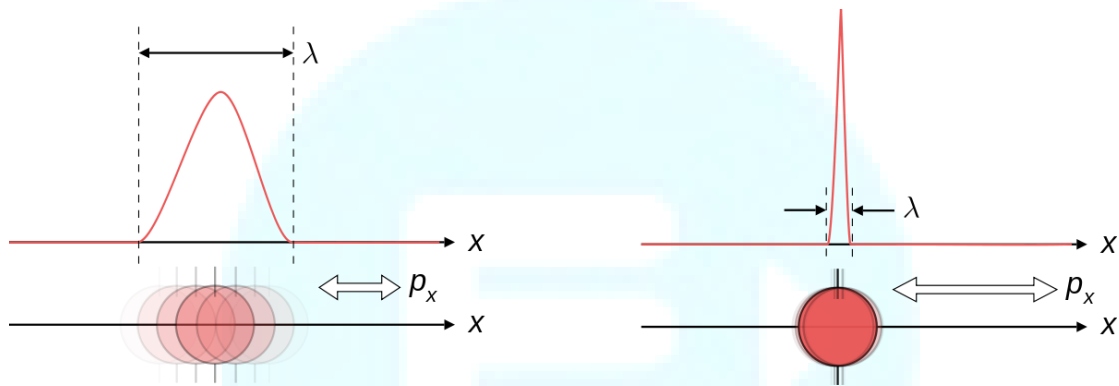


Wave properties of particles
Part 5
Heisenberg's uncertainty Principle

Uncertainty Principle

- We cannot know the future because we cannot know the present.
- To regard a moving particle as a wave implies that there are fundamental limits to the accuracy with which we can measure such “ Particle “ properties as position and momentum.



- It is impossible to know both the exact position and exact momentum of an object at the same time.

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\hbar = \frac{h}{2\pi} = 1.054 \times 10^{-34} \text{ J.s}$$

Uncertainties in energy and momentum

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

This law help us to know;

- The non - existence of free electrons inside Nucleus.
- Ground state energy of systems like Hydrogen atom, Harmonic oscillator etc.
- Width of spectral lines.

Problems

1. Find the uncertainty in the measurement of position of :

a) A neutron moving with a velocity $5 \times 10^6 \text{ m/s}$

b) A 50kg man moving with a velocity 2 ms^{-1}

Solution :

a) Given $V = 5 \times 10^6 \text{ m/s}$

$$\Delta x = ?$$

$$\Delta x \Delta p \geq \frac{\hbar}{2} \text{ ----- (1)}$$

$$P = mv$$

$$\Delta p = m \Delta V$$

$$\Delta x m \Delta V = \frac{\hbar}{2}$$

$$\Delta x = \frac{\hbar}{2mV}$$

$$= \frac{1.054 \times 10^{-34}}{2 \times 1.6 \times 10^{-27} \times 5 \times 10^6}$$

$$\Delta x = 0.65 \times 10^{-16} = 6.5 \times 10^{-15} \text{ m} = 6.5 \text{ fm}$$

b) $V = 2 \text{ ms}^{-1}$

$$\Delta x = \frac{\hbar}{2mV}$$

$$= \frac{1.054 \times 10^{-34}}{2 \times 50 \times 2}$$

$$= 0.5 \times 10^{-36} \text{ m}$$

2. A ball of mass 145g is moving at a speed 42.5 m/s. Assume that its speed can be measured to a precision of 1%. With what precision we can measure its position?

Solution :

Given $m = 145 \text{ g} = 145 \times 10^{-3} \text{ Kg}$

$$V = 42.5 \text{ m/s}$$

$$\Delta V = \frac{1}{100} \times 42.5 = 0.425$$

$$\Delta x = ?$$

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x m \Delta V = \frac{\hbar}{2}$$

$$\Delta x = \frac{\hbar}{2m\Delta V}$$

$$\Delta x = \frac{1.05 \times 10^{-36}}{2 \times 145 \times 10^{-3} \times 0.625}$$

$$\Delta x = 8.5 \times 10^{-36} \text{ m}$$

3. A measurement establishes the position of a proton with accuracy of $\pm 1 \times 10^{-11}$ m. Find the uncertainty in the proton's position 1.00 s later.

Assume $v \ll c$

Solution :

Given $\Delta x_{t=0} = 10^{-11} \text{ m}$

$$\Delta x_{t=1} = ?$$

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta p = \frac{\hbar}{2\Delta x}$$

$$m\Delta V = \frac{\hbar}{2\Delta x}$$

$$\Delta v = \frac{\hbar}{2m\Delta x}$$

$$\Delta x_{t=1} = \Delta v t$$

$$= \frac{\hbar}{2m\Delta x}$$

$$\Delta x_{t=1} = \frac{1.05 \times 10^{-36}}{2 \times 1.6 \times 10^{-27} \times 10^{-11}}$$

$$\Delta x_{t=1} = 3.15 \times 10^3 \text{ m}$$

4. A hydrogen atom is 5.3×10^{-11} m in radius. Use the uncertainty principle to estimate the minimum energy an electron can have in this atom.

Solution :

$$\text{Given } \Delta x = 5.3 \times 10^{-11}$$

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

From the above relation we can find that $\Delta x \propto \frac{1}{\Delta p}$

$$\Delta x_{\max} \times \Delta p_{\min} = \frac{\hbar}{2}$$

$$\Delta x_{\min} = 5.3 \times 10^{-11}$$

$$\begin{aligned} \Delta p_{\min} &= \frac{\hbar}{2\Delta x} \\ &= \frac{1.05 \times 10^{-36}}{2 \times 5.3 \times 10^{-11}} \end{aligned}$$

$$\Delta p_{\min} = 9.9 \times 10^{-25} \text{ kg ms}^{-1}$$

$$\begin{aligned} K_{\min} &= \frac{p_{\min}^2}{2m} = \frac{(9.9 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}} \\ &= \frac{5.6 \times 10^{-19} \text{ J}}{1.6 \times 10^{-19}} \end{aligned}$$

$$K_{\min} = 3.4 \text{ eV}$$