ENTRI

Postulates of Quantum Mechanics
Part 4
Commutator Algebra

Properties of quantum Commutators

- $[\hat{A}, \hat{B}] = -[\hat{B}, \hat{A}]$
- $[\hat{A}, K] = 0$, K= Constant

$[x^2, 2] = 0$

- $[\hat{A}, \hat{B} K] = K[\hat{A}, \hat{B}]$, K= Constant
- $[\hat{A}, \hat{B} K] = \hat{A}\hat{B}K - \hat{B}K\hat{A}$

$= K[\hat{A}\hat{B} - \hat{B}\hat{A}]$

$= K[\hat{A}, \hat{B}]$

Example ; $[x, 2\frac{d}{dx}] = 2[x, \frac{d}{dx}]$

$= 2 \times -1 = 2$

- $[K\hat{A}, \hat{B}] = K[\hat{A}, \hat{B}]$
- $[K\hat{A}, K\hat{B}] = K^2[\hat{A}, \hat{B}]$
- $[\hat{A}, \hat{B}]^+ = [\hat{B}^+, \hat{A}^+]$
- $[\hat{A}, \hat{B}\hat{C}] = [\hat{A}, \hat{B}]\hat{C} + \hat{B}[\hat{A}, \hat{C}]$

$[\hat{A}\hat{B}, \hat{C}] = [\hat{A}, \hat{C}]\hat{B} + \hat{A}[\hat{B}, \hat{C}]$

$[\hat{A}\hat{B}, \hat{C}\hat{D}] = [\hat{A}, \hat{C}\hat{D}]\hat{B} + \hat{A}[\hat{B}, \hat{C}\hat{D}]$

- $[\hat{A}, f(\hat{B})] = [\hat{A}, \hat{B}] f'(\hat{B})$

$f(\hat{B})$ is a function of B

Eg ; B^2, B^3

$f'(\hat{B})$ is the first derivative

- $[f(\hat{A}), \hat{B}] = [\hat{A}, \hat{B}] f'(\hat{A})$
- $[\hat{A} + \hat{B}, \hat{C}] = [\hat{A}, \hat{C}] + [\hat{B}, \hat{C}]$

Problems

1. $[\hat{x}, \hat{P}_x] = ?$

$\hat{p}_x = -i\hbar \frac{\partial}{\partial x}$

$[\hat{x}, \hat{P}_x] = [\hat{x}, -i\hbar \frac{\partial}{\partial x}]$

$= -i\hbar [x, \frac{\partial}{\partial x}] = -i\hbar \times -1$

$[\hat{x}, \hat{P}_x] = i\hbar$

$[\hat{y}, \hat{P}_y] = i\hbar$

$[\hat{Z}, \hat{P}_z] = i\hbar$

Note

$[x, y] = 0$

$[y, z] = 0$

$[x, z] = 0$

$[x, p_x] = 0$

$[y, p_z] = 0$

$\delta_{ij} = \begin{cases} 0, i \neq j \\ 1, i < j \end{cases}$

$[q_j, p_x] = i\hbar \delta_{ix}$

2. $[x^2, p_x] = ?$

$[f(x), p_x] = [x, p_x] f'(x)$

$[x^2, p_x] = i\hbar 2x = 2i\hbar x$

3. $[x^2, p_x] = [x\,x, p_x]$

$= [x, p_x] x + x [x, p_x]$

$= i\hbar x + i\hbar x = 2i\hbar x$

4. $[e^x, p_x] = ?$

$[e^x, p_x] = [x, p_x] e^x$

$= i\hbar e^x$

5. $[x+y, p_x]$

$= [x, p_x] + [y, p_x]$

$= i\hbar + 0$

6. $[x, p_x^2] = ?$

$= [x, p_x] 2p_x$

$= 2i\hbar p_x$

7. $[x^2, p^2{}_x] = ?$

$= [x, p_x^2] x + x [x, p_x^2] 2p_x$

$$= [\, x, p_x \,]\, 2p_x x + x\, [\, x, p_x \,]\, 2p_x$$

$$= i\hbar 2p_x x + x i\hbar 2p_x$$

$$[\, x^2, p^2{}_x \,] = 2i\hbar [\, p_x x + x p_x \,]$$