ENTRI

Harmonic Perturbation

Consider a system in which an atom or molecule is exposed to harmonic perturbation. For convenience, we shall assume that the involved states are discrete. A harmonic perturbation with angular frequency ω has the form

$$H'(r,t) = 2H'(r)\,cos(\omega t) = H'(r)\,[exp(i\omega t) + exp(-i\omega t)]$$

The involved integral over the space coordinates.

$$H_{kn}{}' = \langle k|\, H'|n\rangle[exp(i\omega t) + exp(-i\omega t)]$$

Then $c_k{}^1(t)$ becomes

$$c_k{}^1(t) = \frac{H_{kn}{}'}{i\hbar}\int_o^t \{exp[i(\omega_{kn} + \omega)t'] + exp[i\,(\omega_{kn} - \omega)t']\}\,dt'$$

$$= -\frac{H_{kn}{}'}{\hbar}\left\{\frac{[exp[i(\omega_{kn}+\omega)t']-1}{\omega_{kn}+\omega} + \frac{[exp[i(\omega_{kn}+\omega)t']-1}{\omega_{kn}-\omega}\right\}$$

The first term on the right hand side has a maximum value when $\omega_{kn} + \omega = 0$ or $E_k \approx E_n - \hbar\omega$, where as the second term is maximum when $E_k = E_n + \hbar\omega$. in other words, the effect of harmonic perturbation with angular frequency ω is to receive from the system or transfer to the system the quantum of energy ħω. The first one corresponds to included or stimulated emission whereas the second to induced absorption or simply absorption. Since the emission takes place in the in the presence of the electromagnetic radiation, it is referred to as stimulated emission. Since the only one term is important at a given time, for discussion we shall consider the second term. Retaining the second term, we get

$$c_k{}^1(t) = \frac{H_{kn}{}'}{\hbar}\,\frac{[exp[i(\omega_{kn}+\omega)t']-1}{\omega_{kn}-\omega}$$

$$= -\frac{H_{kn}{}'}{i\hbar}\,\frac{[exp[i(\omega_{kn}+\omega)t/2]}{\omega_{kn}-\omega}\left\{exp\left[\frac{i(\omega_{kn}-\omega)t}{2}\right] - exp\left[\frac{i(\omega_{kn}-\omega)t}{2}\right]\right\}$$

$$= -\frac{2H_{kn}{}'}{\hbar}\,\frac{[exp[i(\omega_{kn}+\omega)t/2]}{\omega_{kn}-\omega}\,\sin\frac{(\omega_{kn}-\omega)t}{2}$$

If the system is initially in the state $|n\rangle$ (at t =0), the probability $P_{n\rightarrow k}$ of finding the system in the discrete state $|k\rangle$ after a time t is $|\,c_k{}^1(t)|^2$

$$P_{n\rightarrow k} = |\, c_k{}^1(t)|^2 = \frac{4|H_{kn}'|^2}{\hbar^2} sin^2 \frac{(\omega_{kn}-\omega)t/2}{(\omega_{kn}-\omega)^2}$$

An interesting feature of the result is that transition probability oscillates sinusoidally as a function of time. Probability $P_{n\rightarrow k}$ versus ($\omega_{kn} - \omega$) plot is as shown in figure. The central peak has a height $\left|H_{kn}'\right|^2 t^2/\hbar^2$ and a width of about and therefore the area around the main peak is proportional to t. That is probability of finding the system in a state k is proportional to t. it may be noted that the probability of finding the system in a state whose energy differs considerably from initial state E_n is smaller than the probability of finding the neighbouring energy state.

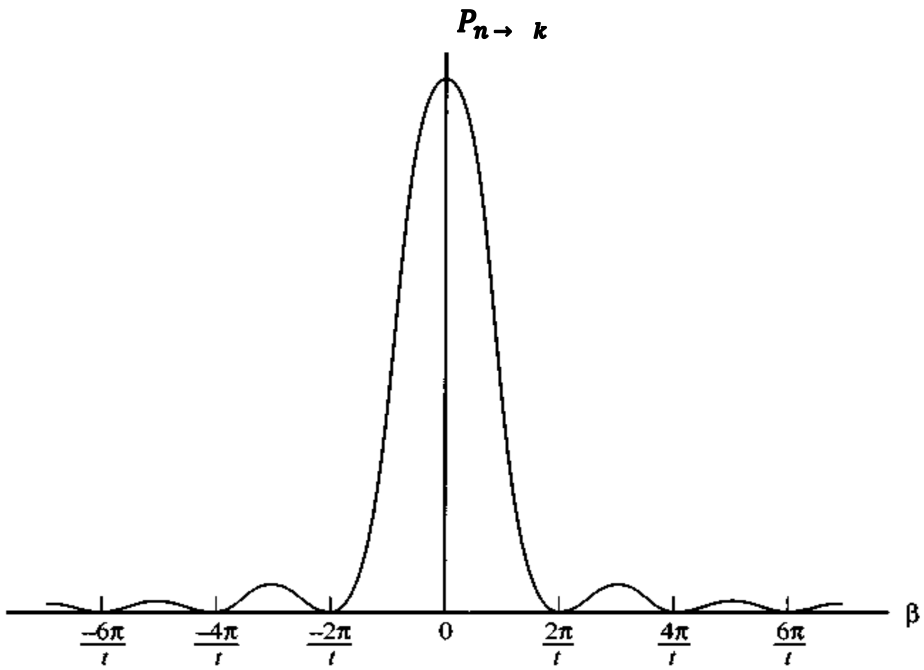

Fig - plot of probability $P_{n\rightarrow k}$ versus ($\omega_{kn} - \omega$)