ENTRI

Operators and its properties

An operator can be defined as a rule by which a different function is obtained from any given function. Therefore in

$$g(x) = \hat{A}\, f(x)$$

The operator $\hat{A}$ operating on f(x) gives the function g(x). so in

$$g(x) = \hat{A}\, f(x) = [f(x)]^2$$

The operator squares the function f(x). the operator $\hat{A}$ differentiates the function f(x) with respect to x if

$$g(x) = \hat{A}\, f(x) = \frac{d}{dx} f(x)$$

An operator is said to be linear if it satisfies the relation

$$\hat{A}[c_2 f_1(x) + c_2 f_2(x)] = c_1 \hat{A} f_1(x) + c_2 \hat{A} f_2(x)$$

Where c_1 and c_2 are constants. In

$$g(x) = \frac{d}{dx}[c_1 f_1(x) + c_2 f_2(x)] = c_1 \frac{d}{dx} f_1(x) + c_2 \frac{d}{dx} f_2(x)$$

The operator (d/dx) is linear. The operator which squares the function is not linear since

$$\hat{A}[c_1 f_1(x) + c_2 f_2(x)] = [c_1 f_1(x) + c_2 f_2(x)]^2$$

$$= c^2{}_1 f^2{}_1 + c^2{}_2 f^2{}_2 + 2c_1 c_2 f_1 f_2$$

$$\neq c^2{}_1 f^2{}_1 + c^2{}_2 f^2{}_2$$

Linear operators are the most important ones in quantum mechanics and therefore we shall consider only such operators.

The sum and difference of operators $\hat{A}$ and $\hat{B}$ are defined by

$$(\hat{A} \pm \hat{B}) f(x) = \hat{A} f(x) + \hat{B} f(x)$$

Addition is commutative

$$\hat{A} + \hat{B} = \hat{B} + \hat{A}$$

Addition is associative

$$(\widehat{A} + \hat{B}) + \hat{C} = \hat{A} + (\hat{B} + \widehat{C})$$

The product of two operators is defined by

$$\hat{A}\hat{B} f(x) = \hat{A}[\hat{B} f(x)]$$

Multiplication is associative

$$\hat{A}(\hat{B}+\widehat{C)}f(x) = (\hat{A}\hat{B}+\hat{A}\hat{C})f(x)$$

Commutator of operators $\hat{A}$ and $\hat{B}$, denoted by $[\,\hat{A},\hat{B}\,]$ is defined as

$$[\,\hat{A},\hat{B}\,] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

It follows that

$$[\,\hat{A},\hat{B}\,] = -\,[\,\hat{B},\hat{A}\,]$$

If $\hat{A}\hat{B}f(x) \;=\; \hat{B}\hat{A}f(x)$, that is $[\,\hat{A},\hat{B}\,]=0$, $\hat{A}$ *and* $\hat{B}$ are said to commute.

If $\hat{A}\hat{B}+\hat{B}\hat{A}=0$, $\hat{A}$ *and* $\hat{B}$ are said to anticommute. The anticommutator of $\hat{A}$ with $\hat{B}$ is usually denoted as $[\,\hat{A},\hat{B}\,]_{+}$. An operator can be applied in succession on the same function. This can be written as follows

$$\hat{\propto}\hat{\propto}\hat{\propto}\ldots\,.\hat{\propto}\; f = \hat{\propto}^{n}\; f$$

The inverse operator $\hat{A}^{-1}$ is defined by the relation.

$$\hat{A}\,\hat{A}^{-1} \;=\; \hat{A}^{-1}\,\hat{A} = 1$$

An operator commutes with its inverse since

$$[\,\hat{A}\,\hat{A}^{-1}] = \hat{A}^{-1}\,\hat{A} - \hat{A}^{-1}\,\hat{A} = 0$$

As an example of commutator, consider the operator $\hat{x}$ *and* (d/dx)

$$\left[\frac{d}{dx},\hat{x}\right] f(\hat{x}) \;=\frac{d(\hat{x},f)}{dx} \;-\; \hat{x}\,\frac{df}{dx} = f$$

$$\left[\frac{d}{dx}\;,\hat{x}\right] = 1 \text{ or } \left[\hat{x},\frac{d}{dx}\right] = -1$$