Pseudo Force

In a situation where the observer is on accelerated frame of reference, the observer will measure acceleration on another mass without any external force. This invalidate Newton's law of motion. So the concept of pseudo force is introduced to modify newtons equations for accelerated frames.

If a_0 is acceleration of observer and he measures the pseudo force F_s on rest mass m, the magnitude of pseudo $F_s = ma_0$ and its direction is opposite to direction of observer.

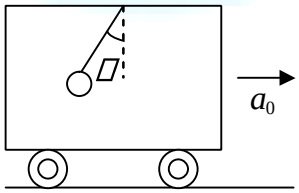
Let us understand the concept of pseudo force from following example:

Consider two observers A and B moving with acceleration $a\hat{i}$ and $-a\hat{i}$ respectively with respect to mass m then observer A measure pseudo force ma_A in $-\hat{i}$ direction and observer B measure pseudo force ma_B in $\hat{i}$ directions respectively. Concept of pseudo force makes Newton's laws and motion valid in non-inertial frames.

Example: A pendulum is hanging from the ceiling of a car having an acceleration a_0 with respect to the road. Find the angle made by the string with the vertical.

Solution:

The situation is shown in figure. Suppose the mass of the bob is m and the string makes an angle θ with the vertical. We shall work from the car frame. This frame is noninertial as it has an acceleration $\vec{a}_0$ with respect to an inertial frame (the road). Observer attached to car will observe pseudo force ma_0 on the bob of pendulum which is in opposite direction of acceleration of car.



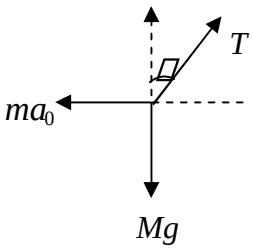
Take the bob as the system.

The forces are:

(a) T along the string, by the string

(b) mg downward, by the earth

(c) ma_0 towards left (pseudo force).



The free body diagram is shown in figure. As the bob is at rest (remember we are discussing the motion with respect to the car) the force in (a),(b) and (c) should add to zero. Take X - axis along the forward horizontal direction and Y -axis along the upward vertical direction. The components of the forces along the X - axis give

$$T\sin\theta - ma_0 = 0 \text{ or } T\sin\theta = ma_0 \qquad \text{(i)}$$

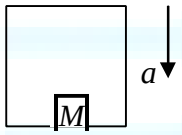
and the components along the Y -axis give

$$T\cos\theta - mg = 0 \text{ or } T\cos\theta = mg. \qquad \text{(ii)}$$

Dividing (i) by (ii) $\tan\theta = a_0/g$

Thus, the string makes an angle $\tan^{-1}[a_0/g]$ with the vertical.

Example: With what acceleration ' a ' should the box of figure descend so that the block of mass M exerts a force $Mg/4$ on the floor of the box?



Solution: The block is at rest with respect to the box which is accelerated with respect to the ground. If observer is attached to box he will attached to accelerated frame which is moving in down ward direction .the observer will measure the Pseudo force in upward direction with magnitude Ma

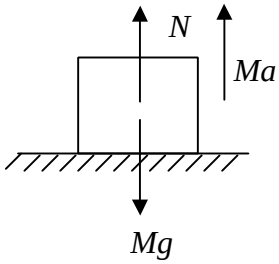
(a) Mg downward (by the earth) and

(b) N upward (by the floor).

(c) Ma Pseudo force in upward direction

The equation of motion of the block is, therefore $Mg - N - Ma = 0$

If $N = \frac{Mg}{4}$, the above equation gives $a = \frac{3g}{4}$.



The block and hence the box should descend with an acceleration $\frac{3g}{4}$.