, add like components.**

$$a\mathbf{A} = (aA_x)\hat{x} + (a A_y)\hat{y} + (aA_z)\hat{z}$$

- **Rule (ii) : To multiply with a scalar, multiply each component**
- Because , $\hat{x}$, $\hat{y}$ and $\hat{z}$ are mutually perpendicular unit vectors,

$$\hat{x}.\hat{x} = \hat{y}.\hat{y}. = \hat{z}.\hat{z} = 1 \qquad \hat{x}.\hat{Y} = .\hat{x}.\hat{z} = \hat{Y}.\hat{z} = 0$$

Accordingly, $\mathbf{A.B} = (A_x \hat{x} + A_y \hat{y} + A_z \hat{z}).\ (B_x \hat{x} + B_y \hat{y} + B_z \hat{z})$

$$= A_x B_x + A_y B_y + A_z B_z$$

- **Rule (iii) : To calculate the dot product, multiply like components, and add**
- In particular $\mathbf{A.A} = A_x^2 + A_y^2 + A_z^2$

So $A = \sqrt{= A_x^2 + A_y^2 + A_z^2}$

$$\hat{x} \times \hat{x} = \hat{Y} \times \hat{Y} = \hat{z} \times \hat{z} = 0$$

$$\hat{x} \times \hat{Y} = -\hat{Y} \times \hat{x} = \hat{z}$$

$$\hat{Y} \times \hat{z} = -\hat{z} \times \hat{Y} = \hat{x}$$

$$\hat{z} \times \hat{x} = -\hat{x} \times \hat{z} = \hat{Y}$$

$$\mathbf{A} \times \mathbf{B} = (A_x \hat{x} + A_y \hat{y} + A_z \hat{z}) \times (B_x \hat{x} + B_y \hat{y} + B_z \hat{z})$$

This can be find using determinant

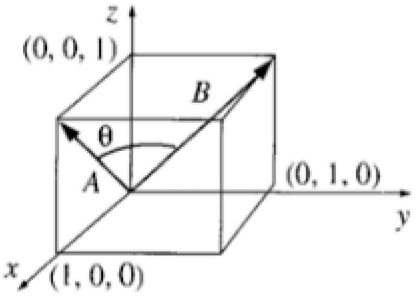
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{x} & \hat{Y} & \hat{z} \\ Ax & Ay & Az \\ Bx & By & Bz \end{vmatrix}$$

$$= (A_y B_z - A_z B_y)\hat{x} + (A_z B_x - A_x B_z)\hat{Y} + (A_x B_y - A_y B_x)\hat{z}$$

- **Rule (iv); To calculate the cross product, form the determinant whose first row is $\hat{x}$, $\hat{y}$, $\hat{z}$ whose second row is A in component from and third row is B**

Problems

1. Find the angle between the face diagonals of a cube of side 1



The face diagonals A and B are

A= 1 $\hat{x}$ + 0 $\hat{Y}$ + 1 $\hat{z}$ B= 0 $\hat{x}$ + 1 $\hat{Y}$ + 1$\hat{z}$

So in component form

A.B = 1.0 + 0.1 + 1.1 = 1

On other hand, in " abstract " form,

A.B = AB cos θ = $\sqrt{2}\,\sqrt{2}$ Cos θ = 2 Cos θ

Therefore Cos θ = $^1/_2$ or θ= 60°

$\hat{z}$ $\hat{y}$

$\hat{x}$