

Phase space

Phase space is an imaginary space which combines the physical space and the momentum space. It is used to represent the state of each system.

To specify the position of one particle in space we need three position coordinates (x, y, z) . Suppose the momentum of particle is p . Then it can be resolved into three components x, y and z axes as p_x, p_y and p_z respectively. These components are called momentum coordinates. Hence the dynamical state of a particle can be represented by six coordinates - 3 position coordinates and 3 momentum coordinates.

A space where the three position coordinates can be represented along three mutually perpendicular axes is called the physical space. Volume of a small element in this space is $dV = dx dy dz$.

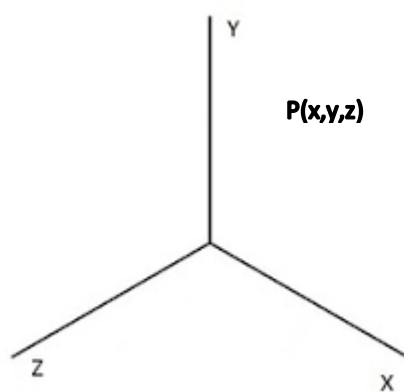


Fig 1.1 (a)

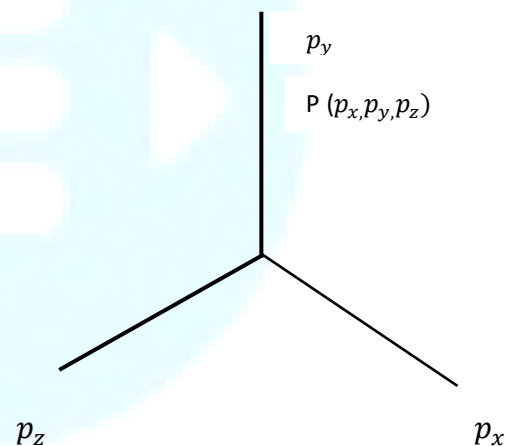


Fig 1.1 (b)

A space where the three momentum coordinates are represented along the three mutually perpendicular directions is called the momentum space. In fig 1.1(b) the point p represents the instantaneous momentum of a particle. Volume of a small element in this space = $dp_x dp_y dp_z$.

Fig 1.1(a) gives only the instantaneous position of a particle and (b) gives the instantaneous momentum. Suppose we want to specify both this position and

momentum at a particular instant in a single space. Such space is called phase space. In phase space a point P has 6 coordinates, (x, y, z, p_x, p_y, p_z) . so it is a 6 dimensional space. Any point P on the space is called phase point. The phase point gives a complete description of the dynamical state of the particle at any instant.

The volume of an infinitesimal element in phase space = $d\tau$

$$d\tau = dx dy dz dp_x dp_y dp_z.$$

Let us now consider a system consisting of N - particles. For one particle there are 6 coordinates. So for N - particles there are 6N coordinates. This means to specify the state of the system completely we need 3N position coordinates and 3N momentum coordinates. The instantaneous position of a system can be represented by a single point on the 6N - dimensional phase space.