
**Tables of Fourier Series and Transform
Properties**



Table A.1 Properties of the continuous-time Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\Omega t} \quad C_k = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jk\Omega t} dt$$

Property	Periodic function $x(t)$ with period $T = 2\pi/\Omega$	Fourier series C_k
Time shifting	$x(t \pm t_0)$	$C_k e^{\pm jk\Omega t_0}$
Time scaling	$x(\alpha t), \alpha > 0$	C_k with period $\frac{T}{\alpha}$
Differentiation	$\frac{d}{dt} x(t)$	$jk\Omega C_k$
Integration	$\int_{-\infty}^t x(t) dt < \infty$	$\frac{1}{jk\Omega} C_k$
	$\sum_i \alpha_i x_i(t)$	$\sum_i \alpha_i C_{ik}$
	$x^*(t)$	C_{-k}^*
Linearity		
Conjugation		
Time reversal	$x(-t)$	C_{-k}
Modulation	$x(t) e^{jK\Omega t}$	$C_{k-i} C_{iK}$
Multiplication	$x(t)y(t)$	$\sum_i C_{k-i} C_i$

Periodic convolution

$$\bar{C}_k = \text{Re } C_{-k},$$

Symmetry

$$\int_T x(\theta)y(t - \theta)d\theta$$

$$TC_{xk}C_{yk}$$

$$\text{Re } \text{Im}C_k = -\text{Im}C_{-k},$$

$$\text{arg}C_k = -\text{arg}C_{-k}$$

$$x(t) = x^*(t) \text{ real}$$

$$\left\{ \begin{array}{l} C_k = C_{-k}^* \\ |C_k| = |C_{-k}| \end{array} \right\} \left\{ \begin{array}{l} C_k = C_{-k} \\ C_k = -C_{-k} \end{array} \right.$$

$$x(t) = x^*(t) = x(-t) \text{ real and even}$$

$$x(t) = x^*(t) = -x(-t) \text{ real and odd}$$

$$\left\{ \begin{array}{l} C_k = C_{-k} \\ C_k = -C_{-k} \end{array} \right. \left\{ \begin{array}{l} \text{real and even} \\ \text{imaginary and odd} \end{array} \right.$$

Parseval's theorem

$$\frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |C_k|^2$$

Table A.2 Properties of the continuous-time Fourier transform

$$\omega \quad X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)e^{j\omega t} d\omega$$



Property	Nonperiodic function $x(t)$	Fourier transform $X(j\omega)$
Time shifting	$x(t \pm t_0)$	$e^{\pm j\omega t_0} X(j\omega)$
Time scaling	$x(\alpha t)$	$\frac{1}{ \alpha } X\left(\frac{j\omega}{\alpha}\right)$
Differentiation		$j\omega X(j\omega)$
Integration		$\frac{1}{j\omega} X(j\omega) + \pi X(j0) \delta(\omega)$
		$\int_{-\infty}^{\infty} X(j\omega) d\omega$
	$\frac{d}{dt} x(t)$	$\sum_i \alpha_i X_i(j\omega)$
	$\int_{-\infty}^t x(t) dt$	$X^*(-j\omega)$
	$\int_{-\infty}^{\infty} x(t) dt$	$X(-j\omega)$
Frequency integration	$\int_{-\infty}^{\infty} x(t) dt$	$X(j\omega - j\omega_0)$
Linearity	$\sum_i \alpha_i x_i(t)$	$\frac{1}{2\pi} X(j\omega) \times Y(j\omega)$
Conjugation	$x^*(t)$	$X(j\omega) Y(j\omega)$
Time reversal	$x(-t)$	$\begin{cases} X(j\omega) = X^*(-j\omega), \\ X(j\omega) = X(-j\omega) , \\ \text{Re } X(j\omega) = \text{Re } X(-j\omega), \\ \text{Im } X(j\omega) = -\text{Im } X(-j\omega) \end{cases}$
Modulation	$x(t)e^{j\omega_0 t}$	
Multiplication	$x(t)y(t)$	
Convolution	$x(t) * y(t)$	
Symmetry	$x(t) = x^*(t)$ real	$\begin{cases} \angle \arg X(j\omega) = -\angle \arg X(-j\omega), \\ X(j\omega) = X^*(-j\omega) \end{cases}$
	$x(t) = x^*(t) = x(-t)$ real and even	$\begin{cases} X(j\omega) = X(j\omega), \\ \text{real and even} \end{cases}$
	$x(t) = x^*(t) = -x(-t)$ real and odd	$\begin{cases} X(j\omega) = -X(-j\omega), \\ X(j\omega) = -X^*(j\omega), \end{cases}$
		$\begin{cases} \text{imaginary and odd} \end{cases}$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

Rayleigh's theorem

B

Tables of Fourier Series and Transform of Basis Signals



Table B.1 The Fourier transform and series of basic signals

Signal $x(t)$	Transform $X(j\omega)$	Series C_k
1	$2\pi\delta(\omega)$	$C_0 = 1, C_{k \neq 0} = 0$ $C_k = \frac{1}{T}$
$\delta(t)$	1	—
$u(t)$	—	—
$u(-t)$	—	—
$e^{j\Omega t}$	$\frac{1}{j\omega} + \pi\delta(\omega)$	$C_1 = 1, C_{k \neq 1} = 0$
$\sum_{k=-\infty}^{\infty} C_k e^{jk\Omega}$	$-\frac{1}{j\omega} + \pi\delta(\omega)$	C_k
$\cos \Omega t$	$\pi\delta(\omega - \Omega)$	$C_1 = C_{-1} = \frac{1}{2}, C_{k \neq \pm 1} = 0$
$\sin \Omega t$	$j\pi \sum_{k=-\infty}^{\infty} C_k \delta(\omega - k\Omega)$	$C_1 = -C_{-1} = \frac{1}{2j}, C_{k \neq \pm 1} = 0$
$\frac{1}{\alpha^2 + t^2}$	$\tau[\delta(\omega - \Omega) + \delta(\omega + \Omega)]$	$\frac{1}{T} e^{-\frac{2\pi\alpha k }{T}}$
Rectangular (Fig. 2.16c)	$\frac{\pi}{j} [\delta(\omega - \Omega) - \delta(\omega + \Omega)]$	$\frac{A}{q} \frac{\sin(k\pi/q)}{k\pi/q}$
Triangular (Fig. 2.21a)	$\frac{e^{-\alpha \omega }}{4T} \frac{\sin(\omega\tau/2)}{\omega\tau/2}$	$\frac{A}{2q} \frac{\sin^2(k\pi/2q)}{(k\pi/2q)^2}$
Trapezoidal (Fig. 2.30)	$\frac{A\tau}{2} \frac{\sin^2(\omega\tau/4)}{(\omega\tau/4)^2}$	$\frac{A}{q} \frac{\sin(k\pi/q)}{k\pi/q} \frac{\sin(k\pi/q_s)}{k\pi/q_s}$
Ramp (Fig. 2.34b)	$4T \frac{\sin(\omega\tau/2)}{\omega\tau/2} \frac{\sin(\omega\tau_s/2)}{\omega\tau_s/2}$	$\frac{A}{j2\pi k} \left[\frac{\sin(k\pi/q)}{k\pi/q} e^{j\frac{k\pi}{q}} - 1 \right]$
Ramp (Fig. 2.34c)	$\frac{A}{j\omega} \left[\frac{\sin(\omega\tau/2)}{\omega\tau/2} e^{j\frac{\omega\tau}{2}} - 1 \right]$	$\frac{A}{j2\pi k} \left[1 - \frac{\sin(k\pi/q)}{k\pi/q} e^{-j\frac{k\pi}{q}} \right]$
$\frac{\sin \alpha t}{\alpha t}$	$\frac{A}{j\omega} \left[1 - \frac{\sin(\omega\tau/2)}{\omega\tau/2} e^{-j\frac{\omega\tau}{2}} \right]$	$\begin{cases} \frac{\pi}{\alpha T}, & k < \frac{\alpha T}{2\pi} \\ 0, & k > \frac{\alpha T}{2\pi} \end{cases}$
$e^{-\alpha t}u(t),$ $\text{Re } \alpha > 0$	$\begin{cases} \frac{\pi}{\alpha}, & \omega < \alpha \\ 0, & \omega > \alpha \end{cases}$	$\frac{1}{\alpha T + j2\pi k}$
$te^{-\alpha t}u(t),$ $\text{Re } \alpha > 0$	$\frac{1}{\alpha + j\omega}$	$\frac{T}{(\alpha T + j2\pi k)^2}$
	$\frac{1}{(\alpha + j\omega)^2}$	

Table B.1 The Fourier transform and series of basic signals (*Contd.*)

$\frac{t^{n-1}}{(n-1)!} e^{-\alpha t} u(t),$ Re $\alpha > 0$	$\frac{1}{(\alpha + j\omega)^n}$	$\frac{T^{n-1}}{(\alpha T + j2\pi k)^n}$
$e^{-\alpha t }, \alpha > 0$	$\frac{2\alpha}{\alpha^2 + \omega^2}$	$\frac{2\alpha T}{\alpha^2 T^2 + 4\pi^2 k^2}$
$e^{-\alpha t^2}$	$\frac{\sqrt{\pi}}{\alpha} e^{-\frac{\omega^2}{4\alpha^2}}$	$\frac{\sqrt{\pi}}{\alpha T} e^{-\frac{\pi^2 k^2}{\alpha^2 T^2}}$

C_k corresponds to $x(t)$ repeated with period T , τ and τ_s are durations, $q = \frac{T}{\tau}$, and $q_s = \frac{T}{\tau_s}$.

Table B.2 The Fourier transform and series of complex signals

Signal $y(t)$	Transform $Y(j\omega)$	Series C_k
Burst of N pulses with known	$\left(\right) \frac{\sin(\frac{2}{\sin(\omega T/2)} \omega NT/2)}{\sin(\omega T/2)}$	$\frac{1}{T_1} X \left(j \frac{2k\pi}{T_1} \right) \frac{\sin(k\pi/q_2)}{\sin(k\pi/Nq_2)}$
Rectangular pulse-burst (Fig. 2.47)	$\frac{A \sin(\omega NT/2)}{\sin(\omega T/2)}$	$\frac{A}{T_1} \frac{\sin(k\pi/q_1)}{k\pi/q_1} \frac{\sin(k\pi/q_2)}{\sin(k\pi/Nq_2)}$
Triangular pulse-burst	$\frac{4}{\pi} \frac{\sin(\omega NT/2)}{\sin(\omega T/2)}$	$\frac{A}{2q_1} \frac{\sin^2(k\pi/2q_1)}{(k\pi/2q_1)^2} \frac{\sin(k\pi/q_2)}{\sin(k\pi/Nq_2)}$
Sinc-shaped pulse-burst		$\begin{cases} \frac{A\pi}{\alpha T_1} \frac{\sin(k\pi/q_2)}{\sin(k\pi/Nq_2)}, & k < \frac{\alpha T_1}{2\pi} \\ 0, & k > \frac{\alpha T_1}{2\pi} \end{cases}$

C_k corresponds to $y(t)$ repeated with period T_1 , τ is pulse duration, T is period of pulse in the burst, T_1 is period of pulse-bursts in the train, $q_1 = \frac{T_1}{\tau}$, and $q_2 = \frac{T_1}{NT}$.

C

Tables of Hilbert Transform and Properties

C Tables of Hilbert Transform and Properties **Table C.1**

Properties of the Hilbert transform

$$y(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\hat{y}(\theta)}{\theta - t} d\theta \quad \hat{y}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{y(\theta)}{t - \theta} d\theta$$

Property	Function $y(t)$	Transform $\hat{y}(t)$
Filtering	$y(t)$ is constant is real is even (odd)	$\hat{y}(t)$ is zero is real is odd (even)
Causality	If $y(t)$ is causal with transform $Y(j\omega) = Y_r(\omega) + jY_i(\omega)$, then: $Y_r(\omega)$ $Y_i(\omega)$	$Y_i(\omega)$ $-Y_r(\omega)$
Linearity	$\sum_i \alpha_i y_i(t)$	$\sum_i \alpha_i \hat{y}_i(t)$
Time shifting	$y(t \pm \theta)$	$\hat{y}(t \pm \theta)$
Time reversal	$y(-t)$ $y(at)$ $y(-at)$	$-\hat{y}(-t)$
Scaling	$y(at)$	$\hat{y}(at)$ $-\hat{y}(-at)$
Multiple transform	$\hat{y}(t)$	$-y(t)$ $H^3 y(t) = H^{-1} y(t)$ $H^4 y(t) = y(t)$
Differentiation	$\frac{d}{dt} y(t)$	$H^n y(t) \Leftrightarrow^F [-j \operatorname{sgn}(\omega)]^n Y(j\omega)$ $\frac{d}{dt} \hat{y}(t)$

Integration	$\int_a^b y(t) dt$, a and b are constants	0
Convolution	$y_1(t) * y_2(t)$	
Autocorrelation	$y(t) * y(t)$	$y_1(t) * \hat{y}_2(t)$ $\widehat{y(t) * y(t)}$ $\hat{y}(t) + \frac{1}{\pi} \int_{-\infty}^{\infty} y(t) dt$
Multiplication	$ty(t)$	

Table C.2 Useful relations between $y(t)$ and its Hilbert transform $\hat{y}(t)$

Property	Relation
Orthogonality	$\int_{-\infty}^{\infty} y(t)\hat{y}(t)dt = 0$
Integration	$\int_{-\infty}^{\infty} y_1(t)\hat{y}_2(t)dt = \int_{-\infty}^{\infty} \hat{y}_1(t)y_2(t)dt$
	$\int_{-\infty}^{\infty} y_1(t)y_2(t)dt = \int_{-\infty}^{\infty} \hat{y}_1(t)\hat{y}_2(t)dt$
	$\int_{-\infty}^{\infty} y^2(t)dt = \int_{-\infty}^{\infty} \hat{y}^2(t)dt$
Energy	$\int_{-\infty}^{\infty} y(t) ^2 dt = \int_{-\infty}^{\infty} \hat{y}(t) ^2 dt$
Autocorrelation	$\int_{-\infty}^{\infty} y(t)y(t-\theta)dt = \int_{-\infty}^{\infty} \hat{y}(t)\hat{y}(t-\theta)dt$

Table C.3 The Hilbert transform of analytic signals

Property	Signal	Transform
Analytic signal	$y_a(t) = y(t) + jy'(t)$	$-jy_a(t) = y'(t) - jy(t)$
Multiplication	$y_{a1}(t)y_{a2}(t)$	$-jy_{a1}(t)y_{a2}(t)$ $= y_{a1}(t)y_{a2}(t)$ $= y_{a1}(t)y_{a2}(t)$

Power ⁿ	j^n	$-jy_a^n(t)$
Real product	$\text{Re}[y_{a1}(t)y_{a2}(t)]$	$\text{Im}[y_{a1}(t)y_{a2}(t)]$
Imaginary product	$\text{Im}[y_{a1}(t)y_{a2}(t)]$	$-\text{Re}[y_{a1}(t)y_{a2}(t)]$

Table C.4 Products of the analytic signals

Product	Relation
$y_{a1}(t)y_{a2}(t)$	$= j\hat{y}_{a1}(t)y_{a2}(t) = jy_{a1}(t)\hat{y}_{a2}(t)$
$\hat{y}_{a1}(t)y_{a2}(t)$	$= -jy_{a1}(t)y_{a2}(t) = y_{a1}(t)\hat{y}_{a2}(t)$

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Table C.5 The Hilbert transform of basic signals

Signal	Transform
$\delta(x)$	$\frac{1}{\pi x}$
$\delta'(x)$	$-\frac{1}{\pi x^2}$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\cos(ax)J_n(bx) \quad [0 < b < a]$	$-\sin(ax)J_n(bx)$
$\frac{\sin x}{x}$	$\frac{\sin^2 x/2}{x/2}$
e^{jx}	je^{jx}

$\frac{a}{x^2+a^2} \quad [a > 0]$	$e^{-x} \quad [x > 0]$	$\frac{x}{x^2+a^2}$
$\frac{ab-x^2}{(ab-x^2)^2+x^2(a+b)^2}$	$\text{sign } xe^{- x } e^{- x }$	$\frac{x(a+b)}{(ab-x^2)^2+x^2(a+b)^2}$
$\delta(x-a) - \delta(x+a)$		$\frac{2}{\pi} \frac{a}{x^2-a^2}$
$\delta(x-a) + \delta(x+a)$		$\frac{2}{\pi} \frac{x}{x^2-a^2}$
$\text{sign } x$		$\left(\frac{\pi v}{2}\right) x ^{v-1} \text{sign } x$
Even rectangular pulse $u(x + \tau) - u(x - \tau)$		$\frac{1}{\pi} e^{-x} \text{Ei}(x)$
$ x ^{v-1} \quad [0 < \text{Re } v < 1]$		$\frac{1}{\pi} [e^{-x} \text{Ei}(x) + e^x \text{Ei}(-x)]$
$ x ^{v-1} \text{sign } x \quad [0 < \text{Re } v < 1]$		$\frac{1}{\pi} [e^{-x} \text{Ei}(x) - e^x \text{Ei}(-x)]$
		\cot
		$-\tan \frac{\pi v}{2} x ^{v-1}$
		$-\infty$
		$\frac{1}{\pi} \ln \frac{x+\tau}{x-\tau}$
		$-\frac{1}{\pi} \ln \frac{x+\tau}{x} + \ln \frac{x-\tau}{x}$
		$\frac{1}{\pi} + \frac{x-\tau}{\tau} \ln \frac{x-\tau}{x} + 1$
		$\frac{1}{\pi} + \frac{x+\tau}{\tau} \ln \frac{x+\tau}{x} + \frac{x-\tau}{\tau} \ln \frac{x-\tau}{x}$
Sawtooth pulse $\frac{1}{\tau}(\tau - x)[-u(x + \tau) + 2u(x) - u(x - \tau)]$		$\frac{1}{\pi} + \frac{x-\tau}{\tau} \ln \frac{x-\tau}{x} - \frac{x+\tau}{\tau} \ln \frac{x+\tau}{x} + 2$
Odd rectangular pulse $-u(x + \tau) + 2u(x) - u(x - \tau)$		
Ramp pulse $\tau^{-1}(\tau - x)[u(x) - u(x - \tau)]$		
Triangular pulse $\tau^{-1}(\tau - x)[u(x + \tau) - u(x - \tau)]$		

Mathematical Formulas

Limits:

- ▷ $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{\partial f(x)/\partial x}{\partial g(x)/\partial x}$ (L'Hospital's rule)
- ▷ $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$
- ▷ $\lim_{x \rightarrow 0} \frac{\sin Nx}{\sin x} = N$
- ▷ $\int_0^{\infty} \sin bx \, dx = \lim_{\alpha \rightarrow 1} \int_0^{\infty} x^{\alpha-1} \sin bx \, dx = \frac{\Gamma(\alpha)}{b^\alpha} \sin \frac{\alpha\pi}{2} \Big|_{\alpha=1} = \frac{1}{b}$

Trigonometric identities:

- ▷ $e^{jx} = \cos x + j \sin x$ (Euler's formula)

$$e^{(\alpha+j)x} = e^\alpha (\cos x + j \sin x)$$

- ▷ $\cos x = \frac{e^{jx} + e^{-jx}}{2}$
- ▷ $\sin x = \frac{e^{jx} - e^{-jx}}{2j}$
- ▷ $\cos^2 x + \sin^2 x = 1$
- ▷ $\cos^2 x - \sin^2 x = \cos 2x$
- ▷ $2 \cos x \sin x = \sin 2x$
- ▷ $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- ▷ $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- ▷ $\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$
- ▷ $\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$

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- ▷ $\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$
- ▷ $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
- ▷ $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
- ▷ $\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$
- ▷ $\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$
- ▷ $\sin x \pm \sin y = 2 \sin \frac{x \pm y}{2} \cos \frac{x \mp y}{2}$

$$\triangleright a \cos x + b \sin x = r \sin(x + \varphi) = r \cos(x - \psi),$$

$$r = \sqrt{a^2 + b^2}, \sin \varphi = \frac{a}{r}, \cos \varphi = \frac{b}{r}, \sin \psi = \frac{b}{r}, \cos \psi = \frac{a}{r}$$

$$\triangleright \frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$\triangleright \frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\triangleright \frac{d}{dx} \arctan x = \frac{1}{1+x^2}$$

$$\triangleright \frac{d}{dx} \operatorname{arccot} x = -\frac{1}{1+x^2}$$

Hyperbolic identities:

$$\triangleright \sinh x = -\sinh(-x) = \pm \sqrt{\cosh^2 x - 1} = \pm \sqrt{\frac{1}{2}(\cosh 2x - 1)} = \frac{e^x - e^{-x}}{2}$$

$$\triangleright \cosh x = \cosh(-x) = \sqrt{\sinh^2 x + 1} = \sqrt{\frac{1}{2}(\cosh 2x + 1)} = 2 \cosh^2 \frac{x}{2} - 1 = \frac{e^x + e^{-x}}{2}$$

$$\triangleright \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{coth} x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\triangleright \cosh^2 x - \sinh^2 x = 1$$

$$\triangleright \cosh x + \sinh x = e^x$$

$$\triangleright \cosh x - \sinh x = e^{-x}$$

Exponents:

$$\triangleright \frac{e^x}{e^y} = e^{x-y}$$

Logarithms:

$$\triangleright \ln \frac{x}{y} = \ln x - \ln y$$

$$\triangleright \ln \alpha x = \ln \alpha + \ln x$$

$$\triangleright \ln x^\alpha = \alpha \ln x$$

Extension to series:

$$\triangleright e^x e^y = e^{x+y}$$

$$\triangleright (e^x)^\alpha = e^{\alpha x}$$

- ▷ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \dots$
- ▷ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + \dots$
- ▷ $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$
- ▷ $e^{jz \cos \phi} = \sum_{n=-\infty}^{\infty} j^n J_n(z) e^{jn\phi}$

Series:

- ▷ $\sum_{n=0}^{N-1} x^n = \frac{1-x^N}{1-x}, \quad x \neq 1 \quad \text{(by geometric progression)}$
- ▷ $\sum_{n=0}^{N-1} e^{\alpha n} = \frac{1-e^{\alpha N}}{1-e^{\alpha}}$
- ▷ $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \quad |x| < 1$
- ▷ $\sum_{k=1}^{\infty} \frac{\sin^2(k\pi/q)}{k^2} = \frac{\pi^2(q-1)}{2q^2}$

Indefinite integrals:

- ▷ $\int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)dx \quad \text{(integration by parts)}$
 - ▷ $\int f(x)dx = \int f[g(y)]g'(y)dy \quad [x = g(y)]$
 - ▷ $\int \frac{dx}{x} = \ln|x|$
 - ▷ $\int e^x dx = e^x \quad \text{]]} \quad \text{(change of variable)}$
 - ▷ $\int a^x dx = \frac{a^x}{\ln a}$
 - ▷ $\int \sin x dx = -\cos x$
 - ▷ $\int \cos x dx = \sin x$
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- ▷ $\int x \begin{Bmatrix} \sin x \\ \cos x \end{Bmatrix} dx = \begin{Bmatrix} \sin x \\ \cos x \end{Bmatrix} \mp x \begin{Bmatrix} \cos x \\ \sin x \end{Bmatrix}$

- ▷ $\int \frac{dx}{x(ax^r+b)} = \frac{1}{rb} \ln \left| \frac{x^r}{ax^r+b} \right|$
- ▷ $\int \sin^2 x \, dx = -\frac{1}{4} \sin 2x + \frac{x}{2}$
- ▷ $\int \cos^2 x \, dx = \frac{1}{4} \sin 2x + \frac{x}{2}$
- ▷ $\int x e^{\alpha x} \, dx = e^{\alpha x} \left(\frac{x}{\alpha} - \frac{1}{\alpha^2} \right)$
- ▷ $\int x^2 e^{\alpha x} \, dx = e^{\alpha x} \left(\frac{x^2}{\alpha} - \frac{2x}{\alpha^2} + \frac{2}{\alpha^3} \right)$
- ▷ $\int x^\lambda e^{\alpha x} \, dx = \frac{1}{\alpha} x^\lambda e^{\alpha x} - \frac{\lambda}{\alpha} \int x^{\lambda-1} e^{\alpha x} \, dx$
- ▷ $\int e^{-(ax^2+bx+c)} \, dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} e^{\frac{b^2-4ac}{4a}} \operatorname{erf} \left(x\sqrt{a} + \frac{b}{2\sqrt{a}} \right)$
- ▷ $\int \frac{1}{x} e^{\alpha x} \, dx = \operatorname{Ei}(\alpha x) \quad [\alpha \neq 0]$
- ▷ $\int \frac{1}{\sqrt{x}} e^{-\alpha x} \, dx = \sqrt{\frac{\pi}{\alpha}} \operatorname{erf}(\sqrt{\alpha x}) \quad [\alpha > 0]$
- ▷ $\int \frac{e^x}{x^2+\alpha^2} \, dx = \frac{1}{\alpha} \operatorname{Im}[e^{j\alpha} \operatorname{Ei}(x-j\alpha)]$

Definite integrals:

- ▷ $\int_{-\infty}^{\infty} \frac{\sin \alpha x}{x} \, dx = \pi$
- ▷ $\int_{-\infty}^{\infty} e^{-\alpha x^2} \, dx = \sqrt{\frac{\pi}{\alpha}}$
- ▷ $\int_{-\infty}^{\infty} x^2 e^{-\alpha x^2} \, dx = \sqrt{\pi} \alpha^{-3/2}$
- ▷ $\int_0^{\infty} \frac{\sin \alpha x}{x} \, dx = \frac{\pi}{2} \operatorname{sgn} \alpha$
- ▷ $\int_0^{\infty} \frac{\sin^2 \alpha x}{x^2} \, dx = \frac{\pi \alpha}{2}$
- ▷ $\int_{-\infty}^{\infty} \frac{\sin^4 \alpha x}{x^2} \, dx = \frac{\pi \alpha}{2}$
- ▷ $\int_0^{\infty} \frac{dx}{\alpha^2+x^2} = \frac{\pi}{2\alpha}$
- ▷ $\int_0^{\infty} x^{\alpha-1} \sin bx \, dx = \frac{\Gamma(\alpha)}{b^\alpha} \sin \frac{\alpha\pi}{2}$

- ▷ $\int_0^{\infty} \sin bx \, dx = \frac{1}{b}$
- ▷ $\int_{-\infty}^{\infty} \frac{\sin x}{x(x-\alpha)} dx = \frac{\pi}{\alpha} (\cos \alpha - 1)$, α is real
- ▷ $\int_{-\infty}^{\infty} \frac{\cos(ax)}{b^2-x^2} dx = \frac{\pi}{2b} \sin(ab)$, $a, b > 0$

Special functions:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad (\text{Error function})$$

$$\operatorname{Ei}(x) = - \int_x^{\infty} \frac{e^{-t}}{t} dt = \int_{-\infty}^x \frac{e^t}{t} dt \quad [x < \infty]$$

(Exponential-integral function)

$$\operatorname{Ei}(x) = e^x \left[\frac{1}{x} + \int_0^{\infty} \frac{e^{-t}}{(x-t)^2} dt \right] \quad [x > 0]$$

(Exponential-integral function)

$$\operatorname{S}(x) = \frac{2}{\sqrt{2\pi}} \int_0^x \sin t^2 dt = \int_0^x \sin \frac{\pi t^2}{2} dt \quad (\text{Fresnel integral})$$

$$\operatorname{C}(x) = \frac{2}{\sqrt{2\pi}} \int_0^x \cos t^2 dt = \int_0^x \cos \frac{\pi t^2}{2} dt \quad (\text{Fresnel integral})$$

$$J_n(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-j(n\theta - z \sin \theta)} d\theta \quad (\text{Bessel function of the first kind})$$

