Electronics -Part 2

Filters

Filters are electronic circuits that allow certain frequency components and / or reject some other.

Active Filters

Active filters are the electronic circuits, which consist of active element like op-amp(s) along with passive elements like resistor(s) and capacitor(s).

Types of Active Filters

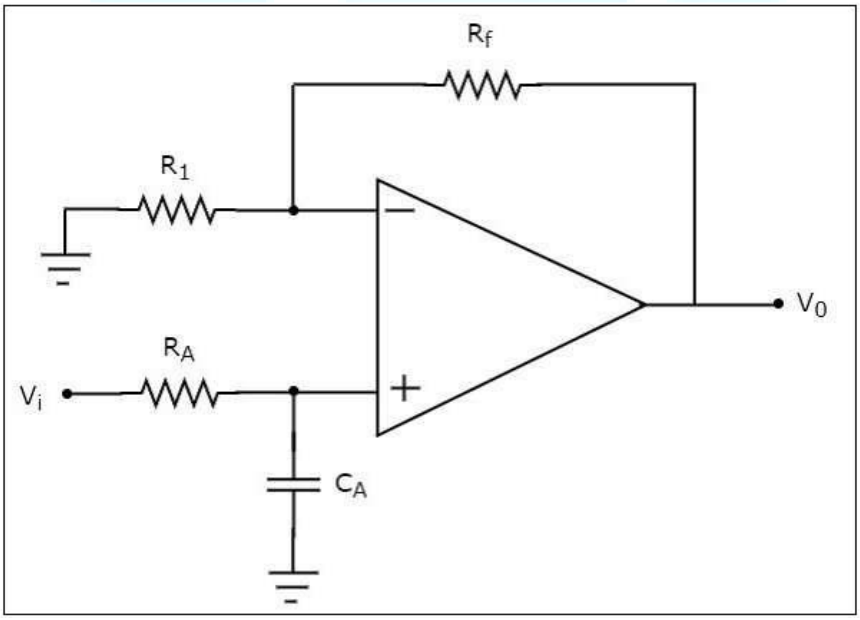
Active filters are mainly classified into the following **four types** based on the band of frequencies that they are allowing and / or rejecting −

- Active Low Pass Filter
- Active High Pass Filter
- Active Band Pass Filter
- Active Band Stop Filter

Active Low Pass Filter

If an active filter allows (passes) only **low frequency** components and rejects (blocks) all other high frequency components, then it is called as an **active low pass filter**.

The **circuit diagram** of an active low pass filter is shown in the following figure −



The electric network, which is connected to the non-inverting terminal of an op-amp is a **passive low pass filter**. So, the input of a non-inverting terminal of an op-amp is the output of a passive low pass filter.

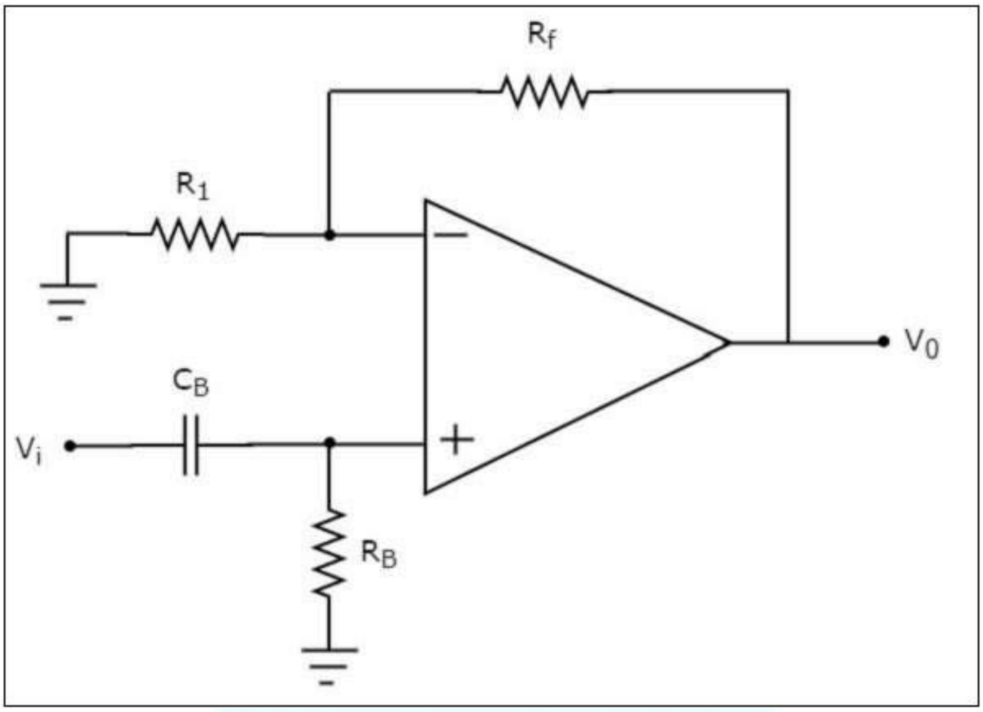
Observe that the above circuit resembles a **non-inverting amplifier**. It is having the output of a passive low pass filter as an input to the non-inverting terminal of op-amp.

Hence, it produces an output, which is $(1+R_fR_1)$ times the input present at the non-inverting terminal.
We can choose the values of Rf and R1 suitably in order to obtain the **desired gain** at the output. Suppose, if we consider the resistance values of Rf and R1 as zero ohms and infinity ohms, then the above circuit will produce a **unity gain** low pass filter output.

Active High Pass Filter

If an active filter allows (passes) only **high frequency** components and rejects (blocks) all other low frequency components, then it is called an **active high pass filter**.

The **circuit diagram** of an active high pass filter is shown in the following figure –



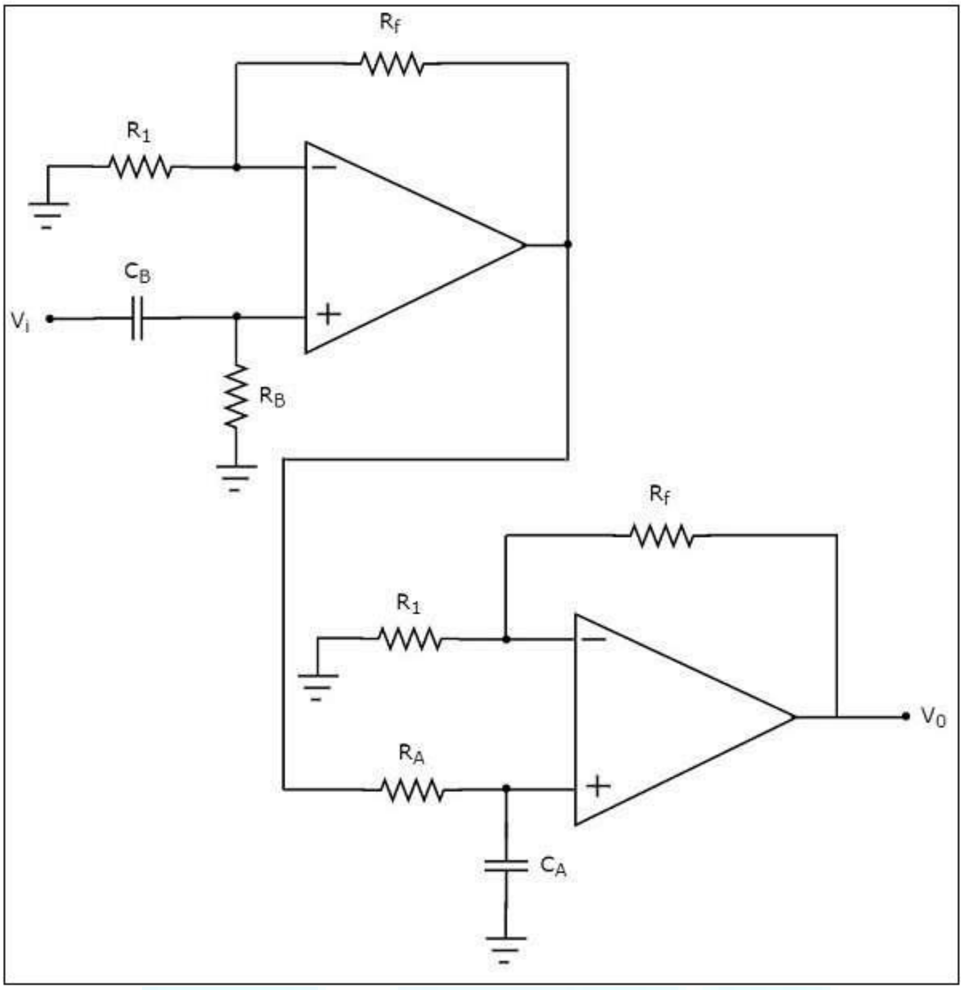
The electric network, which is connected to the non-inverting terminal of an op-amp is a **passive high pass filter**. So, the input of a non-inverting terminal of op-amp is the output of passive high pass filter.

Now, the above circuit resembles a **non-inverting amplifier**. It is having the output of a passive high pass filter as an input to non-inverting terminal of op-amp. Hence, it produces an output, which is $(1+R_fR_1)$ times the input present at its non-inverting terminal.
We can choose the values of Rf and R1 suitably in order to obtain the **desired gain** at the output. Suppose, if we consider the resistance values of Rf and R1 as zero ohms and infinity ohms, then the above circuit will produce a **unity gain** high pass filter output.

Active Band Pass Filter

If an active filter allows (passes) only one band of frequencies, then it is called as an **active band pass filter**. In general, this frequency band lies between low frequency range and high frequency range. So, active band pass filter rejects (blocks) both low and high frequency components.

The **circuit diagram** of an active band pass filter is shown in the following figure



Observe that there are **two parts** in the circuit diagram of active band pass filter:

i. The first part is an active high pass filter,

ii. The second part is an active low pass filter.

The output of the active high pass filter is applied as an input of the active low pass filter. That means, both active high pass filter and active low pass filter are **cascaded** in order to obtain the output in such a way that it contains only a particular band of frequencies.

The **active high pass filter**, which is present at the first stage allows the frequencies that are greater than the **lower cut-off frequency** of the active band pass filter. So, we have to choose the values of RB and CB suitably, to obtain the desired **lower cut-off frequency** of the active band pass filter.

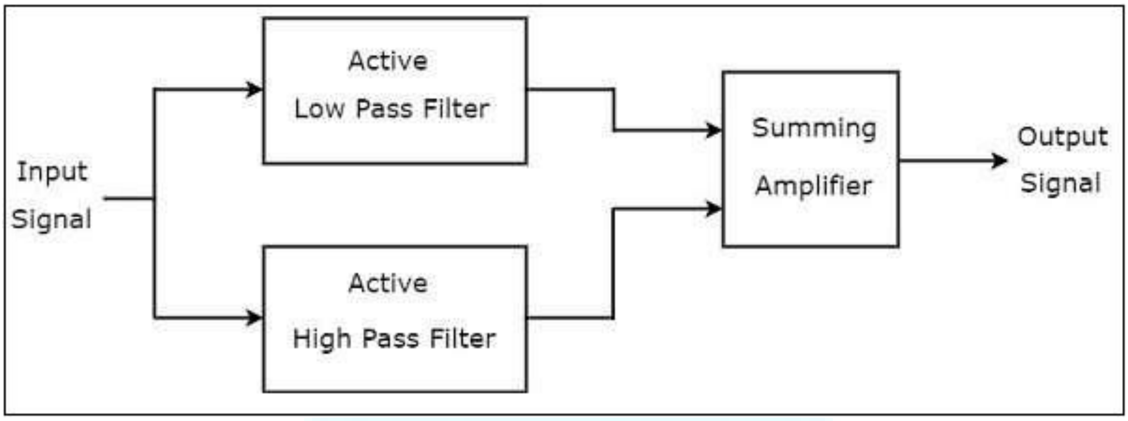
Similarly, the **active low pass filter**, which is present at the second stage allows the frequencies that are smaller than the higher cut-off frequency of the active band pass filter. So, we have to choose the values of RA and CA suitably in order to obtain the desired **higher cut-off frequency** of the active band pass filter.

Hence, the circuit in the diagram discussed above will produce an active band pass filter output.

Active Band Stop Filter

If an active filter rejects (blocks) a particular band of frequencies, then it is called as an **active band stop filter**. In general, this frequency band lies between low frequency range and high frequency range. So, active band stop filter allows (passes) both low and high frequency components.

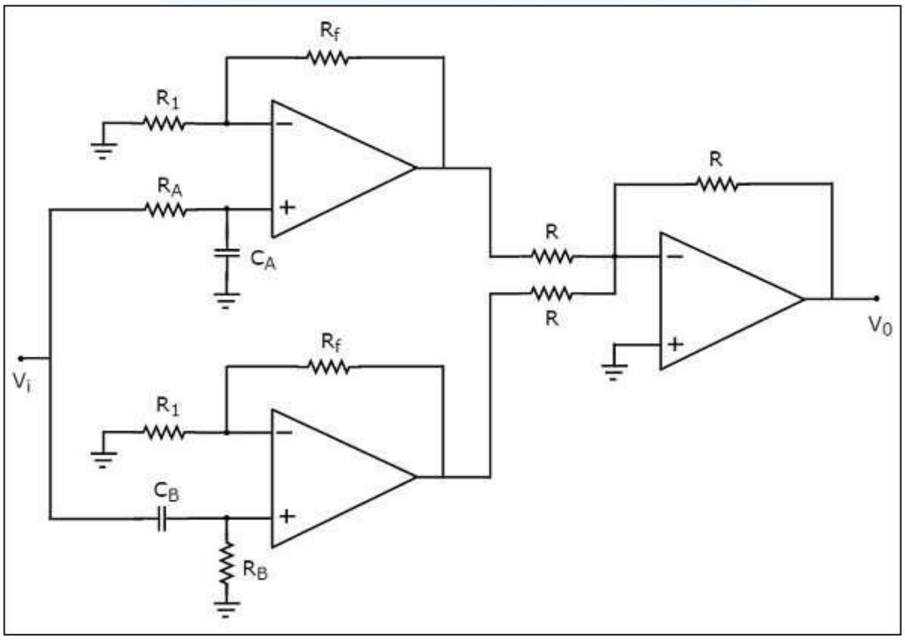
The **block diagram** of an active band stop filter is shown in the following figure –



Observe that the block diagram of an active band stop filter consists of two blocks in its first stage: an active low pass filter and an active high pass filter. The outputs of these two blocks are applied as inputs to the block that is present in the second stage. So, the **summing amplifier** produces an output, which is the amplified version of sum of the outputs of the active low pass filter and the active high pass filter.

Therefore, the output of the above block diagram will be the **output of an active band stop**, when we choose the cut-off frequency of low pass filter to be smaller than cut-off frequency of a high pass filter.

The **circuit diagram** of an active band stop filter is shown in the following figure –



Butterworth Filter

A Butterworth filter is a type of signal processing filter designed to have a frequency response as flat as possible in the pass band. Hence the Butterworth filter is also known as "maximally flat magnitude filter". It was invented in 1930 by the British engineer and physicist Stephen Butterworth in his paper titled "On the Theory of Filter Amplifiers".

The frequency response of the Butterworth filter is flat in the passband (i.e. a bandpass filter) and roll-offs towards zero in the stopband. The rate of roll-off response depends on the order of the filter. The number of reactive elements used in the filter circuit will decide the order of the filter.

The inductor and capacitor are reactive elements used in filters. But in the case of Butterworth filter only capacitors are used. So, the number of capacitors will decide the order of the filter.

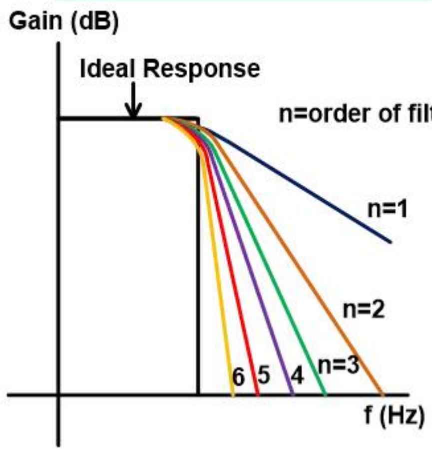
High pass filters are formed by simply interchanging the positions of the frequency determining components (resistors and capacitors) in the equivalent low pass filter.

Butterworth Low Pass Filter

In Butterworth filter, mathematically it is possible to get flat frequency response from 0 Hz to the cut-off frequency at -3dB with no ripple. If the frequency is more than the cut-off frequency, it will roll-off towards zero with the rate of -20 dB/decade for the first-order filter.

If you increase the order of the filter, the rate of a roll-off period is also increased. And for second-order, it is -40 dB/decade. The quality factor for the Butterworth filter is 0.707.

The below figure shows the frequency response of the Butterworth filter for various orders of the filter.



The generalized form of frequency response for n^{th}-order Butterworth low-pass filter is;

$$H(j\omega) = \frac{1}{\sqrt{1+\varepsilon^2(\frac{\omega}{\omega_C})^{2n}}}$$

Where,
n = order of the filter,
ω = operating frequency (passband frequency) of circuit
ωC = Cut-off frequency
ε = maximum passband gain = Amax

The below equation is used to find the value of ε.

$$H_1 = \frac{H_0}{\sqrt{1+\varepsilon^2}}$$

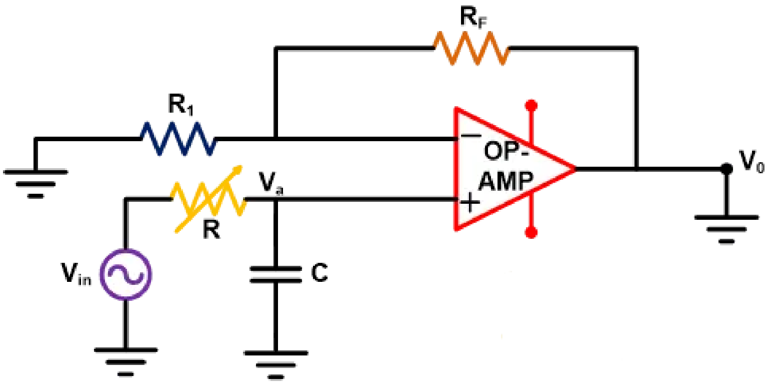
Where,

H_1 = minimum passband gain
H_0 = maximum passband gain

First-order Lowpass Butterworth Filter

The lowpass filter is a filter that allows the signal with the frequency is lower than the cut-off frequency and attenuates the signals with the frequency is more than cut-off frequency.

In the first-order filter, the number of reactive components is only one. The below figure shows the circuit diagram of the first-order lowpass Butterworth filter.



The low pass Butterworth filter is an active Low pass filter as it consists of the op-amp. This op-amp operates on non-inverting mode. Hence, the gain of the filter will decide by the resistor R1 and RF. And the cut-off frequency decides by R and C.

Now, if you apply the voltage divider rule at point Va and find the voltage across a capacitor. It is given as;

$$V_a = \frac{-jX_C}{R - jX_C} V_{in}$$

$$V_a = \frac{-j(\frac{1}{2\pi fC})}{R - j(\frac{1}{2\pi fC})} V_{in}$$

$$V_a = \frac{-j}{2\pi fRC - j} V_{in}$$

$$V_a = \frac{V_{in}}{1 - \frac{2\pi fRC}{j}}$$

$$V_a = \frac{V_{in}}{1 + j2\pi fRC}$$

Because of the non-inverting configuration of an op-amp,

$$V_0 = \left(1 + \frac{R_f}{R_1}\right) V_a$$

$$V_0 = \left(1 + \frac{R_f}{R_1}\right) \frac{V_{in}}{1 + j2\pi fRC}$$

$$\frac{V_0}{V_a} = \frac{A_f}{1 + j\frac{f}{f_c}}$$

Where,

$$A_f = 1 + \frac{R_F}{R_1}$$

A_f = Gain of filter in Passband

$$f_c = \frac{1}{2\pi RC}$$

f_c = Cut off Frequency

f = Operating Frequency

$$\frac{V_0}{V_a} = \left|\frac{V_0}{V_a}\right| \angle\phi$$

$$\left|\frac{V_0}{V_a}\right| = \frac{A_f}{\sqrt{1 + j\left(\frac{f}{f_c}\right)^2}}$$

$$\phi = -\tan^{-1}\left(\frac{f}{f_c}\right)$$

1. At very low frequency, f<<fc

   $$\left|\frac{V_0}{V_a}\right| \approx A_f(Constant)$$

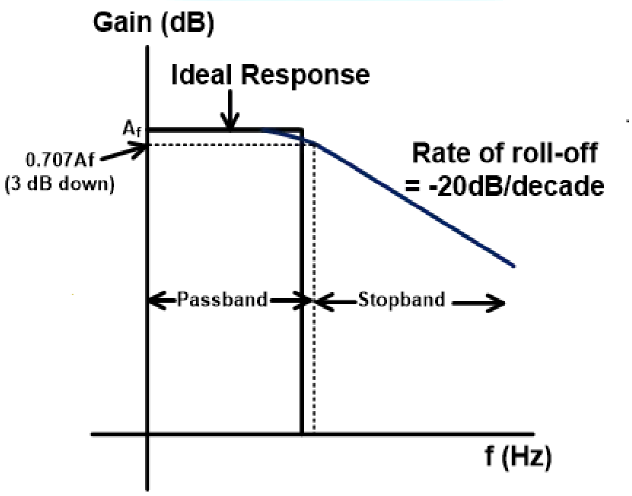
2. At cut-off frequency, f= fc

   $$\left|\frac{V_0}{V_a}\right| = \frac{A_f}{\sqrt{2}} = 0.707A_f$$

3. At high frequency, f> fc

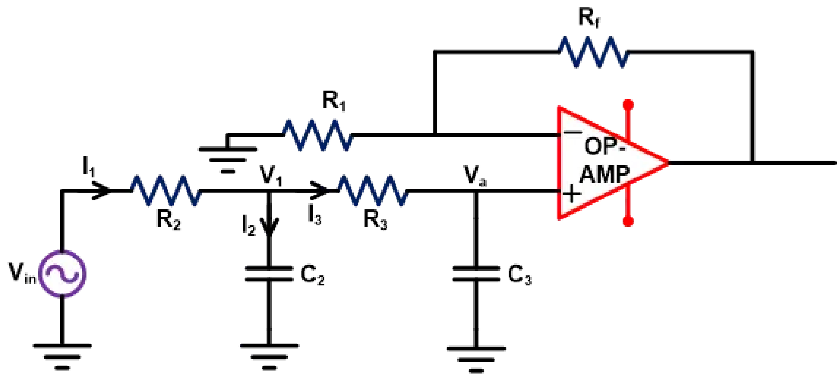
   $$\left|\frac{V_0}{V_a}\right| < A_f$$

The below figure shows the frequency response of first-order lowpass Butterworth filter.



Second-order Butterworth Filter

The second-order Butterworth filter consists of two reactive components. The circuit diagram of a second-order low pass Butterworth filter is as shown in the below figure.



In this type of filter, resistor R and RF are the negative feedback of op-amp. And the cut off frequency of the filter decides by R2, R3, C2, and C3.

The second-order lowpass Butterworth filter consists of two back-to-back connected RC networks. And RL is the load resistance.

First-order and second-order Butterworth filters are very important. Because we can get higher-order Butterworth filter by just cascading of the first-order and second-order Butterworth filters.

Let's analyse the circuit of second-order Butterworth filter,

Apply Kirchhoff's Current Law at point V1.

$$I_1 = I_2 + I_3$$

$$\frac{V_{in} - V_1}{R_2} = \frac{V_1 - V_0}{\frac{1}{sC_2}} + \frac{V_1 - V_a}{R_3}$$

Using potential divider rule at point Va

$$V_a = V_1 \left[\frac{\frac{1}{sC_3}}{R_3 + \frac{1}{sC_3}} \right]$$

$$V_a = V_1 \left[\frac{\frac{1}{sC_3}}{\frac{R_3 sC_3 + 1}{sC_3}} \right]$$

$$V_a = \frac{V_1}{1 + sR_3 C_3}$$

$$V_1 = V_a (1 + sR_3 C_3)$$

Put the value of V1 in equation-(1)

$$\frac{V_{in}-V_a(1+sR_3C_3)}{R_2}=\frac{V_a(1+sR_3C_3)-V_0}{\frac{1}{sC_2}}+\frac{V_a(1+sR_3C_3)-V_a}{\frac{1}{R_3}}$$

$$\frac{V_{in}}{R_2}-\frac{V_a(1+sR_3C_3)}{R_2}=\frac{V_a(1+sR_3C_3)}{\frac{1}{sC_2}}-\frac{V_0}{\frac{1}{sC_2}}+\frac{V_a(1+sR_3C_3)}{R_3}-\frac{V_a}{R_3}$$

$$\frac{V_{in}}{R_2}+\frac{V_0}{\frac{1}{sC_2}}=\frac{V_a(1+sR_3C_3)}{\frac{1}{sC_2}}+\frac{V_a(1+sR_3C_3)}{R_2}+\frac{V_a(1+sR_3C_3)}{R_3}-\frac{V_a}{R_3}$$

$$\frac{V_{in}}{R_2}+V_0sC_2=V_a\left[sC_2(1+sR_3C_3)+\frac{(1+sR_3C_3)}{R_2}+\frac{(1+sR_3C_3)}{R_3}-\frac{1}{R_3}\right]$$

$$\frac{V_{in}+V_0sC_2R_2}{R_2}=V_a\left[\frac{R_3R_2sC_2(1+sR_3C_3)+R_3(1+sR_3C_3)+R_2(1+sR_3C_3)-R_2}{R_2R_3}\right]$$

$$R_3(V_{in}+V_0sC_2R_2)=V_a\left[R_3R_2sC_2(1+sR_3C_3)+R_3(1+sR_3C_3)+R_2(1+sR_3C_3)-R_2\right]$$

$$R_3V_{in}+V_0sC_2R_2R_3=V_a\left[(1+sR_3C_3)\left(R_3R_2sC_2+R_3+R_2\right)-R_2\right]$$

$$V_a=\frac{R_3V_{in}+V_0sC_2R_2R_3}{(1+sR_3C_3)\left(R_3R_2sC_2+R_3+R_2\right)-R_2}$$

Because of the non-inverting configuration of an op-amp,

$$V_0=A_fV_a$$

Where,

$$A_f=1+\frac{R_f}{R_1}=Gain\ of\ filter\ in\ passband$$

$$V_0=A_f\left[\frac{R_3V_{in}+V_0sC_2R_2R_3}{(1+sR_3C_3)\left(R_3R_2sC_2+R_3+R_2\right)-R_2}\right]$$

$$V_0-\frac{A_fV_0sC_2R_2R_3}{(1+sR_3C_3)\left(R_3R_2sC_2+R_3+R_2\right)-R_2}=\frac{A_fR_3V_{in}}{(1+sR_3C_3)\left(R_3R_2sC_2+R_3+R_2\right)-R_2}$$

$$V_0\left[(1+sR_3C_3)(R_3R_2sC_2+R_3+R_2)-R_2-A_fsC_2R_2R_3\right]=A_fR_3V_{in}$$

$$\frac{V_0}{V_{in}}=\frac{A_fR_3}{\left[(1+sR_3C_3)(R_3R_2sC_2+R_3+R_2)-R_2-A_fsC_2R_2R_3\right]}$$

Rearrange this equation,

$$\frac{V_0}{V_{in}}=\frac{A_fR_3}{\left[(1+sR_3C_3)(R_2+R_3+sR_2R_3C_2)-R_2-sA_fR_2R_3C_2\right]}$$

$$\frac{V_0}{V_{in}}=\frac{A_fR_3}{\left[(R_2+R_3+sR_2R_3C_2+sR_2R_3C_3+sR_3^2C_3+s^2R_2R_3^2C_2C_3)-R_2-sA_fR_2R_3C_2\right]}$$

$$\frac{V_0}{V_{in}} = \frac{A_f R_3}{s^2 R_2 R_3^2 C_2 C_3 + s(R_2 R_3 C_2 + R_2 R_3 C_3 + R_3^2 C_3 - A_f R_2 R_3 C_2) + R_3}$$

$$\frac{V_0}{V_{in}} = \frac{A_f R_3}{R_2 R_3^2 C_2 C_3 \left(s^2 + s\frac{R_2 R_3 C_2 + R_2 R_3 C_3 + R_3^2 C_3 - A_f R_2 R_3 C_2}{R_2 R_3^2 C_2 C_3} + \frac{R_3}{R_2 R_3^2 C_2 C_3}\right)}$$

$$\frac{V_0}{V_{in}} = \frac{A_f}{R_2 R_3 C_2 C_3 \left(s^2 + s\frac{R_2 C_2 + R_2 C_3 + R_3 C_3 - A_f R_2 C_2}{R_2 R_3 C_2 C_3} + \frac{1}{R_2 R_3 C_2 C_3}\right)}$$

$$\frac{V_0}{V_{in}} = \frac{\frac{A_f}{R_2 R_3 C_2 C_3}}{\left(s^2 + s\frac{R_2 C_2 + R_2 C_3 + R_3 C_3 - A_f R_2 C_2}{R_2 R_3 C_2 C_3} + \frac{1}{R_2 R_3 C_2 C_3}\right)}$$

Compare this equation with the standard form transfer function for second-order Butterworth filter. And that is,

$$\frac{V_0}{V_{in}} = \frac{A}{s^2 + 2\zeta\omega_c s + \omega_c^2}$$

By comparing above equations, we can find the equation of cut off frequency and overall gain for the second-order lowpass Butterworth filter.

The gain of filter is,

$$A_{max} = \frac{A_f}{R_2 R_3 C_2 C_3}$$

And the Cut-off frequency of filter is ,

$$\omega_c^2 = \frac{1}{R_2 R_3 C_2 C_3}$$

$$\omega_c = \frac{1}{\sqrt{R_2 R_3 C_2 C_3}}$$

$$f_c = \frac{1}{2\pi\sqrt{R_2 R_3 C_2 C_3}}$$

Now, if we consider the value of R2 is same as R3 and the value of C2 is same as C3.

$$R_2 = R_3 = R \quad and \quad C_2 = C_3 = C$$

$$f_c = \frac{1}{2\pi RC}$$

Now if we put above values in transfer function,

$$\frac{V_0}{V_{in}} = \frac{\frac{A_f}{R^2 C^2}}{s^2 + s\frac{RC + RC + RC - A_f RC}{R^2 C^2} + \frac{1}{R^2 C^2}}$$

$$\omega_c = \frac{1}{RC}$$

$$\frac{V_0}{V_{in}} = \frac{A_f \omega^2}{s^2 + s(3 - A_f)\omega + \omega^2}$$

From above equation, the quality factor Q is equal to,

$$Q = \frac{1}{3 - A_f}$$

The quality factor is only depends on the gain of filter. And the value of gain should not more than 3. If the value of gain is more than 3, the system will be unstable.

The value of quality factor is 0.707 for the Butterworth filter. And if we put this value in equation of quality factor, we can find the value of gain.

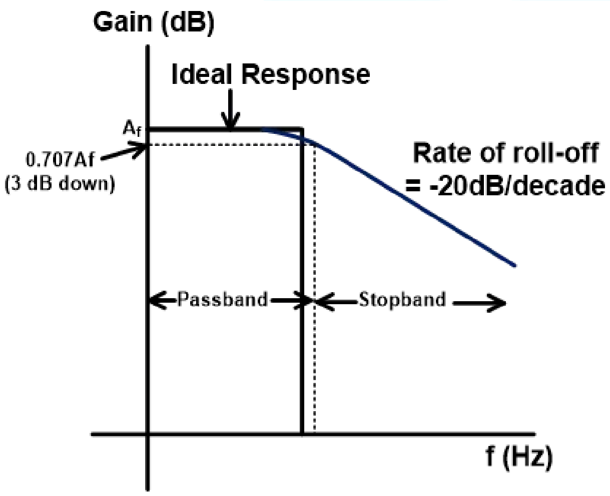
$$0.707 = \frac{1}{3 - A_f}$$

Af = 1.586

1+RfR1 = 1.586

RfR1 = 0.586

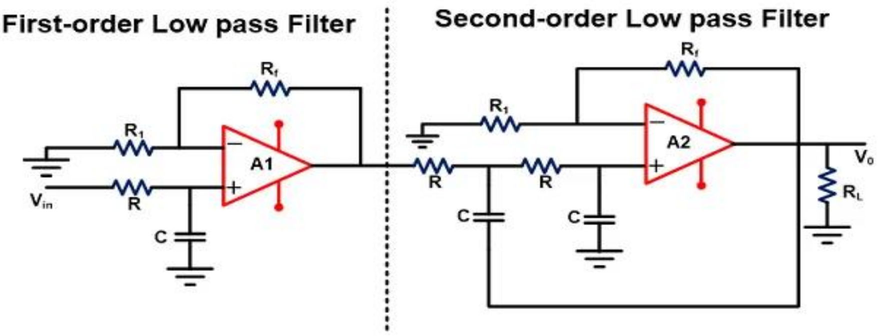
While designing the second-order Butterworth filter above relation must be satisfy. The frequency response of this filter is as shown in below figure.



Third-order Lowpass Butterworth Filter

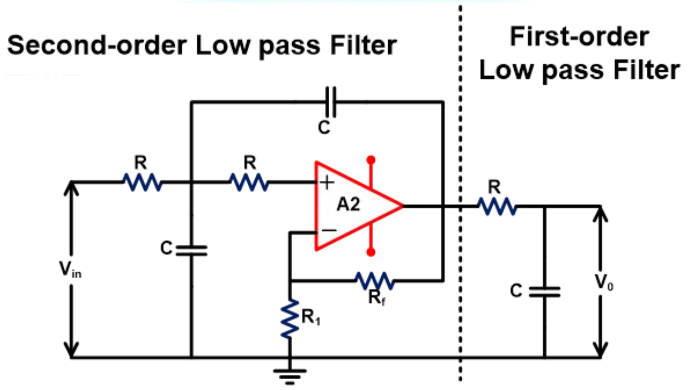
Third-order lowpass Butterworth filter can design by cascading the first-order and second-order Butterworth filter.

The below figure shows the circuit diagram of the third-order lowpass Butterworth filter.



In this figure, the first part shows the first-order lowpass Butterworth filter, and the second part shows the second-order lowpass Butterworth filter.

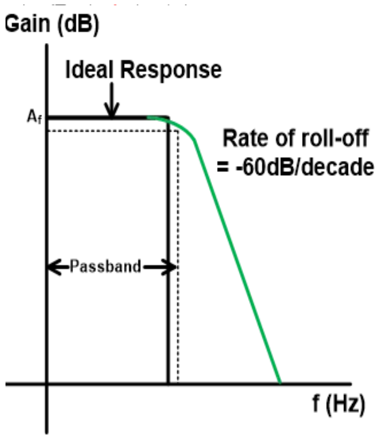
But in this condition, the voltage gain of the first part is optional and it can be set at any value. Therefore, the first op-amp is not taking part in voltage gain. Hence, the figure for the third-order low pass filter can be expressed as below figure also;



The voltage gain of a second-order filter affects the flatness of frequency response. If the gain of the second-order filter is kept at 1.586, the gain will down 3db for each part. So, the overall gain will down 6dB at the cut off frequency.

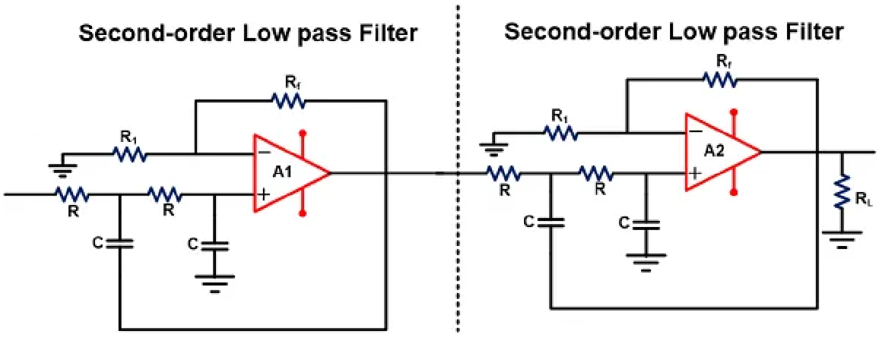
By increasing the voltage gain of the second-order filter, we can offset the cumulative loss of voltage gain.

In the third-order Butterworth filter, the rate of a roll-off period is -60dB/decade. And the frequency response of this filter is nearer to the ideal Butterworth filter compared to the first and second-order filters. The frequency response of this filter is as shown in the below figure.



Fourth-order Lowpass Butterworth Filter

Fourth-order Butterworth filter is established by the cascade connection of two second-order low pass Butterworth filters. The circuit diagram of the fourth-order lowpass Butterworth filter is as shown in the below figure.



If the gain of both filters is set at 1.586, the voltage gain will be down 6 dB at the cut-off frequency. We can get a flatter response by choosing different values of voltage gain for both stages. According to the advanced research, we get maximum flat response, if we use the voltage gain 1.152 for the first stage and 2.235 for the second stage.

The below figure shows the frequency response of the fourth-order lowpass Butterworth filter.