Dirac's Notation

The state of a system can be represented by a vector called state vector in the vector space. Dirac introduced the symbol $|\ \rangle$ called the ket vector or simply ket to denote the state vector which will take different form in different representations. To distinguish the ket vectors corresponding to different states a label is introduced in the ket. Thus a state vector corresponding to $\Psi(a)_r$ is denoted by ket $|a\rangle$. Corresponding to every vector $|a\rangle$ is defined a conjugate vector $|a\rangle^*$ for which Dirac used the notation $\langle a|$ which is called a bra vector or simply bra. The conjugate of a ket vector is bra vector and vice versa. A scalar in the kept space becomes its complex conjugate in the bra space. The bra - ket notation is distorted from the bracket notation. Thus, the bracket symbol (|) is distorted to $\langle\ \ |$ and $|\ \ \rangle$ in the Dirac notation.

Operation by an operator A on a ket vector produces another ket vector.

$$A\ |a\rangle = |a'\rangle$$

Operation on a bra vector from the right by A gives another bra vector

$$\langle b|A = \langle b'|$$

In terms of bra and ket vectors, the definition of the inner product of the state vectors Ψ_a and Ψ_b takes the form.

$$(\Psi_a , \Psi_b) = \int \Psi^*{}_a\, \Psi_b\, d\tau = \langle a|b\rangle$$

The norm of a ket $|a\rangle$, denoted by $\langle a|a\rangle$ is a real non negative number. That is

$$\langle a|a\rangle \geq 0$$

The equality sign holds only if $|a\rangle = 0$. The ket $|a\rangle$ is said to be normalized if

$$\langle a|a\rangle = 1$$

Kets $|a\rangle$ and $|b\rangle$ are orthogonal if

$$\langle a|b\rangle = 0$$

The orthonormality relation is expressed as

$$\langle a_i|a_j\rangle = \delta_{ij}$$

In this notation, the condition for an operator to be Hermitian is

$$\langle a|A\,|b\rangle = \langle b|A|a\rangle^*$$