A

Tables of Fourier Series and Transform Properties

Table A.1 Properties of the continuous-time Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\Omega t} \qquad C_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t) e^{-jk\Omega t} dt$$

Property	Periodic function $x(t)$ with period $T = 2\pi/\Omega$	Fourier series C_k
Time shifting	$x(t \pm t_0)$	$C_k e^{\pm jk\Omega t_0}$
Time scaling	$x(\alpha t)$, $\alpha > 0$	C_k with period $\frac{T}{\alpha}$
Differentiation	$\frac{d}{dt}x(t)$	$jk\Omega C_k$
Integration	$\int_{-\infty}^{t} x(t)dt < \infty$	$\frac{1}{jk\Omega} C_k$
Linearity	$\sum_i \alpha_i x_i(t)$	C_{ik}
Conjugation	$x^*(t)$	
Time reversal	$x(-t)$	
Modulation	$x(t)e^{jK\Omega t}$	
Multiplication	$x(t)y(t)$	$C_{xi}C_{y(k-i)}$

Periodic convolution	$\int_T x(\theta)y(t-\theta)\mathrm{d}\theta$	$TC_{xk}C_{yk}$	
Symmetry	$x(t) = x^*(t)$ real	$\left\{ C_k = C^*_{-k}, \lvert C \right.$	$\lvert C_k \rvert = \lvert C_{-k} \rvert$, Re $C_k = \mathrm{Re}\, C_{-k}$, Im$C_k$ = −ImC_{-k}, argC_k = −argC_{-k}
	$x(t) = x^*(t) = x(-t)$ real and even		$\left\{ C_k = C_{-k}, C_k = C_k^* \right.$, realandeven
	$x(t) = x^*(t) =$ real and odd		$= -C_{-k}, C_k = -C_{k}^*$, imaginary and odd
Parseval's theorem	$\frac{1}{T}\int_{-T/2}^{T/2} \lvert x(t)\rvert^2 \mathrm{d}t = \sum_{k=-\infty}^{\infty} \lvert C_k \rvert^2$		

Table A.2 Properties of the continuous-time Fourier transform

ω $$X(j\omega) = \int_{-\infty}^{\infty} x(t)\mathrm{e}^{-j\omega t}\mathrm{d}t x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\mathrm{e}^{j\omega t}\mathrm{d}$$

Property	Nonperiodic function $x(t)$	Fourier transform $X(j\omega)$
Time shifting	$x(t \pm t_0)$	$e^{\pm j\omega t_0}X(j\omega)$
Time scaling	$x(\alpha t)$	$\frac{1}{\lvert\alpha\rvert} X\left(\frac{j\omega}{\alpha}\right)$
Differentiation	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^{t} x(t)\mathrm{d}t$	$\frac{1}{j\omega}X(j\omega) + \pi X(j0)\delta(\omega)$
	$\int_{-\infty}^{\infty} x(t)\mathrm{d}t$	$X(j0)$
Frequency integration	$2\pi x(0)$	$\int_{-\infty}^{\infty} X(j\omega)\mathrm{d}\omega$
Linearity	$\sum_i \alpha_i x_i(t)$	$\sum_i \alpha_i X_i(j\omega)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Time reversal	$x(-t)$	$X(-j\omega)$
Modulation	$x(t)e^{j\omega_0 t}$	$X(j\omega - j\omega_0)$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}X(j\omega) \times Y(j\omega)$
Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
Symmetry	$x(t) = x^*(t)$ real	$X(j\omega) = X^*(-j\omega)$, $\lvert X(j\omega)\rvert = \lvert X(-j\omega)\rvert$, $\mathrm{Re}\,X(j\omega) = \mathrm{Re}\,X(-j\omega)$, $\mathrm{Im}\,X(j\omega) = -\mathrm{Im}\,X(-j\omega)$, $\arg X(j\omega) = -\arg X(-j\omega)$
	$x(t) = x^*(t) = x(-t)$ real and even	$X(j\omega) = X(-j\omega)$, $X(j\omega) = X^*(j\omega)$, real and even
	$x(t) = x^*(t) = -x(-t)$ real and odd	$X(j\omega) = -X(-j\omega)$, $X(j\omega) = -X^*(j\omega)$, imaginary and odd

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \mathrm{d}t = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 \mathrm{d}\omega$$

Rayleigh's theorem

B

Tables of Fourier Series and Transform of Basis Signals

Table B.1 The Fourier transform and series of basic signals

Signal $x(t)$	Transform $X(j\omega)$	Series C_k
1	$2\pi\delta(\omega)$	$C_0 = 1$, $C_{k\neq 0} = 0$ $C_k = \frac{1}{T}$
$\delta(t)$	1	
$u(t)$		—
$u(-t)$		—
$e^{j\Omega t}$	$\frac{1}{j\omega} + \pi\delta(\omega)$	$C_1 = 1$, $C_{k\neq 1} = 0$
$\sum_{k=-\infty}^{\infty} C_k e^{jk\Omega}$	$-\frac{1}{j\omega} + \pi\delta(\omega)$	C_k
$\cos\Omega$	$2\pi\delta(\omega - \Omega)$	$C_1 = C_{-1} = \frac{1}{2}$, $C_{k\neq\pm 1} = 0$
$\sin\Omega t$	$2\pi \sum_{k=-\infty}^{\infty} C_k\delta(\omega - k\Omega)$	$C_1 = -C_{-1} = \frac{1}{2j}$, $C_{k\neq\pm 1} = 0$
$\frac{1}{\alpha^2+t^2}$	$\pi[\delta(\omega-\Omega) + \delta(\omega+\Omega)]$	$\frac{1}{T}e^{-\frac{2\pi\alpha\|k\|}{T}}$
Rectangular (Fig. 2.16c)	$\frac{\pi}{j}[\delta(\omega-\Omega) - \delta(\omega+\Omega)]$	$\frac{A}{q}\frac{\sin(k\pi/q)}{k\pi/q}$
Triangular (Fig. 2.21a)	$e^{-\alpha\|\omega\|}$ $A\tau\frac{\sin(\omega\tau/2)}{\omega\tau/2}$	$\frac{A}{2q}\frac{\sin^2(k\pi/2q)}{(k\pi/2q)^2}$
Trapezoidal (Fig. 2.30)	$\frac{A\tau}{2}\frac{\sin^2(\omega\tau/4)}{(\omega\tau/4)^2}$	$\frac{A}{q}\frac{\sin(k\pi/q)}{k\pi/q}\frac{\sin(k\pi/q_s)}{k\pi/q_s}$
Ramp (Fig. 2.34b)	$A\tau\frac{\sin(\omega\tau/2)}{\omega\tau/2}\frac{\sin(\omega\tau_s/2)}{\omega\tau_s/2}$	$\frac{A}{j2\pi k}\left[\frac{\sin(k\pi/q)}{k\pi/q}e^{j\frac{k\pi}{q}} - 1\right]$
Ramp (Fig. 2.34c)	$\frac{A}{j\omega}\left[\frac{\sin(\omega\tau/2)}{\omega\tau/2}e^{j\frac{\omega\tau}{2}} - 1\right]$	$\frac{A}{j2\pi k}\left[1 - \frac{\sin(k\pi/q)}{k\pi/q}e^{-j\frac{k\pi}{q}}\right]$
$\frac{\sin\alpha t}{\alpha t}$	$\frac{A}{j\omega}\left[1 - \frac{\sin(\omega\tau/2)}{\omega\tau/2}e^{-j\frac{\omega\tau}{2}}\right]$	$\begin{cases}\frac{\pi}{\alpha T}, & \|k\| < \frac{\alpha T}{2\pi}\\ 0, & \|k\| > \frac{\alpha T}{2\pi}\end{cases}$
$e^{-\alpha t}u(t)$, Re$\alpha > 0$	$\begin{cases}\frac{\pi}{\alpha}, & \|\omega\| < \alpha\\ 0, & \|\omega\| > \alpha\end{cases}$	$\frac{1}{\alpha T + j2\pi k}$
$te^{-\alpha t}u(t)$, Re$\alpha > 0$	$\frac{1}{\alpha+j\omega}$	$\frac{T}{(\alpha T+j2\pi k)^2}$
	$\frac{1}{(\alpha+j\omega)^2}$	

Table B.1 The Fourier transform and series of basic signals (*Contd.*)

$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$, Re$\alpha > 0$	$\frac{1}{(\alpha+j\omega)^n}$	$\frac{T^{n-1}}{(\alpha T+j2\pi k)^n}$
$e^{-\alpha\|t\|}$, $\alpha > 0$	$\frac{2\alpha}{\alpha^2+\omega^2}$	$\frac{2\alpha T}{\alpha^2T^2+4\pi^2k^2}$
$e^{-\alpha^2t^2}$	$\frac{\sqrt{\pi}}{\alpha}e^{-\frac{\omega^2}{4\alpha^2}}$	$\frac{\sqrt{\pi}}{\alpha T}e^{-\frac{\pi^2k^2}{\alpha^2T^2}}$

C_k corresponds to $x(t)$ repeated with period T, τ and τ_s are durations, $q = \frac{T}{\tau}$, and $q_s = \frac{T}{\tau_s}$.

Table B.2 The Fourier transform and series of complex signals

Signal $y(t)$	Transform $Y(j\omega)$	Series C_k
Burst of N pulses with known	$(\quad)\frac{\sin(\omega NT/2)}{\sin(\omega T/2)}$	$\frac{1}{T_1}X\left(j\frac{2k\pi}{T_1}\right)\frac{\sin(k\pi/q_2)}{\sin(k\pi/Nq_2)}$
Rectangular pulse-burst (Fig. 2.47)	$\frac{\sin(\omega NT/2)}{\sin(\omega T/2)}$	$\frac{A}{q_1}\frac{\sin(k\pi/q_1)}{k\pi/q_1}\frac{\sin(k\pi/q_2)}{\sin(k\pi/Nq_2)}$
Triangular pulse-burst	$\frac{\sin(\omega NT/2)}{\sin(\omega T/2)}$	$\frac{A}{2q_1}\frac{\sin^2(k\pi/2q_1)}{(k\pi/2q_1)^2}\frac{\sin(k\pi/q_2)}{\sin(k\pi/Nq_2)}$
Sinc-shaped pulse-burst		$\begin{cases}\frac{A\pi}{\alpha T_1}\frac{\sin(k\pi/q_2)}{\sin(k\pi/Nq_2)}, & \|k\| < \frac{\alpha T_1}{2\pi} \\ 0, & \|k\| > \frac{\alpha T_1}{2\pi}\end{cases}$

C_k corresponds to $y(t)$ repeated with period T_1, τ is pulse duration, T is period of pulse in the burst, T_1 is period of pulse-bursts in the train, $q_1 = \frac{T_1}{\tau}$, and $q_2 = \frac{T_1}{NT}$.

C

Tables of Hilbert Transform and Properties

C Tables of Hilbert Transform and Properties **Table C.1**

Properties of the Hilbert transform

$$y(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\hat{y}(\theta)}{\theta - t} \mathrm{d}\theta \qquad \hat{y}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{y(\theta)}{t - \theta} \mathrm{d}\theta$$

Property	Function $y(t)$	Transform $\hat{y}(t)$
Filtering	$y(t)$ is constant	$\hat{y}(t)$ is zero
	is real	is real
	is even (odd)	is odd (even)
Causality	If $y(t)$ is causal with transform $Y(j\omega) = Y_r(\omega) + jY_i(\omega)$, then:	
	$Y_r(\omega)$	
	$Y_i(\omega)$	$Y_i(\omega)$
		$-Y_r(\omega)$
Linearity	$\sum_i \alpha_i y_i(t)$	$\sum_i \alpha_i \hat{y}_i(t)$
Time shifting	$y(t \pm \theta)$	$\hat{y}(t \pm \theta)$
Time reversal	$y(-t)$	$-\hat{y}(-t)$
Scaling	$y(at)$	$\hat{y}(at)$
	$y(-at)$	$-\hat{y}(-at)$
Multiple transform	$\hat{y}(t)$	$-y(t)$
		$H^3 y(t) = H^{-1} y(t)$
		$H^4 y(t) = y(t)$
		$H^n y(t) \overset{F}{\Leftrightarrow} [-j\,\mathrm{sgn}(\omega)]^n Y(j\omega)$
Differentiation	$\frac{d}{dt} y(t)$	$\frac{d}{dt} \hat{y}(t)$

Integration	$\int_a^b y(t)\mathrm{d}t$, a and b are constants	0
Convolution	$y_1(t) * y_2(t)$	
Autocorrelation	$y(t) * y(t)$	$y_1(t) * \hat{y}_2(t)$
		$\widehat{y(t) * \hat{y}(t)}$
		$\hat{y}(t) + \frac{1}{\pi}\int_{-\infty}^{\infty} y(t)\mathrm{d}t$
Multiplication	$ty(t)$	

Table C.2 Useful relations between $y(t)$ and its Hilbert transform $\hat{y}(t)$

Property	Relation
Orthogonality	$\int_{-\infty}^{\infty} y(t)\hat{y}(t)\mathrm{d}t = 0$
Integration	$\int_{-\infty}^{\infty} y_1(t)\hat{y}_2(t)\mathrm{d}t = \int_{-\infty}^{\infty} \hat{y}_1(t)y_2(t)\mathrm{d}t$
	$\int_{-\infty}^{\infty} y_1(t)y_2(t)\mathrm{d}t = \int_{-\infty}^{\infty} \hat{y}_1(t)\hat{y}_2(t)\mathrm{d}t$
	$\int_{-\infty}^{\infty} y^2(t)\mathrm{d}t = \int_{-\infty}^{\infty} \hat{y}^2(t)\mathrm{d}t$
Energy	$\int_{-\infty}^{\infty} \lvert y(t)\rvert^2(t)\mathrm{d}t = \int_{-\infty}^{\infty} \lvert\hat{y}(t)\rvert^2\mathrm{d}t$
Autocorrelation	$\int_{-\infty}^{\infty} y(t)y(t-\theta)\mathrm{d}t = \int_{-\infty}^{\infty} \hat{y}(t)\hat{y}(t-\theta)\mathrm{d}t$

Table C.3 The Hilbert transform of analytic signals

Property	Signal	Transform
Analytic signal	$y_a(t) = y(t) + j\hat{y}(t)$	$-jy_a(t) = \hat{y}(t) - jy(t)$
Multiplication	$y_{a1}(t)y_{a2}(t)$	$-jy_{a1}(t)y_{a2}(t)$
		$= \hat{y}_{a1}(t)y_{a2}(t)$
		$= y_{a1}(t)\hat{y}_{a2}(t)$

Power^n		$-jy_a^n(t)$
Real product	$\text{Re}[y_{a1}(t)y_{a2}(t)]$	$\text{Im}[y_{a1}(t)y_{a2}(t)]$
Imaginary product	$\text{Im}[y_{a1}(t)y_{a2}(t)]$	$-\text{Re}[y_{a1}(t)y_{a2}(t)]$

Table C.4 Products of the analytic signals

Product	Relation
$y_{a1}(t)y_{a2}(t)$	$= j\hat{y}_{a1}(t)y_{a2}(t) = jy_{a1}(t)\hat{y}_{a2}(t)$
$\hat{y}_{a1}(t)y_{a2}(t)$	$= -jy_{a1}(t)y_{a2}(t) = y_{a1}(t)\hat{y}_{a2}(t)$

C Tables of Hilbert Transform and Properties

Table C.5 The Hilbert transform of basic signals

Signal	Transform
$\delta(x)$	$\frac{1}{\pi x}$
$\delta'(x)$	$-\frac{1}{\pi x^2}$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\cos(ax)J_n(bx) \quad [0 < b < a]$	$-\sin(ax)J_n(bx)$
$\frac{\sin x}{x}$	$\frac{\sin^2 x/2}{x/2}$
e^{jx}	je^{jx}

$\frac{a}{x^2+a^2}$ $[a > 0]$ e^{-x} $[x > 0]$ $\frac{x}{x^2+a^2}$

$\frac{ab-x^2}{(ab-x^2)^2+x^2(a+b)^2}$ sign $x e^{-|x|}$ $e^{-|x|}$ $\frac{x(a+b)}{(ab-x^2)^2+x^2(a+b)^2}$

$\delta(x-a)-\delta(x+a)$ $\frac{2}{\pi}\frac{a}{x^2-a^2}$

$\delta(x-a)+\delta(x+a)$ $\frac{2}{\pi}\frac{x}{x^2-a^2}$ $\left(\frac{\pi v}{2}\right)|x|^{v-1}$ sign x

sign x $\frac{1}{\pi}e^{-x}\mathrm{Ei}(x)$

Even rectangular pulse $u(x+\tau)-u(x-\tau)$ $\frac{1}{\pi}[e^{-x}\ \mathrm{Ei}\ (x)+e^{x}\mathrm{Ei}(-x)]$

$\frac{1}{\pi}[e^{-x}\mathrm{Ei}(x)-e^{x}\mathrm{Ei}(-x)]$

$|x|^{v-1}$ $[0 < \mathrm{Re}\,v < 1]$ cot

$|x|^{v-1}$ sign x $[0 < \mathrm{Re}\,v < 1]$ $-\tan\frac{\pi v}{2}\ |x|^{v-1}$

$-\infty$

$\frac{1}{\pi}\ln\frac{x+\tau}{x-\tau}$

$-\frac{1}{\pi}+\ln\frac{x+\tau}{x}+\ln\frac{x-\tau}{x}$,

$\frac{1}{\pi}+\frac{x-\tau}{\tau}\ln\frac{x-\tau}{x}+1$,

$\frac{1}{\pi}+\frac{x+\tau}{\tau}\ln\frac{x+\tau}{x}+\frac{x-\tau}{\tau}\ln\frac{x-\tau}{x}$

Sawtoothpulse $\frac{1}{\tau}(\tau-|x|)[-u(x+\tau)+2u(x)-u(x-\tau)]$ $\frac{1}{\pi}+\frac{x-\tau}{\tau}\ln\frac{x-\tau}{x}-\frac{x+\tau}{\tau}\ln\frac{x+\tau}{x}+2$,

---

Odd rectangular pulse
$-u(x+\tau)+2u(x)-u(x-\tau)$

Ramp pulse $\frac{1}{\tau}(\tau-x)[u(x)-u(x-\tau)]$

Triangular pulse

$\frac{1}{\tau}(\tau-|x|)[u(x+\tau)-u(x-\tau)]$,

D

Mathematical Formulas

Limits:

- $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{\partial f(x)/\partial x}{\partial g(x)/\partial x}$ (L'Hospital's rule)
- $\lim_{x\to 0} \frac{\sin x}{x} = 1$
- $\lim_{x\to 0} \frac{\sin Nx}{\sin x} = N$
- $\int_0^\infty \sin bx \, dx = \lim_{\alpha\to 1} \int_0^\infty x^{\alpha-1} \sin bx \, dx = \frac{\Gamma(\alpha)}{b^\alpha} \sin \frac{\alpha\pi}{2} \Big|_{\alpha=1} = \frac{1}{b}$

Trigonometric identities:

- $e^{jx} = \cos x + j \sin x$ (Euler's formula)

  $e^{(\alpha+jx)} = e^{\alpha}(\cos x + j \sin x)$
- $\cos x = \frac{e^{jx}+e^{-jx}}{2}$
- $\sin x = \frac{e^{jx}-e^{-jx}}{2j}$
- $\cos^2 x + \sin^2 x = 1$
- $\cos^2 x - \sin^2 x = \cos 2x$
- $2 \cos x \sin x = \sin 2x$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos x \cos y = \frac{1}{2}[\cos(x + y) + \cos(x - y)]$
- $\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$

D Mathematical Formulas

- $\sin x \cos y = \frac{1}{2}[\sin(x + y) + \sin(x - y)]$
- $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
- $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
- $\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$
- $\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$
- $\sin x \pm \sin y = 2 \sin \frac{x\pm y}{2} \cos \frac{x\mp y}{2}$

▷ $a\cos\overline{x+b\sin}\,x = r\sin(x+\varphi) = r\cos(x-\psi)$,
$r = \sqrt{a^2+b^2}$, $\sin\varphi = \frac{a}{r}$, $\cos\varphi = {}^{b}$, $\sin\psi = {}^{b}$ ψ ^{a}

▷ $\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$

▷ $\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$

▷ $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$ r_ r_, cos = _r

▷ $\frac{d}{dx}\text{arccot}\,x = -\frac{1}{1+x^2}$

Hyperbolic identities:

▷ $\sinh x = -\sinh(-x) = \pm\sqrt{\cosh^2 x - 1} = \pm\sqrt{\frac{1}{2}(\cosh 2x - 1)} = \frac{e^x - e^{-x}}{2}$

▷ $\cosh x = \cosh(-x) = \sqrt{\sinh^2 x + 1} = \sqrt{\frac{1}{2}(\cosh 2x + 1)} = 2\cosh^2\frac{x}{2} - 1 = \frac{e^x + e^{-x}}{2}$

▷ $\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

▷ $\cosh^2 x - \sinh^2 x = 1$

▷ $\cosh x + \sinh x = e^x$

▷ $\cosh x - \sinh x = e^{-x}$

Exponents:

$e^{\ln x} = x$

▷ $\frac{e^x}{e^y} = e^{x-y}$

Logarithms:

$\ln e^x = x$

▷ $\ln\frac{x}{y} = \ln x - \ln y$

▷ $\ln \alpha x = \ln\alpha + \ln x$

▷ $\ln x^\alpha = \alpha\ln x$

Extension to series:

▷ $e^x e^y = e^{x+y}$

▷ $(e^x)^\alpha = e^{\alpha x}$

ENTRI

- $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots$
- $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots + (-1)^n \frac{x^{2n}}{(2n)!} + \ldots$
- $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots$
- $e^{jz\cos\phi} = \sum_{n=-\infty}^{\infty} j^n J_n(z) e^{jn\phi}$

Series:

- $\sum_{n=0}^{N-1} x^n = \frac{1-x^N}{1-x}$, $x \neq 1$ (by geometric progression)
- $\sum_{n=0}^{N-1} e^{\alpha n} = \frac{1-e^{\alpha N}}{1-e^{\alpha}}$
- $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$, $|x| < 1$
- $\sum_{k=1}^{\infty} \frac{\sin^2(k\pi/q)}{k^2} = \frac{\pi^2(q-1)}{2q^2}$

Indefinite integrals:

- $\int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)dx$ (integration by parts)
- $\int f(x)dx = \int f[g(y)]g'(y)dy \quad [x = g(y)]$ (change of variable)
- $\int \frac{dx}{x} = \ln|x|$
- $\int e^x dx = e^x$
- $\int a^x dx = \frac{a^x}{\ln a}$
- $\int \sin x \, dx = -\cos x$
- $\int \cos x \, dx = \sin x$

D Mathematical Formulas

- $\int x \begin{Bmatrix} \sin x \\ \cos x \end{Bmatrix} dx = \begin{Bmatrix} \sin x \\ \cos x \end{Bmatrix} \mp x \begin{Bmatrix} \cos x \\ \sin x \end{Bmatrix}$

ENTRI

- ▷ $\int \frac{\mathrm{d}x}{x(ax^r+b)} = \frac{1}{rb} \ln \left| \frac{x^r}{ax^r+b} \right|$
- ▷ $\int \sin^2 x \,\mathrm{d}x = -\frac{1}{4} \sin 2x + \frac{x}{2}$
- ▷ $\int \cos^2 x \,\mathrm{d}x = \frac{1}{4} \sin 2x + \frac{x}{2}$
- ▷ $\int x\mathrm{e}^{\alpha x} \mathrm{d}x = \mathrm{e}^{\alpha x} \left(\frac{x}{\alpha} - \frac{1}{\alpha^2} \right)$
- ▷ $\int x^2\mathrm{e}^{\alpha x} \mathrm{d}x = \mathrm{e}^{\alpha x} \left(\frac{x^2}{\alpha} - \frac{2x}{\alpha^2} + \frac{2}{\alpha^3} \right)$
- ▷ $\int x^\lambda \mathrm{e}^{\alpha x} \mathrm{d}x = \frac{1}{\alpha} x^\lambda \mathrm{e}^{\alpha x} - \frac{\lambda}{\alpha} \int x^{\lambda-1} \mathrm{e}^{\alpha x} \mathrm{d}x$
- ▷ $\int \mathrm{e}^{-(ax^2+bx+c)} \mathrm{d}x = \frac{1}{2} \sqrt{\frac{\pi}{a}} \mathrm{e}^{\frac{b^2-4ac}{4a}} \operatorname{erf} \left(x\sqrt{a} + \frac{b}{2\sqrt{a}} \right)$
- ▷ $\int \frac{1}{x} \mathrm{e}^{\alpha x} \mathrm{d}x = \mathrm{Ei}(\alpha x) \quad [\alpha \neq 0]$
- ▷ $\int \frac{1}{\sqrt{x}} \mathrm{e}^{-\alpha x} \mathrm{d}x = \sqrt{\frac{\pi}{\alpha}} \operatorname{erf} (\sqrt{\alpha x}) \quad [\alpha > 0]$
- ▷ $\int \frac{\mathrm{e}^x}{x^2+\alpha^2} \mathrm{d}x = \frac{1}{\alpha} \mathrm{Im}[\mathrm{e}^{j\alpha} \,\mathrm{Ei}(x - j\alpha)]$

Definite integrals:

- ▷ $\int_{-\infty}^{\infty} \frac{\sin \alpha x}{x} \mathrm{d}x = \pi$
- ▷ $\int_{-\infty}^{\infty} \mathrm{e}^{-\alpha x^2} \mathrm{d}x = \sqrt{\frac{\pi}{\alpha}}$
- ▷ $\int_{-\infty}^{\infty} x^2 \mathrm{e}^{-\alpha x^2} \mathrm{d}x = \sqrt{\pi} \alpha^{-3/2}$
- ▷ $\int_{0}^{\infty} \frac{\sin \alpha x}{x} \mathrm{d}x = \frac{\pi}{2} \operatorname{sgn} \alpha$
- ▷ $\int_{0}^{\infty} \frac{\sin^2 \alpha x}{x^2} \mathrm{d}x = \frac{\pi\alpha}{2}$
- ▷ $\int_{-\infty}^{\infty} \frac{\sin^4 \alpha x}{x^2} \mathrm{d}x = \frac{\pi\alpha}{2}$
- ▷ $\int_{0}^{\infty} \frac{\mathrm{d}x}{\alpha^2+x^2} = \frac{\pi}{2\alpha}$
- ▷ $\int_{0}^{\infty} x^{\alpha-1} \sin bx \,\mathrm{d}x = \frac{\Gamma(\alpha)}{b^\alpha} \sin \frac{\alpha\pi}{2}$

- $\int\limits_0^\infty \sin bx \, \mathrm{d}x = \frac{1}{b}$
- $\int\limits_{-\infty}^{\infty} \frac{\sin x}{x(x-\alpha)} \mathrm{d}x = \frac{\pi}{\alpha}(\cos\alpha - 1)$, α is real
- $\int\limits_{-\infty}^{\infty} \frac{\cos(ax)}{b^2-x^2} \mathrm{d}x = \frac{\pi}{2b}\sin(ab), \quad a, b > 0$

Special functions:

erf $(x) = \frac{2}{\sqrt{\pi}} \int\limits_0^x \mathrm{e}^{-t^2} dt$ (Error function)

Ei$(x) = -\int\limits_{-x}^{\infty} \frac{\mathrm{e}^{-t}}{t} \mathrm{d}t = \int\limits_{-\infty}^{x} \frac{\mathrm{e}^{t}}{t} \mathrm{d}t \quad [x < 0]$ (Exponential-integral function)

Ei$(x) = \mathrm{e}^x \left[\frac{1}{x} + \int\limits_0^\infty \frac{\mathrm{e}^{-t}}{(x-t)^2} \mathrm{d}t \right] \quad [x > 0]$ (Exponential-integral function)

- $S(x) = \frac{2}{\sqrt{2\pi}} \int\limits_0^x \sin t^2 \mathrm{d}t = \int\limits_0^x \sin \frac{\pi t^2}{2} \mathrm{d}t$ (Fresnel integral)
- $C(x) = \frac{2}{\sqrt{2\pi}} \int\limits_0^x \cos t^2 \mathrm{d}t = \int\limits_0^x \cos \frac{\pi t^2}{2} \mathrm{d}t$ (Fresnel integral)
- $J_n(z) = \frac{1}{2\pi} \int\limits_{-\pi}^{\pi} \mathrm{e}^{-j(n\theta - z\sin\theta)} \mathrm{d}\theta$ (Bessel function of the first kind)