

Wave Properties of Particles
Part 2
de- Broglie Waves

Problems

1. What is the wavelength of a particle of mass 2 mg with velocity 0.5c ?

Solution : Given

$$\begin{aligned} m &= 2 \text{ mg} = 2 \times 10^{-6} \text{ kg} \\ V &= 0.5c \\ \lambda &= \frac{h}{m_0 \gamma v} \\ &= \frac{h}{m_0 v} \sqrt{1 - \frac{v^2}{c^2}} \\ &= \frac{6.626 \times 10^{-34}}{2 \times 10^{-6} \times \frac{c}{2}} \sqrt{1 - \frac{c^2}{4c^2}} \\ \lambda &= 1.9 \times 10^{-36} \end{aligned}$$

2. What is the de-Broglie wavelength of 1 MeV proton ?

Solution : Given

$$\begin{aligned} K &= 1 \text{ MeV} \\ \text{Rest mass energy of proton, } E_0 &= \frac{mc^2}{e} \\ E_0 &= \frac{1.6 \times 10^{-27} (3 \times 10^8)^2}{1.6 \times 10^{-19}} = 939.4 \text{ Mev} \end{aligned}$$

Since $k \ll E_0$, it is non-relativistic case

$$\begin{aligned} \lambda &= \frac{h}{\sqrt{2mk}} = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 1.6 \times 10^{-27} \times 10^6 \times 1.6 \times 10^{-19}}} \\ \lambda &= 2.96 \times 10^{-14} \text{ m} \end{aligned}$$

3. What is the de- Broglie wavelength of 1 MeV Electron ?

Solution : Given

$$\begin{aligned} K &= 1 \text{ MeV} \\ \text{Rest mass energy of electron } E_0 &= \frac{mc^2}{e} = 0.511 \text{ MeV} \end{aligned}$$

Since $k \gg E_0$, it is relativistic

$$\begin{aligned}\lambda &= \frac{hc}{\sqrt{k^2 + 2kE_0}} \\ &= \frac{1.26 \times 10^{-6} \text{ eV m}}{\sqrt{1 \text{ MeV}^2 + 2 \times \text{MeV} \times 0.5 \text{ MeV}}} \\ &= 0.87 \text{ pm}\end{aligned}$$

4. Find the kinetic energy of a proton whose de Broglie wavelength is 1 fm

Solution ; Given

$$\lambda = 1 \text{ fm}$$

If $Pc > E_0$, it is relativistic

$$\lambda = \frac{h}{p}$$

$$Pc = \frac{hc}{\lambda}$$

As $E_0 = 939.6 \text{ MeV}$ pc must be in eV

$$Pc = \frac{1.26 \times 10^{-6} \text{ eV m}}{10^{-15} \text{ m}} = 1.26 \times 10^9 \text{ eV} = 1.26 \text{ GeV}$$

$$E_0 = 0.939 \text{ GeV}$$

Clearly If $Pc > E_0$, it is relativistic

Also $E = K + E_0$

$$E = \sqrt{p^2 c^2 + E_0^2}$$

$$\begin{aligned}E &= \sqrt{(1.26^2)(\text{GeV})^2 + (0.939^2)(\text{GeV})^2} \\ &= \sqrt{1.53 + 0.9} \text{ GeV} = \sqrt{2.5} \text{ GeV} = 1.55 \text{ GeV}\end{aligned}$$

Then Kinetic energy, $K = E - E_0$

$$K = 1.55 - 0.939 = 0.617 \text{ GeV} = 617 \text{ MeV}$$