Of Quarks and Gluons...

Even though much of nuclear physics is concerned with point nucleons interacting through a nuclear force, this picture is at best a decent approximation.

As you are all aware, the nucleon is not a fundamental particle in the normal sense of the word, but all strongly interacting matter is made up of quark and gluons. In this part of the course we shall look at the effects of substructure on nuclear physics. In other words, we are thus concentrating on the Quantum Chromo Dynamics (QCD) part of the Standard Model, Fig. 2.1.

2.1 Principles of QCD

There are some basic principles that summarize QCD

- Confinement:
  Quarks and gluons can not be liberated: they interact more strongly at lower energies

- Asymptotic freedom: The higher the energy we probe a strong-force system at, the more the response is like a system of free particles

- Self-interactions: The force carriers (gluons) interact amongst themselves. Can we have glueballs?

- Colour Charge: Quarks carry a colour charge (red, green, blue); gluons charge-anticharge, but not neutral.

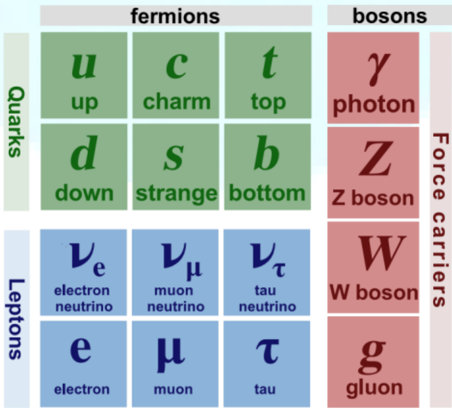

Figure 2.1: The particles making up the standard model. We have not included the putative Higgs particle.

- Flavour symmetry: Quarks come in flavours linked to the families: 6 flavours in 3 families

- Chiral symmetry breaking: the symmetry of the theory for light quarks (between left and righthanded quarks) is broken in vacuum

2.2 Essentials of QCD

Let us summarise the basics of QCD, and then try and capture those sing some simple models

1. flavoured and coloured quarks interacting through gluons

   - The known number of flavours $N_f = 6$ (in three families of two), but usually we concentrate on the light flavours. Depending on the situation that is 2 (*u* and *d*) or 3 (also including *s*)
   - The number of colours is exactly equal to $N_c = 3$.
   - gluons are "flavourless", and carry a colour-anticolour index, excluding the scalar combination, which gives $9 - 1 = 8$ allowed combinations.

2. Light quarks in the basic theory are really light

   - The "current quark masses" of *u* and *d* are in the order of 3 MeV/c^2
   - The "current quark masses" of *s* quark is about 100 MeV/c^2

3. Relativity plays an important role. Relativistic fermions satisfy the Dirac wave equation

$$(i\hbar c\,\boldsymbol{\alpha}\cdot\nabla+\beta mc^2)\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t) \qquad (2.1)$$

   where we require that if we repeat the left hand operation we get the standard relativistic energymomentum relation,

$$(i\hbar c\,\boldsymbol{\alpha}\cdot\nabla+\beta mc^2)^2 = (c^2(i\hbar\nabla)^2 + mc^2)^2 = E^2. \qquad (2.2)$$

   This last relation requires $\alpha_{1,2,3}$ and β to be four by four matrices. That sounds rather esoteric, but means that ψ has four components. We shall interpret this as a factor of 2 from spin up or down, and a second 2 from particle or antiparticle: Since the Dirac equation is linear, we can either have the positive or negative solution for E in the relation $E^2 = p^2c^2 + m^2c^4$.

   All negative energy states are assumed to be filled ("the Dirac sea" in analogy with the Fermi sea), and holes in the Dirac sea are interpreted as antiparticles (in our case antiquarks); particles in the positive energy states are assumed to be quarks.

4. For a massless quark spin is either parallel or antiparallel to the motion: Chiral symmetry Since particles of zero mass "move with the speed of light", the direction of movement is a Lorentz invariant. Thus it makes sense to define our basis states to be the so-called chiral (handed) states, where the spin of the electron is either parallel or anti-parallel to the direction of motion, rather than up or down. There is a symmetry that transforms the right-handed into left-handed particles: Chiral symmetry

5. Chiral symmetry breaking and structure of the vacuum
   Chiral symmetry is only present in the resonance spectrum if the vacuum itself is chirally symmetric. A famous mechanism called "spontaneous symmetry breaking" ensures this is not the case.

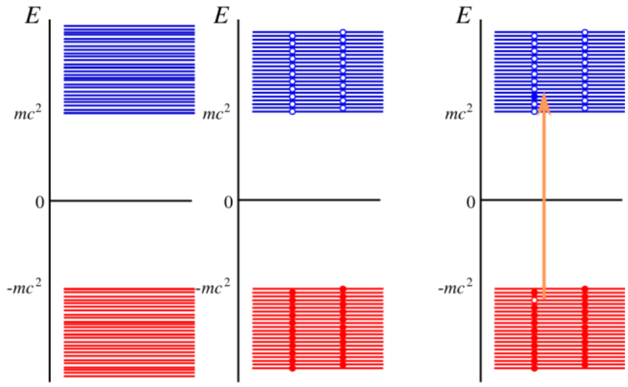

Figure 2.2: The relativistic spectrum on the left. It is interpreted as filed and empty states as seen in the middle; the excitation of an electron from the filed to empty states is interpreted as the excitation of a particle-hole pair.

2.3 Spontaneous symmetry breaking

The model for the disappearance of chiral symmetry is spontaneous symmetry breaking. This occurs when a system has a set of symmetries, but the vacuum state is not symmetric. In that case we loose some of the more obvious consequences of the symmetry. It is a phenomenon that occurs in many situations: the most familiar one is probably a magnet, where rotational symmetry is broken sinze all microscopic magnets order to point in the same direction.

A common example to help explain this phenomenon is a ball on top of a hill. This ball is in a completely symmetric state: a small movement in any direction is equivalent. However, its position is unstable: the slightest perturbing force will cause the ball to roll down the hill in some particular direction. At

that point, symmetry has been broken because we have selected one directions to roll along!

In particle physics we must look at "field theory" describing the physics in terms of a space and time dependent field. This field has a certain shape decribing the vacuum, and the familiar particles are described by small time-dependent wiggles in the vauum field. The symmetry breaking of the vauum is caused by the potential energy of the field. An example of a potential is illustrated in Fig. 2.3,

$$V(\varphi) = -10|\varphi|^2 + |\varphi|^4$$

This potential has many possible (vacuum states) given by

$$\varphi = \sqrt{5}e^{i\vartheta}$$

for any real ϑ between 0 and 2π. The system also has an unstable vacuum state corresponding to $\varphi = 0$. This symmetry is called "$U(1)$" the group of complex phases. It corresponds to a choice of phase of φ. However, once the system falls into one specific stable vacuum state (corresponding to a choice of ϑ) this symmetry will be lost or spontaneously broken. There is still a crucial remnant of the symmetry: if we look for changes in the field along the bottom of the valley, we see that they do not change the energy. Such fluctuation give rise to a massless (i.e., $m = 0$) mode called a "Goldstone mode". Fluctuations in the radial direction take energy, and thus correspond to massive modes. The appearance of Goldstone modes is the smoking gun for spontaneous symmetry breaking.

In the case of QCD the basic model sketched above holds for *light* quarks: for a zero-mass fermion we can define our Dirac basis states as chiral (handed) states. Left-handed states occur when the spin of a particle is parallel to its direction of motion; right-handed when they are antiparallel. In the case we consider only the *u* and *d* quarks (since they are indeed approximately massless), chiral symmetry is the statement of how we can mix left and right-handed states of quarks and antiquarks without changing the physics: there are two rotational symmetries associated with this, and one of those two, called $SU(2)_V$

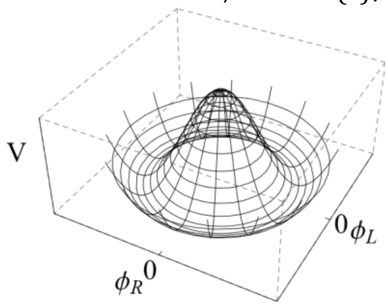

Figure 2.3: Spontaneous symmetry breaking.

(the vector $SU(2)$) is spontaneously broken. The number of flat directions at the bottom of the potential landscape is now three, and we interpret the three pions as the three Goldstone modes. Pions are clearly not massless (their mass is about 138 MeV/c^2) but they do come in an isospin 1 multiplet, so there are three of them. Also their mass is much lower than any other state (by about a factor of 4), so in some

sense they are approximately massless. We do understand qualitatively and quantitatively how to derive the pion mass from the quark mass, the Gell-Mann–Oakes–Renner relation, so this connection is pretty watertight.

2.3.1 Symmetries

2.3.1.1 Isospin

The basic symmetry due to QCD that plays a crucial role in nuclear physics is the isotopic spin symmetry (isospin). Basically, if we look at the nucleon and the proton masses they are remarkably close,

$$M_p = 939.566 \text{ MeV/c}^2 \qquad M_n = 938.272 \text{ MeV/c}^2,$$

which is a hint of possible symmetry! If we further study the mass of the lightest meson, the pion, we see that these come in three charge states, and once again their masses are remarkably similar,

$$M_{\pi^+} = M_{\pi^-} = 139.567 \text{ MeV/c}^2, \qquad M_{\pi^0} = 134.974 \text{ MeV/c}^2.$$

Most importantly the Interactions between nucleons (p and n) is independent of charge, they only depend on the nucleon character of these particles: “the strong interactions see only one nucleon and one pion”.

In that case a continuous transformation between the neutron an a proton, and between the pions is a symmetry– the physics is unchanged

The symmetry that was proposed (by Wigner) is an internal symmetry like spin symmetry called isotopic spin or isospin. A rotation of spin and angular momentum is linked to a rotation of space; isospin is an abstract rotation in isotopic space, and leads to states with isotopic spin I = 1/2, 1, 3/2, ... Define third component of isospin of a “fundamental” particle as

$$Q = e(I_3 + B),$$

where B is the baryon number (B = 1 for n, p, 0 for π).

We thus find

	B	Q/e	I	I_3
n	1	0	1/2	−1/2
p				

1 1 1/2 1/2

π 0 −1 1 −1 0 0 0 1 0 π

+0 1 1 1

π

2.3. SPONTANEOUS SYMMETRY BREAKING

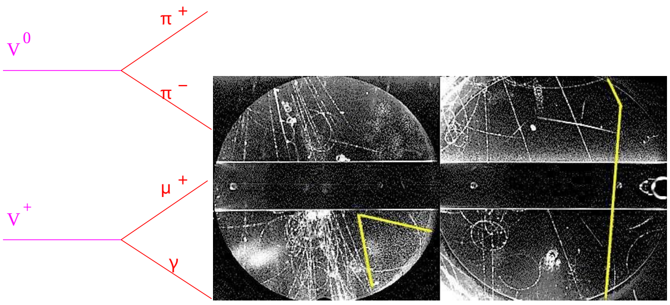

Figure 2.4: Left: decay of a neutral kaon. The neutral kaon leaves no track, but a “V” of tracks appears when it decays into two charged pions . The right-hand image shows the decay of a charged kaon into a muon and a neutrino. The decay occurs where the track appears to bend to the left abruptly: The neutrino is invisible.

Notice that the energy levels of these particles are split by an electric force, as ordinary spins split under a magnetic force.

2.3.1.2 Strange particles

In 1947 Rochester and Butler (Manchester) observed new particles in cosmic ray events. These particles came in two forms: a neutral one that decayed into a π^+ and a π^-, and a positively charge one that decayed into a μ^+ and a neutrino,

These particles have long lifetimes. The decay times due to strong interactions are very fast, of the order of an fs (10^{-15} s). Decay time of the K mesons is about 10^{-10} s, (weak decay). Many similar particles, collectively know as strange particles. These are typically formed in pairs, e.g., $\pi^+ + p \to \underbrace{\Lambda^0}_{\text{baryon}} + \underbrace{K^0}_{\text{meson}}$:

Implies additive conserved quantity called strangeness. If we assume that the Λ^0 has strangeness −1, and the K^0 +1,

$$\pi^+ + p \to \Lambda^0 + K^0$$

$$0 + 0 = -1 + 1$$

The weak decay

$$\Lambda^0 \to \pi^- + p$$
$$-1 \neq 0 + 0$$

does not conserve strangeness (but it conserves baryon number). Is found to take much longer, about 10^{-10} s.

We can accommodate this quantity in the charge-isospin relation,

$$Q = e\left(I_3 + \frac{B + S}{2}\right)$$

Clearly for $S = -1$ and $B = 1$ we get a particle with $I_3 = 0$. This allows us to identify the Λ^0 as an

$I = 0$, $I_3 = 0$ particle, which agrees with the fact that there are no particles of different charge and a similar mass and strong interaction properties.

The kaons come in three charge states $K^{\pm}$, K^0 with masses $m_{K^{\pm}} = 494$ MeV, $m_{K^0} = 498$ MeV. Further analysis shows that the the K^+ is the antiparticle of K^-, but K^0 is not its own antiparticle! So we need four particles, and the assignments are $S = 1$, $I = 1/2$ for K^0 and K^-, $S = -1$, $I = 1/2$ for K^+ and K^{-0}.

It was argued by Gell-Mann and Ne'eman in 1961 that a natural extension of isospin symmetry would be an SU(3) symmetry. One of the simplest representations of SU(3) is 8 dimensional. A particle with

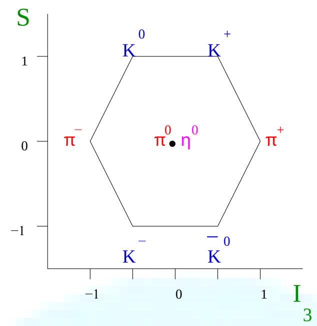

Figure 2.5: Octet of mesons

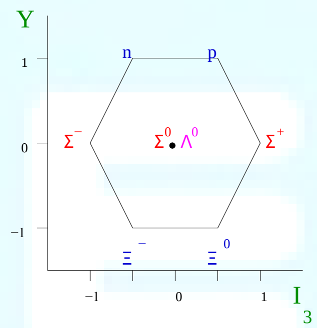

Figure 2.6: Octet of nucleons

$I = I_3 = S = 0$ is missing. Such a particle is known, and is called the η^0. The breaking of the symmetry can be seen from the following mass table:

$m_{\pi^{\pm}} = 139$ MeV $m_{\pi^0} = 134$ MeV

$m_{K^{\pm}} = 494$ MeV $M_{\overset{(-)}{K0}} = 498$ MeV

$m_{\eta^0} = 549$ MeV

In order to have the scheme make sense we need to show its predictive power. This was done by studying the nucleons and their excited states. Since nucleons have baryon number one, they are labelled with the "hyper-charge" Y, $Y = (B + S)$. The nucleons form an octet with the single-strangeness particles Λ and σ and the doubly-strange cascade particle Ξ.

$M_n = 938$ MeV

$M_p = 939$ MeV

$M_{\Lambda^0} = 1115$ MeV

$M_{\Sigma^+} = 1189$ MeV

$$M_{\Sigma^0} = 1193 \text{ MeV}$$

$$M_{\Sigma^-} = 1197 \text{ MeV}$$

$$M_{\Xi^0} = 1315 \text{ MeV}$$

$$M_{\Xi^-} = 1321 \text{ MeV}$$

All these particles were known before the idea of this symmetry. The first confirmation came when studying the excited states of the nucleon. Nine states were easily incorporated in a decuplet, and the

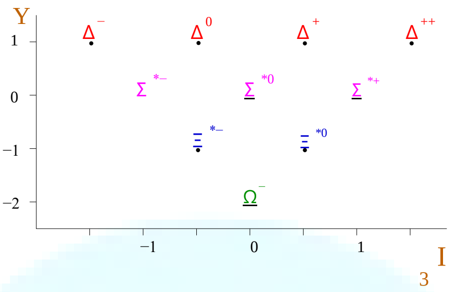

Figure 2.7: decuplet of excited nucleons

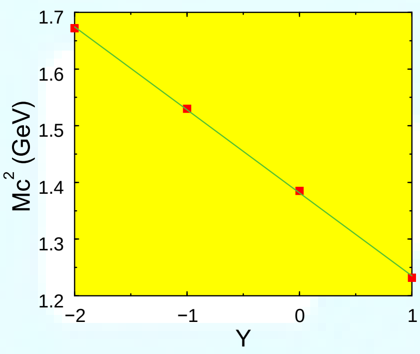

Figure 2.8: A linear fit to the mass of the decuplet

tenth state (the Ω^-, with strangeness -3) was predicted. It was found soon afterwards at the predicted value of the mass.

The masses are

$$M_{\Delta} = 1232 \text{ MeV}$$

$$M_{\Sigma^*} = 1385 \text{ MeV}$$

$$M_{\Xi^*} = 1530 \text{ MeV}$$

$$M_{\Omega} = 1672 \text{ MeV}$$

(Notice almost that we can fit these masses as a linear function in *Y*. This was of great help in finding the Ω.)

2.4 The quark model of strong interactions

Once the eightfold way (as the SU(3) symmetry was poetically referred to) was discovered, the race was on to explain it.

The decuplet and two octets occur in the product **3** ⊗ **3** ⊗ **3** = **1** ⊕ **8** ⊕ **8** ⊕ **10**. Introduce a quark in three "flavours" called up, down and strange (*u*, *d* and *s*, respectively). Assume that the baryons are made from three of such particles, and the mesons from a quark and anti-quark (**3** ⊗ **3¯** = **1** ⊕ **8**). Each quark carries one third a unit of baryon number:

2.4.0.3 Meson octet

Make all possible combinations of a quark and antiquark, apart from the scalar one $\eta^0 = u\bar{u} + d\bar{d} + c\bar{c}$. A similar assignment can be made for the nucleon octet, and the nucleon decuplet

Table 2.1: a

Quark	*f*	*S*	*Q/e*	*I*	I_3	*S*	*B*
Up	*u*	$\frac{1}{2}$					
Down	*d*		$+\frac{2}{3}$ $-\frac{1}{3}$ $-\frac{1}{3}$	$\frac{1}{2}$ $\frac{1}{2}$ 0	$+\frac{1}{2}$ $-\frac{1}{2}$ 0	0 0 -1	$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$
Strange	*s*	$\frac{1}{2}$ $\frac{1}{2}$					

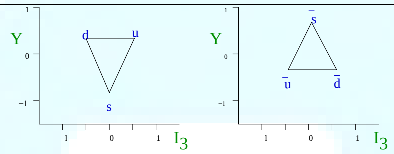

Figure 2.9: a

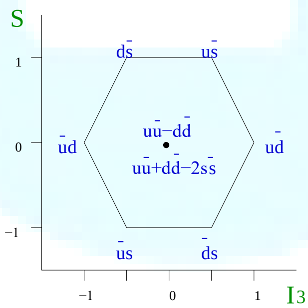

Figure 2.10: quark assignment of the meson octet

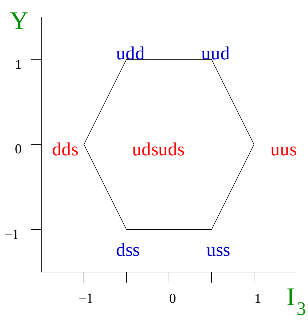

Figure 2.11: quark assignment of the nucleon octet

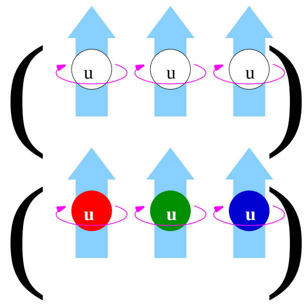

Figure 2.12: The Δ^{++} in the quark model.

Once we have three flavours of quarks, we can ask the question whether more flavours exists. At the moment we know of three generations of quarks, corresponding to three generations (pairs). These give rise to SU(4), SU(5), SU(6) flavour symmetries. Since the quarks get heavier and heavier, the symmetries get more-and-more broken as we add flavours.

Quark	label	spin	Q/e	mass (GEV/c^2)
Down	*d*			
Up	*u*			
Strange	*s*			
Charm	*c*		$-\frac{1}{3}$ $+\frac{2}{3}$ $-\frac{1}{3}$ $+\frac{2}{3}$ $-\frac{1}{3}$ $+\frac{2}{3}$	0.35 0.35 0.5 1.5 4.5 93
Bottom	*b*			
Top	*t*			

2.4.1 Colour symmetry

So why don't we see fractional charges in nature? If quarks are fermions– spin 1/2 particles– what about antisymmetry? Investigate the Δ^{++}, which consists of three *u* quarks with identical spin and flavour and symmetric spatial wavefunction,

$$\psi_{\text{total}} = \psi_{\text{space}} \times \psi_{\text{spin}} \times \psi_{\text{flavour}}.$$

This would be symmetric under interchange, which is unacceptable. Assume that there is an additional quantity called colour, and take the colour wave function to be antisymmetric:

$$\psi_{\text{total}} = \psi_{\text{space}} \times \psi_{\text{spin}} \times \psi_{\text{flavour}} \times \psi_{\text{colour}}$$

Assume that quarks come in three colours. Another $SU(3)$ symmetry, linked to the gauge symmetry of strong interactions, QCD. New question: why can't we see coloured particles?

The only particles that have been seen are colour neutral ("white") ones. This leads to the assumption of confinement – We cannot liberate coloured particles at "low" energies and temperatures!

2.4.2 Feynman diagrams

There are two key features that distinguish QCD from QED:

1. Quarks interact more strongly the further they are apart, and more weakly as they are close by – asymptotic freedom.

2. Gluons interact with themselves

The first point can only be found through detailed mathematical analysis. It means that free quarks can't be seen, but at high energies quarks look more and more like free particles. The second statement makes QCD so hard to solve. The gluon comes in 8 colour combinations (since it carries a colour and anti-colour index, minus the scalar combination). The relevant diagrams are sketched below. Try to work out yourself how we can satisfy colour charge conservation!

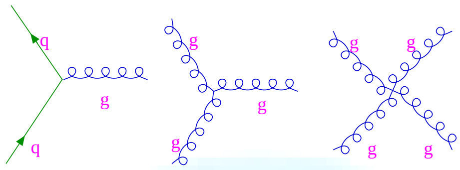

Figure 2.13: The basic building blocks for QCD Feynman diagrams

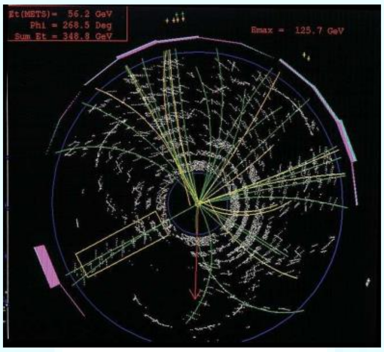

Figure 2.14: Top quark and anti top quark pair decaying into jets, visible as collimated collections of particle tracks, and other fermions in the CDF detector at Tevatron.

2.4.3 Jets and QCD

One way to see quarks is to use the fact that we can liberate quarks for a short time, at high energy scales. One such process is $e^+e^- \to q\bar{q}$, which use the fact that a photon can couple directly to $q\bar{q}$. The quarks don't live very long and decay by producing a "jet" a shower of particles that results from the decay of the quarks. These are all "hadrons", mesons and baryons, since they must couple through the strong interaction. By determining the energy in each if the two jets we can discover the energy of the initial quarks, and see whether QCD makes sense.

2.5 Experimental evidence

At this point it makes sense to look back at the evidence for quantum chromo-dynamics (QCD), the theory of strong interactions. This is one few theories where we have never directly seen the basic particles in the theory. We can try to use a probe whose wave length (optical or de Broglie) is small enough to resolve the sub-structure of the nucleon. The classical experiment [ref], first performed at SLAC, did exactly that, see Fig. 2.15.

Here we scatter an electron, or other lepton, off a nucleon (in its simples form a proton, i.e., hydrogen nucleus). This process is electromagnetic, in other words it is mediated by a virtual[1] photon. If the momentum transfer is

[1] Which doesn't satisfy the relativistic energy-momentum relationship $p^2c^2 - E^2 = 0$

large enough, we would normally expect to be able to resolve the individual quarks. We take the typical size of a proton to be 1 fm = 10^{-15} m, and assume that the size of a quarks is at least a factor of 100 smaller. The momentum transfer should be of the order of the de Broglie wave length, $pc\lambda \approx \hbar$, or

$$pc \approx \hbar/\lambda = 20 \times 10^3 \text{ MeV} = 20 \text{ GeV} = 0.2 \text{ TeV}.$$

For such highly relativistic momentum, this would also be the electron's energy.

The idea underlying such an approach is at least as old as Rutherford's analysis of the classical Geiger and Marsden experiment. Remember that initially the atom was thought of using J.J. Thomson's "plum

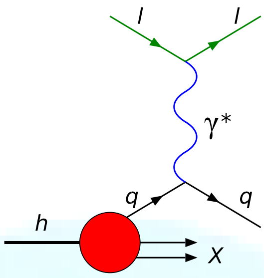

Figure 2.15: The process of deep-inelastic scattering

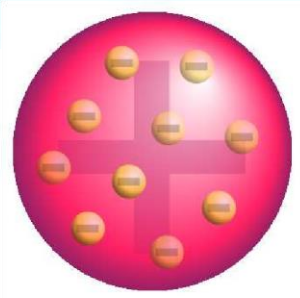

Figure 2.16: The plum pudding model

pudding" model, Fig. 2.16 a homogeneously positively charged sphere with electrons (the plums) at certain positions, maybe in a shell/shells [3]. This predicts a rather weak scattering of other charged particles, such as the α particle, by such atoms. After initial exciting experiments by Geiger and Marsden [5], which showed Thompson was wrong, Rutherford [4] analysed this process in terms of Rutherford scattering from a tiny positively charged nucleus, mainly ignoring the weak effect of the electrons. In 1913, Geiger and Marsden [6] produced a data set that shows that Rutherford's analysis was likely correct, see Fig. 2.17.

This figure shows an excess of events at large scattering angles which is not predicted by Thomson. The technique to derive the cross section is illustrative, and Rutherford's derivation is remarkable simple [I'll leave it as an exercise to derive the first part]; from the expression relating the impact parameter b to the scattering angle ϑ,

$$b = \frac{ZZ_0 e^2}{4\pi\epsilon_0 E} \cot \vartheta/2$$

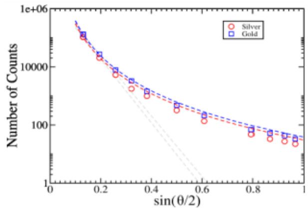

Figure 2.17: The original data from Ref. [6] compared with the Rutherford scattering formula . The grey line is an extrapolation from the small angle data, and corresponds roughly to what we would expect in Thomson's model.

we can derive the expression for the differential cross-section

$$d\sigma = -2\pi b db$$

$$= 2\pi \frac{ZZ0e2\ 2}{4\pi e0E} \cot \vartheta/2 \frac{1}{2} \frac{\sec 2\ \vartheta/2}{d} \vartheta$$

$$= \left(\frac{ZZ'e^2}{4\pi\epsilon_0 E}\right)^2 \sec^4\theta \ /24\pi sin\vartheta/2 \cos\vartheta/2 d\vartheta/2$$

$$= \left(\frac{ZZ'e^2}{4\pi\epsilon_0 E}\right)^2 \sec^4\theta/2(4\pi sin\theta d\theta)$$

$$= \left(\frac{ZZ'e^2}{4\pi\epsilon_0 E}\right)^2 \sec^4\theta/2 d\Omega$$

$$= \left(\frac{ZZ'\alpha\hbar c}{E}\right)^2 \sec^4\theta/2 d\Omega.$$

This formula is identical for the quantum mechanical elastic scattering of two spinless particles. The last line is particularly useful since it is independent of the electromagnetic units used; many papers use Gauss rather than SI units (where $4\pi e_0 = 1$).

In early accelerator driven high-energy physics, there were important attempts to try to do similar experiments by scattering electrons from a proton (or even a proton of a proton, but we shall not consider this here). Once again, we shall ignore centre-of-mass effects, since an electron is much lighter than a proton.

Of course we shall have to take into account relativity, which leads to a modification of the cross section, essentially only due to the effect of the spin of the electron. The standard formula is due to Mott [**?**]. The derivation, using the Dirac equation, can be found in text books (e.g., [**?**], pg 173/174).

$$\frac{d\sigma}{d\Omega_{\text{Mott}}} = \frac{Z\alpha)^2 E^2}{} \left(- v^2 \quad 2\ \frac{\theta}{} \right) \quad 4k^2 \sin^4\frac{\theta}{4} \qquad {}^2{}_1 \qquad \sin$$

,

with $k = \left|k_f\right| = |k_i|,\ v = \frac{k}{E}$ [Units... hc^-]. If we scatter electrons from an extended charged object elastically, we get

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega_{\text{Mott}}} |F(q)|^2$$

Here the form-factor $F(\boldsymbol{q})$ is the Fourier transform of the charge density,

$$F(\boldsymbol{q}) = \quad \rho(\boldsymbol{r})e^{i\boldsymbol{q}\cdot\boldsymbol{r}}d^3r.$$

We are actually not able to directly measure the Mott cross section in *ep* scattering–the proton picks up momentum as well, and has more structure than we allow for above (especially, since the proton has a magnetic moment there are magnetic as well as electric interactions).

We define two basic variables v and x (after Broken)

$$2Mc^2v = W^2c^4 + Q^2c^2 - M_p^2c^4, \quad x = Q^2/(2Mv).$$

Here W is the invariant mass (see appendix) of the hadron after scattering (since it could have been excited internally), Q^2 is the four-momentum transfer in the reaction (again see appendix), and M_p is the proton mass. Note that v has the dimension of energy, and that [exercise] in the proton's rest frame $v = E - E^0$, the energy transfer in the reaction. As we shall see in a minute, it is x we are really interested in!

2.5. EXPERIMENTAL EVIDENCE

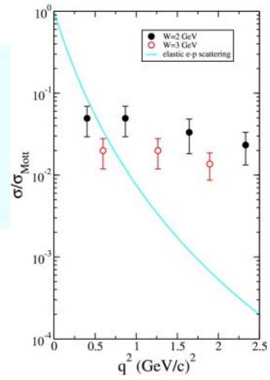

Figure 2.18: The 6° data from Ref. [7, 8] compared with the elastic scattering cross section

Bjorken first proposed showed that in inelastic (i.e., the outgoing energy E^0 is not equal to E) relativistic scattering of a proton the most general behaviour of the cross section is given by

$$\frac{d^2\sigma}{d\Omega dE^0} \quad \frac{\alpha^2}{2E} \quad \frac{1}{2\sin^4(\vartheta/2)v}\left[\cos^2(\vartheta/2)F_2(x,Q^2) + \sin^2(\theta/2)\frac{2Q^2}{xM^2c^2}F_1(x,Q^2)\right].$$