ENTRI

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SET MATHS
100 FREE QUESTIONS

1. Let A, B and C be non -empty sets and let X = (A-B)- C and Y = (A - C) - (B - C). Which one of the following is TRUE?

a) **X = Y**

b) X ⊂⊂ Y

c) Y ⊂⊂ X

d) None of these

Solution

X = (A - B) - C

= (A ∩∩ B') - C

= (A ∩∩ B') ∩∩ C'

= AB'C'

Y = (A - C)-(B-C)

= (A ∩∩ C')-(B ∩∩ C)

= (A C')-(BC')

= (A C') ∩∩ (BC')'

= (A C') ∩∩ (B' + C)

= (A C') .(B' + C)

= AC'B' + AC'C

= AC'B' (Since CC'=0)

= AB'C' (Commutative propery)

Therefore X = Y

2. Let T: $R^2 \rightarrow R^2$ **be a linear transformation such that** T(1,2)=(2,3) and T(0,1)=(1,4)**.**
Then T (5,6) **is ?**

a. (6,-1)

b. (-6,1)

c. (-1,6)

d. (1,-6)

Solution

we have to find: T(1,0)

T(1,0) = T{ (1,2)-2(0,1)}

= T (1,2)-2 T (0,1)

= (2,3)-2(1,4)

= (2, 3)- (2,8)

= (0, -5)

T(5,6)=T{5(1,0)+6(0,1)}

= 5T(1,0) + 6T(0,1)

= 5(0, -5) + 6(1,4)

= (0, -25) + (6,24)

= (6, -1)

3. Remainder when 97! is divided by 101 is

A. 15
B. 16
C. 17
D. None of these

Solution

If p is prime then $(p-1)! \equiv -1 \pmod{p}$

101 is prime

$(101-1)! \equiv -1(\text{mod}101)$

$(100)! \equiv -1(\text{mod}101)$

$100\times99\times98\times97! \equiv -1(\text{mod}101)$

ie, $-1\times-2\times-3\times97! \equiv -1(\text{mod}101)$

since 100(mod 101) = -1, 99 (mod 101) = -2 and 98(mod 101) = -3

$6\times97! \equiv 102(\text{mod}101)$, remove negative

$97! \equiv 17(\text{mod}101)$ and $\frac{102}{6} = 17$

4. An insurance company classifies insured policyholders into a accident prone or non-accident prone. Their current risk model works with the following probabilities. The probability that an accident prone insured has an accident within a year is 0.4.The probability that a non- accident prone insured has an accident within a year is 0.2 .If 30% of the population is accident prone, What is the probability that a policy holder will have an accident within a year ?

A. 0.12
B. **0.26**
C. 0.65
D. 0.75

Solution

A1:1: Policy holder will have an accident within a year

A: Policy holder is accident prone.

$P(A1=P(A1/A)P(A)+P(A1/Ac)(1-P(A)$

$=0.4\times0.3+0.2(1-0.3)0.4\times0.3+0.2(1-0.3) = 0.26$

5. An insurance company classifies insured policyholders into a accident prone or non-accident prone. Their current risk model works with the following probabilities.
The probability that an accident prone insured has an accident within a year is 0.4.
The probability that a non- accident prone insured has an accident within a year is 0.2.
If 30% of the population is accident prone .
Suppose now that the policy holder has had an accident within one year. What is the probability that he/she is accident prone?

a. $\frac{1}{14}$

b. $\frac{3}{14}$

c. $\frac{5}{14}$

d. $\frac{6}{14}$

Solution

A1: Policy holder will have an accident within a year.
A: Policy holder is accident prone
$P(A1) = P(A1/A)\,P(A) + P(A1/A^C)(1-P(A))$
$= 0.4\times0.3+0.2(1-0.3) = 0.26$

$P(A/A1) = \frac{P(A)P(A1-A)}{P(A1)}$

$= \frac{0.3*0.4}{0.26} = \frac{6}{14}$

6. Two dice are rolled. Consider the events A={sum of two dice equals 3}, B={Sum of two dice equals 7} and C={at least one of the dice shows a 1}
What is P(A/C) ?

a. $\frac{2}{11}$

b. $\frac{3}{11}$

c. $\frac{5}{11}$

d. $\frac{10}{11}$

Solution

Sample space, S{(i, j)| i,j=1,2,3,4,5,6} with each outcome equally likely.
A={(1,2),(2,1)}
C={(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),((3,1),(4,1),(5,1),(6,1)}

$P(A/C) = \frac{P(A \cap C)}{P(C)}$

$= \frac{2}{11}$

7. Two dice are rolled. Consider the events A={sum of two dice equals 3}, B={Sum of two dice equals 7} and C={at least one of the dice shows a 1}
What is P(B/C) ?

a. $\frac{1}{11}$

b. $\frac{2}{11}$

c. $\frac{3}{11}$

d. $\frac{4}{11}$

solution

Sample space, S{(i,j)| i,j=1,2,3,4,5,6} with each outcome equally likely.
B={(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)}
C={(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),((3,1),(4,1),(5,1),(6,1)}
P(B/C)= $= \frac{P\ (B\cap C)}{P(C)}$

$= \frac{2}{11}$

8. The value of $\sqrt{(-16)}$ is ?

a. -4i

b. 4i
c.-2i
d. 2i

Solution

$\sqrt{(-16)} = \sqrt{16} \times \sqrt{(-1)} = 4i$

9. The value of $\sqrt{(-49)} + 3\sqrt{(-4)} + 2\sqrt{(-9)}$ is ?

A. 19i
B. -19i
C. 17i
D. -17i

Solution

$\sqrt{(-49)} + 3\sqrt{(-4)} + 2\sqrt{-9}$
$= \sqrt{(49)} \times \sqrt{(-1)} + 3\sqrt{(4)} \times \sqrt{(-1)} + 2\sqrt{(9)} \times \sqrt{(-1)}$

$=7i+6i+6i$
$=19\,i$

10. The curve represented by $im(z^2)=k$, where k is a non zero real number is ?

A. A pair of straight line
B. An ellipse
C. A parabola
D. **A hyperbola**

Solution

let $z=x+iy$
$z^2=(x+iy)$
$z^2=x^2-y^2+2xyi$
$Im(z)=k$
$2xy=k$
$xy=k/2$ which is a hyperbola.

11. How many prime numbers lie between 1 and 30 ?

A. 8
B. 9
C. **10**
D. 11

Solution

Prime numbers between 1 and 30 are:
2,3,5,7,11,13,17,19,23,29

View Details

12. The sum of squares of distinct common prime factors of 120, 210 and 330 is ?

A. 34
B. **38**
C. 39
D. 46

Solution

Prime factors of $120=2\times2\times2\times3\times52\times2\times2\times3\times5$
Prime factors of $210=2\times3\times5\times72\times3\times5\times7$
Prime factors of $330=2\times5\times5\times12\times5\times5\times1$
Common distinct primes=2,3,5
The sum of squares of distinct common prime factors $=2^2+3^2+5^2=38$

13. Which of the following group of numbers has least prime numbers ?

A. **From 20 to 40**
B. From 30 to 50
C. From 40 to 60
D. From 60 to 80

Solution

- From 20 to 40 primes: 23, 29, 31 and 37
- From 30 to 50 primes:31, 37, 41, 43 and 47
- From 40 to 60 primes:41, 43, 47, 53 and 59
- From 60 to 80 primes:61, 67, 71, 73 and 79

14. A polynomial of degree p has ....

A. Only one zero
B. At least p zeroes
C More than p zeroes
D. Atmost p zeroes

Solution

A polynomial's maximum number of zeroes equals the polynomial's degree.

15. Zeroes of $p(x)=x^2-27$ are ?

A. $\pm 3-\sqrt{3}$
B. $\pm 9-\sqrt{3}$
C. $\pm 7\sqrt{3}$
D. None of the above

Solution

$X^2-27=0$
$\Rightarrow x^2=27$
$\Rightarrow x=\sqrt{(27)}$
$=\pm 3\sqrt{3}$

16. If a quadratic polynomial's discriminant is D, is greater than zero, the polynomial has ....

A. Two real and equal roots
B. **Two real and unequal roots**
C. Imaginary roots
D. No roots

Solution

If the discriminant of a quadratic polynomial $D > 0$ then the polynomial has real and unequal roots.

If the discriminant of a quadratic polynomial D = 0 then the polynomial has real and equal roots.

If the discriminant of a quadratic polynomial D < 0 then the polynomial has imaginary and unequal roots.

17. What is the domain of the function $f(x)=x^{1/3}$?

A. Domain is (2,∞)
B. Domain is (−∞,1)
C. **Domain is [**0,∞)0,∞)
D .None of the above

Solution

A square root function is not defined for negative real numbers.

Hence any value less then zero is not possible. Hence the domain is [0,∞].

18. The domain of a function is ....

A. **The maximal set of numbers for which a function is defined**
B. The maximal set of numbers which a function can take values
C. It is a set of natural numbers for which a function is defined
D. None of the above

Solution

The domain of a function is the maximal set of numbers for which a function is defined.

19. The range of a function is ......

A. The maximal set of numbers for which a function is defined
B. **The maximal set of numbers which a function can take values**
C. It is a set of natural numbers for which a function is defined
D. None of the above

Solution

The range of a function is the maximal set of numbers which a function can take values.

20. If f: A→B will be an into function if

A. **Range(f) ⊂B**
B. f(a)=B
C. B ⊂f(a)
D. f(b) ⊂A

Solution

If f: A→B will be an into function if range(f) ⊂B.

21. Consider the nonempty set consisting of children in a family and a relation R defined as aRb if a is brother of b. Then R is ?

A. Symmetric but not transitive
B. **Transitive but not symmetric**
C. Neither symmetric nor transitive
D. Both symmetric and transitive

Solution

Consider the non empty set consisting of children in a family and a relation R defined as aRb if a is brother of b. Then R is Transitive but not symmetric.

22. The maximum number of equivalence relation on the set A={1,2,3} are ?

A. 12
B. 2
C. 3
D. **5**

Solution

Ans : 5
R_1= {(1,1),(2,2),(3,3)}
R_2 = {(1,1),(2,2),(3,3),(1,2),(2,1)}
R_3= {(1,1),(2,2),(3,3),(1,3),(3,1)}
R_4= {(1,1),(2,2),(3,3),(2,3),(3,2)}
R_5= {(1,1),(2,2),(3,3),(1,2),(2,1),(1,3),(3,1),(2,3),3,2)}}

23. An injection is a function which is ......

A. Many-one
B. **One-one**
C. Onto
D. None of these

Solution

One-one functions are known as injection.

Onto function are known as surjective function.

A function which is both surjective and injective is called bijective

24. A function f:X→Y is one-one if , ...

A. $f(x_1)\neq f(x_2) \forall x_1,x_2 \in X$
B. **if** $f(x_1) = f(x_2)$ **then** $x_1 = x_2 \ \forall x_1,x_2 \in X$
C. $f(x_1)=f(x_2), \forall x_1,x_2 \in X$

D. None of the above

Solution

In one-one function every element in A should have unique image in B. Thus two images are equal means their pre images are same.

25. A function is defined by mapping f:A→B such that A contains m elements and B contains n elements and $m \le n$, then the number of one-one functions are ....

A. **$nC_m \times m!$**
B. $nC_m \times n!$
C. 0
D. n + m

Solution

From n elements in B we need to select m elements and arrange them in all ways. So the answer is $nC_m \times m!$.

26.A function is defined by mapping f:A→B such that A contains m elements and B contains n elements and m>n, then the number of one-one functions are .....

A. $nC_m \times m!$
B. $nC_m \times n!$
C. (mn)!
D. **0**

Solution

Since m > n at least some elements should have same image. So there will not any one-one function.

27. Let X be a non-empty set and R be set of real numbers then d:X×X→R is called.....

A. Metric
B. Distance function
C. Metric space
D. **Both (a) and (b)**

Solution

Let X be a non-empty set and R be set of real numbers then d:X×X→R is called metric or distance function.
(X, d) is called metric space, when the distance function d satisfies some reasonable properties.

28. Which of the following is incorrect for a distance function d ?

A. $d(x,y) \ge 0$
B. $d(x,x)= d(y,y)$
C. $d(x,y)=d(y,x)$
D. $d(x,y)+d(y,z)\le d(x,z)$

Solution

Properties of distance function d:
For any $x,y \in X$

- $d(x,y) \geq 0$
- $d(x,y)=0 \Rightarrow x=y$
- $d(x,y)=d(y,x)$
- $d(x,z) \leq d(x,y)+d(y,z)$

29. For a metric d on a non empty set X, the metric space is represented as:

A. **(X, d)**
B. (d,X)
C. $(X, d) \to R$
D. $X:X \to d$

Solution

A metric space is a pair (X,d), where X is a set and $d:X \times X$ is a map which satisfies some properties.For any $x,y \in X$

- $d(x,y) \geq 0$
- $d(x,y)=0 \Rightarrow x=y$
- $d(x,y)=d(y,x)$
- $d(x,z) \leq d(x,y)+d(y,z)$

30. The space (R_n, d) is called.... ?

A. Real metric space
B. **n-dimensional Euclidean space**
C. n-dimensional real space
D. None of these

Solution

The space (R_n,d) is called an n-dimensional Euclidean space.
$d:R_n \times R_n \to R$ is defined by,
$d(x,y)=||x-y||$

31.What is the radius of the circle $x^2+ y^2-6y =0$?

A. 2
B. **3**
C. 4
D. 5

Solution

$X^2+y^2-6y =0$
By completing the square

$x^2 + y^2 - 6y + 9 = 9$

$x^2 + (y-3)^2 = 3^2$

Standard equation of a circle with centre at (h, k) is

$(x-h)^2 + (y-k)^2 = r^2$

So, r=3

32. What are the coordinates of the centre of the curve $x^2 + y^2 - 2x - 4y - 31 = 0$?

a. (-1,-1)
b. (-2,-2)
c. **(1,2)**
d. (2,1)

Solution

$x^2 + y^2 - 2x - 4y - 31 = 0$

$x^2 - 2x + y^2 - 4y - 31 = 0$

By completing the square

$(x-1)^2 + (y-2)^2 = 36$

Standard equation of the circle with centre (h, k) is

$(x-h)^2 + (y-k)^2 = r^2$

h=1 and k=2 centre= (1,2)

33. A circle whose equation is $x^2 + y^2 + 4x + 6y - 23 = 0$ has its centre at ?

A. (2,3)
B. (3,2)
C. (-3,2)
D. **(-2,-3)**

Solution

$X^2 + y^2 + 4x + 6y - 23 = 0$

By completing the square

$X^2 + 4x + 2^2 + y^2 + 6y + 3^2 = 23 + 2^2 + 3^2$

$(x+2)^2 + (y+3)^2 = 6^2$

Standard equation of a circle with centre at (h,k) is $(x-h)^2 + (y-k)^2 = r^2$

So, h= -2 and k= -3 centre= (-2,-3)

34. What is the radius of the circle with equation $X^2 - 6x + y^2 - 4y - 12 = 0$?

A. 3.46
B. 7
C. **5**
D. 6

Solution

$X^2 - 6x + y^2 - 4y - 12 = 0$
By completing the square
$X^2 - 6x + 3^2 + y^2 - 4y + 2^2 = 12 + 2^2 + 3^2$
$(x-3)^2 + (y-2)^2 = 5^2$
Standard equation of a circle with centre at (h,k) is

$(x-h)^2 + (y-k)^2 = r^2$
So, r=5

35. What is the value of $\lim_{y\to 2} (y^2 - 4)/(y-2)$

A. 2
B. **4**
C. 1
D. 0

Solution

$\lim_{y\to 2} (y^2 - 4)/(y-2)$
$= \lim_{y\to 2} (y-2)(y+2)/(y-2)$
=2+2=4

36. What is the value of $\lim_{y\to \infty 2y} 2/y$?

A. **0**
B. 1
C. 2
D. ∞

Solution

Any number divided by ∞ gives 0.
$\lim_{y\to\infty} 2/y$

$= 2/\infty = 0$

37. What is the value of $\lim_{x\to 4} (x^2 - 2x - 8)/(x-4)$?

A. 0
B. 2
C. 8

D. **6**

Solution

$\lim_{x\to4} (x^2 - 2x - 8)/(x-4)$
$\lim_{x\to4} (x-4)(x-2))/(x-4) = 6$

38. What is the value of $\lim_{x\to3} (x^2-9)/(x-3)$?

A. 0
B. 3
C. ∞
D. **6**

Solution

$\lim_{x\to3} (x^2-9)/(x-3)$
$\lim_{x\to3} (x-3)(x+3)/(x-3) = 6$

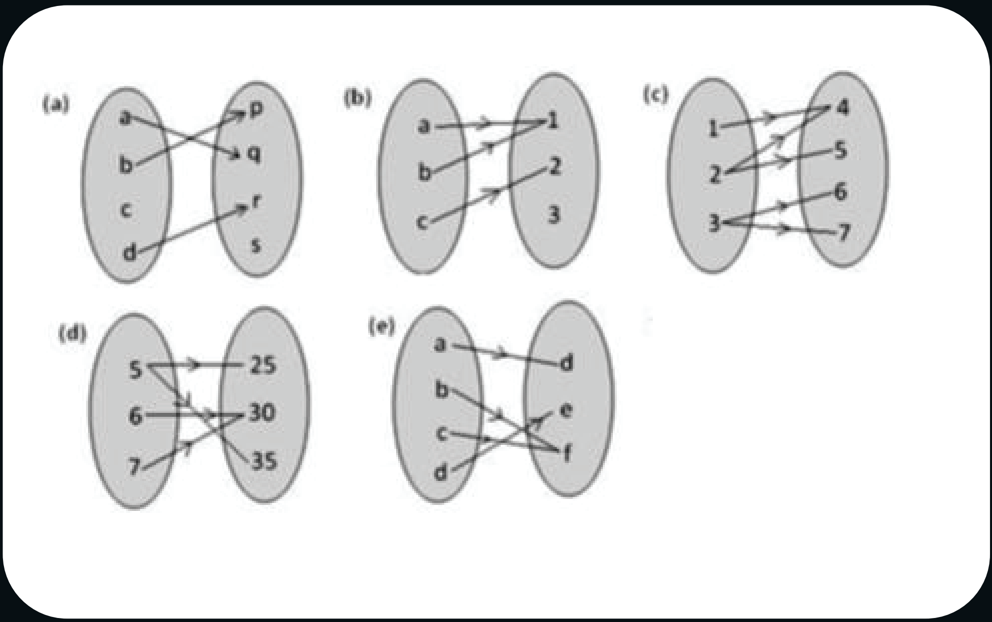
39. Which of the following relation is functions ?

A. R_1={(1,7),(2,7),(4,7),(6,7)}
B. R_2={(1,2),(1,3),(1,4),(1,5)}
C. R_3 **={(x,y),(y,z),(z,t),(t,v)}**
D. None of the above

Solution

R_3={(x,y),(y,z),(z,t),(t,v)} every element in the domain has unique image in the range.

40.which of the arrow diagram represent functions



A. a, b, c, e

B. b, c, d, e
C.c, d
D. **b, e**

Solution

A function is a mapping which assigns for every value in the domain one and only one value in the range.
In (a) not all value in the domain are assigned a value in range.
In (b) and (e) every value in the domain one and only one value in the range.
In (c) and (d) value in the domain has multiple range values.

41. Given f(x)=3x-1, find the value of f(-2) ?

A. -5
B. **-7**
C. 7
D. -5

Solution

f(-2) = $3\times-2-1 3\times-2-1$
=-6-1= -7

42. If A, B and C are any three sets then $A-(B\cup C)$ = ?

A. $(A-B)\cup(A-C)$
B. $(A-B)\cup C$
C. $(A-B)\cap C$
D. $(A-B)\cap(A-C)$

Solution

$A-(B\cup C)$ $=A\cap(B\cup C)'$
$=A\cap B'\cap C'$..(1)
$(A-B)\cap(A-C)=(A\cap B')\cap(A\cap C')$
$=A\cap B'\cap C'$..(2)
(1)=(2)

43.Which of the following statement is true ?

(i)If a subset $A\subset X$�⊂� is closed in X, then every sequency of points of A that converges must converge to a point of A

(ii)Both ϕ� and X are closed in X

A. Only (i) follows
B. Only (ii) follows
C. **Both are true**
D. None of the above is true

Solution

(i)If a subset $A \subset X$ is closed in X, then every sequence of points of A that converges must converge to a point of A

(ii)Both ϕ and X are closed in X

A set is open if every point in is an interior point.

A set is closed if it contains all of its boundary points.

44. Which of the following statement is true ?

A. $A \subset \bar{A}$ for any set A
B. If $A \subset B$ then $\bar{A} \subset \bar{B}$ as well
C. The set A is closed iff $\bar{A}=A$
D. **All of the above**

Solution

- $A \subset \bar{A}$ for any set A
- If $A \subset B \subset$ then $\bar{A} \subset \bar{B}$ as well
- The set A is closed iff $\bar{A}=A$
- $\bar{A}$ is the smallest closed set that contains A.
- The closure of $\bar{A}$ is itself.

45. For any set A, $(\bar{A})^{-}$ =?

A. **A**
B. $\bar{A}$
C. U
D. ϕ

Solution

Let U={1,2,3,4,5,6}
A={1,2}
A'= U-A={3,4,5,6}
(A')'=U-(U-A)={1,2}=A

46. Let A be any set and A^0 be the interior of A then which of the following is true ?

A. $A^0 \subset A$ for any set A
B. If $A \subset B$ then $A^0 \subset B^0$ as well
C. The set A is open iff $A^0=A$
D. **All the above**

Solution

$A^0 \subset A$ for any set A

If $A \subset B$ then $A^0 \subset B^0$ as well
The set A is open iff $A^0=A$

47. Let A be any set and A^0 be the interior of A then $(A^0)^0$= ?

A. A
B. A^0
C. U
D. ϕ

Solution

- Let A be any set and A^0 be the interior of A then . $(A^0)^0=(A^0)$
- $(A^0)^0$ is an open set so it must be equal to its own interior
- (A^0) is the largest open set contained in A
- (A^0) is equal to A iff A is open

48. Which of the following are groups ?

A. $(Q^+, *)$
B. $(Z,+)$
C. $(R -\{0\} , *)$
D. **All the above**

Solution

- An algebraic system (G, *)consisting of a non empty set G together with a binary composition* is called a group if the following axioms are satisfy.

  1.Associative property
  2.Existance of identity element
  3.existence of inverse

- All the above are examples of groups.

49. Number of elements in a group is called.... of that group.

A. Range
B. Array
C. **Order**
D. Frequency

Solution

Number of elements in a group, G is called order of that group. It is denoted by |G| or O(G).

50. The order of a group G is denoted as..... ?

A. O(G)
B. |G|
C. e(G)

D. **Both A and B**

Solution

Number of elements in a group, G is called order of that group. It is denoted by |G| or O(G).

51. Let G be a group then the least positive integer n is said to be the order of an element if,

A. a=e
B. na=e
C. a^n=e
D. ae=0

Solution

- Let G be a group then the least positive integer n is said to be the order of an element if, a^n=e.
- Let G be a group and g∈G. We say g has finite order if g^n=e for some integer n.
- In abelian group t the order of each element divides the size of the group

52. R is called a ring with unit element if,

A. **If ∃1∈R such that 1.a=a.1 =a** ∀a∈R
B. If ∃1∈R such that 1+a=a+1 =a ∀a∈R
C. If ∃1∈R such that 1.a=a.1 =0 ∀a∈R
D. If ∃1∈R such that 1.a=a.1 =1 ∀a∈R

Solution

R is called a ring with unit element if, If ∃1∈R such that 1.a=a.1 =a ∀a∈R
Usually unit element of R is denoted by 1.
Let (R, +, .) be a ring and a,b,c are any three elements of R, then
a.0=0.a=0
a.(-b)= -(a.b)=(-a).b

53. If the multiplication of the ring R is such that a.b= b.a ∀a,b∈R , then R is known as...?

A. Additive ring
B. Inverse ring
C. **Commutative ring**
D. Non commutative ring

Solution

If the multiplication of the ring R is such that a.b= b.a ∀a,b∈R, then R is known as commutative ring.
Boolean ring: If x^2=x for all x∈R

54. Suppose R is the set of integers positive, negative and 0, + is the usual addition and '.' is the usual multiplication then which of the following is true ?

A. **R is a commutative ring with unit element**
B. R is a commutative ring without unit element
C. R is a not a commutative ring.
D. None of the above

Solution

Suppose R is the set of integers positive, negative and 0, + is the usual addition and '.' is the usual multiplication then R is a commutative ring with unit element.
R is a commutative ring if $a.b=b.a \ \forall a.b \in R$

55. Suppose R is the set of even integers under usual operations of addition and multiplication, then which of the following is true ?

A. R is a commutative ring with unit element
B. **R is a commutative ring but has no unit element**
C. R is not a commutative ring
D. None of the above

Solution

R is a commutative ring but has no unit element.
An element u in R is called a unit element of R if it has a multiplicative inverse in R. Here there is no unit element in R since R is the set of even integers under usual operations of addition and multiplication.

56. If R is a ring in which $a^4 = a \ \forall a \in R$, then,..... ?

A. **R is commutative**
B. R is not commutative
C. R is zero ring
D. None of these

Solution

$(ab)^4=ab.ab.(ab)^2$
$= aabb.ab.ab$
$=a^2b^2.ab.ba$
$= a^2b^3.a.ba$
$= a^2b^2.ba.a$
$= a^2b^4.a.a(b4=b, \forall b \in R)$
$=a^2.b.a.a= a^3ba$

$= a^4b$
$=a.b$

57. Let $T:R^3 \to R^3$ be a linear transformation defined by $T(x,y,z)=(x+y,y+z,z+x)$ for all $(x,y,z) \in R^3$.Then

A. Rank(T)=0 and Nullity(T)=3
B. Rank(T)=2 and Nullity(T)=1
C. Rank(T)=1 and Nullity(T)=2
D. **Rank(T)=3 and Nullity(T)=0**

Solution

$T(x,y,z)=(x+y,y+z,z+x)$

Null space (T)= $\{(x,y,z) \in R^3 : T(x,y,z)=(0,0,0)\}$
= $\{(x,y,z) \in R^3 : (x+y,y+z,z+x)=(0,0,0)\} \Rightarrow y=z=0$
$\therefore$ Null space (T)={(0,0,0)}
$\therefore$ dim Null space(T) = Nullity =0. $\therefore$Rank(T)+" Nullity"(T)=n
Rank(T)+0=3
$\Rightarrow$ Rank(T)=3

58. Consider a function $f(z)=u+iv$ defined on $|z-i|<1|$ where u and v are real valued functions of y. Then $f(z)$�(�) is analytic for $u=x^2+y^2$

A. $u=x^2+y^2$
B. $u=e^{xy}$
C. $u=\ln(u=x^2+y^2)$
D None of the above

Solution

Analytic $\Rightarrow\Rightarrow$ Both real part and imaginary part is harmonic.

A) $u_x=2x, u_y=2y$

$u_{xx}=2, u_{yy}=2$

$u_{xx}+u_{yy}=2+2 \neq 0 \Rightarrow u$ is Not harmonic

$\therefore u=x^2+y^2 \therefore$ is not a real part of any analytic function

B) $u_{xx}+u_{yy} \neq 0 \Rightarrow$ u is Not harmonic

$\Rightarrow u=e^{xy} \Rightarrow$ is not a real part of any analytic function.

C)

$u_{yy}+u_{yy}=0$, u is harmonic

$f'(z)=u_x-u_y$

$= (2x)/(x^2+y^2) - I\,(2y)/(x^2+y^2)$

put x=z, y=0

f'(z)= 2/z.

Option C is true

59. If f(x) is differentiable in the interval (2,5)(2,5) where f(2)=1/5 and f(5)=1/2 , then there exist a number C, 2<C<5 for which f'(c) is

A. 1/2
B. 1/5
C. 1/10
D. 10

Solution

Mean Value Theorem

f is differentiable on (a,b)

By mean value Theorem, ∃ ∈(a,b) ⇒ f'(c)= (f(b)−f(a))/(b−a)
Here f(2)=1/5,f(5)=1/2

∴∃C∈(2,5)

⇒f'(C)=(f(5)−f(2))/(5−2)

=(1/2−1/5)/3=(3/10)/3

=1/10
⇒f'(C)=1/10

ANS :(C)

60.How many four digit even numbers have all four digits distinct ?

A. 2240
B. 2296
C. 2620
D. 4536

Solution

If is fixed in fourth position
⇒ Number of 4 digit even Numbers = 9×8×7=50
Ending with 2,4,6 or 8
⇒ Number of 4 digit even Numbers = 8×8×7×4=1792

∴ Total number of 4 digit Numbers =504+1792=2296
ANS : (A)

61. A card is drawn from a well shuffled pack of 5252 cards. The probability that the card drawn is a queen of clubs or a king of hearts is.

A. 1/26
B. 1/52
C. 1/13
D. 1/2

Solution

Probability of getting a queen of clubs = 1/52

Probability of getting a king of heart = 1/52

P (queen of clubs or king of heart) = 1/52 + 1/52 = 1/26
ANS: (A)

62. If a function f is monotonic on [a,b], then the set of discontinuities of f is.

A. empty
B. finite
C. **countable**
D. [a,b]

Solution

Set of all discontinuities of a monotone functions almost countable.
(C) True.
ANS :(C)

63. Let A be the set of all rational numbers in the interval [0,1], and α be the Lebesgue measure of A, then α is equal to.

A. **zero**
B. one
C. infinity
D. none of these

Solution

A={x: x∈[0,1]∩Q}
A is countable.
Every countable set has measure zero
∴ Measure (A)= α = 0
ANS: (A)

64. The harmonic conjugate of the function e^x cosy+ e^y cosx +xy is.

A. e^x siny− e^y sinx + ½(x^2 + y^2)
B. e^x siny+ e^y sinx + ½(x^2 + y^2)
C. e^x siny + e^y sinx - ½(x^2 + y^2)
D. **none of these**

Solution

$U_x = e^x$ cosy+ e^y cosx +xy

$U_y = - e^x$ siny+ e^y cosx +x

By CR equations, $u_x=u_y$
v = e^x siny− e^y sinx + (y^2/2) + ϕ (x)
$v_x=-u_y$
On solving these eqaution
ϕ′(x)=−x
ϕ(x) = (-x^2/2) +c
∴ v = e^x siny - e^y sinx - ½(x^2 - y^2) + c

65. The value of the integral ∫ 1/(z^2 + 4) dz around the circle |z−i|=2 oriented in counter clockwise direction is,

A. 0
B. π
C. π/2
D. 4

Solution

Singularities =±2i
|2i−1|=1<2 ,2i lie inside |z−1|<2|

|−2i−1|=3>2,−2i lie outside |2−i|<2

By cauchy Integral formula value of integral =2πif(2i)

=2πi.f(2i)
=π/2=π/2

66. Which of the following is not a topological property

A. Openness
B. Closeness
C. Connectedness
D. **Boundedness**

Solution

- Boundedness is not a topological property

- Topological property means property preserved under homeomorphism.
- Eg: (0,1) and $(1,\infty(1,\infty)$ are homeomorphic but (0,1) is bounded and $(1,\infty(1,\infty)$ is not bounded.

67. The residue of $f(z) = e^{2z}/(z+1)^2$ at $z=-1$ is,

A. 2e
B. 2
C. $2/e^2$
D. e

Solution

$\lim_{z\to-1} f(z)$ is Not defined.

$\Rightarrow z=-1$ is a pole.

$\lim_{z\to-1}(z+1)^2 e^{2z}/(z+1)^2$ **not equal to zero**

$\therefore z=-1$ is a pole of order 2 .

$\mathrm{Res}[f(z),z=-1]=\lim_{z\to-1} d/dx(z+1)^2\ e^{2z}/(z+1)^2 = 2/e^2$

ANS:(C)

68. The radius of convergence of the series $\sum 2^{-n}z^{2n}$ is

A. 1
B. $\sqrt{2}$
C. 2
D. ∞

Solution

Radius of convergence $1/R = \lim_{n\to\infty} (a_{2n})^{1/2n}$

$=\lim_{n\to\infty}(2^{-n})^{1/2n}$

$1/R = 1/\sqrt{2}$

Radius of convergence $R=\sqrt{2}$
ANS:(B)

69. The number of abelian groups (up to isomorphism) of order 24 is

A. 2
B. 3

C. 8
D. None of these

Solution

$24 = 2^3 * 3^1$

Hence, the number of abelian groups of order 24 up to isomorphism is,

$p(3)\times p(1)$

$= 3\times 1$

$= 3$

70. Number of left cosets of the subgroup <18> of Z_{36} is

A. 18
B. 36
C. 44
D. none of these

Solution

Z_{36} is a cyclic group.

$H=\langle 18\rangle= \{0,18\}$

$O(H)=O(\langle 18\rangle)=2$

$G = Z_{36}$, $O(G)=36$.

Number of left coset= [G : H] = order(G)/order(H) = 36/2 = 18

71. If U denotes the set of units in the ring of rational numbers Q, then

A. U={1}
B. U is empty
C. U={1,2}
D. **U consists of all non-zero elements of Q**

Solution

$x= 3 \in Q$,

$x^{-1} = 1/3 \in Q$.

$xx^{-1}=1=x^{-1}x$

$\Rightarrow 3$ is a unit

(A), (B),(C) False.

$\forall x \neq 0 \in Q$, $1/x \in Q$

∴ Every non zero element of θ are units.
ANS: (D)

72. The characteristic of the ring ℂ of complex numbers is

A. **zero**
B. one
C. infinity
D. none of these

Solution

Characteristic of a ring is the least integer $n \in \mathbb{Z}$ such that $n \cdot a = 0$ $\forall a \in R$
$\therefore \mathrm{Char}(\mathbb{C}) = 0$
ANS: (A)

73. If the dimensions of the subspaces U and V of the vector space W are respectively 5 and 6 and dim(U ∩ V) = 1, then dim(U + V) is equal to

A. 4
B. **10**
C. 7
D. none of these

Solution

$U+V \subseteq W$ and

$\dim(V_1+V_2)=\dim(U)+\dim(V)-\dim(U\cap V)$

Given, $\dim(U)=5$, $\dim(V)=6$ and $\dim(U\cap V)=1$

$\dim(U+V)=\dim(U)+\dim(V)-\dim(U\cap V)$

$= 5+6-1=10$

74. If gcd(a, b) = d, then gcd $\left(\frac{a}{d}, \frac{b}{d}\right)$ is equal to

A. $\frac{ab}{d}$
B. d
C. d^2
D. **1**

Solution

gcd $\left(\frac{a}{d}, \frac{b}{d}\right) = \frac{\gcd(a,b)}{lcm(d,d)}$

$= \frac{d}{d}$

=1

75. The differential equation of the family of all concentric circles centred at the origin is

A. $y + x\frac{dy}{dx} = c$
B. $y - x\frac{dy}{dx} = c$
C. $x + y\frac{dy}{dx} = c$
D. none of these

Solution

The equation of circles having centred at origin is :

$X^2 + y^2 = r^2$

Differentiating w,r,to x we get

$2x+2y \frac{dy}{dx} = 0$

$x+y \frac{dy}{dx} = 0$

or, $x+y \frac{dy}{dx} = c$, where c is a constant

76. The number of integers n , $1 \le n \le 10$ **such that** $\phi(n)=\phi(2n)$**, where** $\phi(n)$ **is the Euler Totient function , is**

A. 1
B. 2
C. **3**
D. 4

Solution

$\phi(1)=1$, $\phi(2)=1$,$\phi(3)=2$,$\phi(4)=2$,$\phi(5)=4$

$\phi(6)=2$, $\phi(7)=6$, $\phi(8)=4$, $\phi(9)=6$, $\phi(10)=4$

Hence we get ,

$\phi(1)$=phi(2)

ϕ(5)=phi(6)

ϕ(5)=phi(10)

Number of n such that ϕ(n)=ϕ(2n) is 3

77. The differential equation of the family of circles touching the y - axis at the origin is

A. $X^2 + y^2 - 2xy\frac{dy}{dx} = 0$
B. $X^2 + y^2 + 2xy\frac{dy}{dx} = 0$
C. $X^2 - y^2 -2xy\frac{dy}{dx} = 0$
D. $X^2 - y^2 -2xy\frac{dy}{dx} = 0$

Solution

Circle touching y - axis at origin.

⇒⇒ centre of the circle lies on the x -axis.

Centre is of the form (a,0)

Radius = a.

Equation of the circle is given by ,

$(X-a)^2 + (y -)^2 = a^2$

$\Rightarrow x^2 + y^2 -2ax =0$........(1)

To find the differential equation we want to remove the constant 'a' by differentiating (1)

Hence differentiating (1) with respect to x, $2x + 2y\frac{dy}{dx} - 2a = 0$

, $2x + 2y\frac{dy}{dx} = 2a$

$x + y\frac{dy}{dx} = a$

Substituting in (1)

$\Rightarrow x^2 + y^2 -2x(x + y\frac{dy}{dx}) = 0$

$= x^2 - y^2 + 2x y\frac{dy}{dx} = 0$

78. The value of m such that the equation $xu_{xx}+mu_{xy}+yu_{yy}-2u_x=0$ is parabolic is

A. xy
B. $\sqrt{(xy)}$
C. 2xy
D. $-2\sqrt{(xy)}$

Solution

$Au_{xx}+Bu_{xy}+Cuyy=0$ is parabolic if $B2-4AC=0$

Given $xu_{xx}+mu_{xy}+yu_{yy}-2u_x=0$

A=x, B=m, C=y

$B^2-4AC=0\Rightarrow m^2-4xy=$

$m=\pm\sqrt{(4xy)}$

$m=\pm2\sqrt{(xy)}$

79. Let R be a relation on $Z^+ \times Z^+$ such that $((a,b),(c,d))\in R$ iff $a-d=b-c$. Which of the following is true about R?

A. Reflexive but not symmetric
B. **Symmetric but not reflexive**
C. Both reflexive and symmetric
D. Neither reflexive nor symmetric

Solution

$(a,b),(c,d)\in R\Leftrightarrow a-d=b-c$

$\Rightarrow-(d-a)=-(c-b)$

$(d-a)=c-b$

$\Rightarrow((c,d),(a,b))\in R$

$\Rightarrow$R is symmetric

$a-b\neq b-a$

$\Rightarrow$ not reflexive.

80. If α,β,γ are the roots of $2x^3+x^2-2x-1=0$, then the value of $\alpha^2+\beta^2+\gamma^2=?$

A. -1/2
B. ½

C. ¾
D. 9/4

Solution

$2x^3 + x^2 - 2x - 1 = 0$ have roots α,β,γ

Let $f(x) = 2x^3 + x^2 - 2x - 1$

Then f(1)=2+1−2−1=0 .Hence x−1 is a root of f(x).

On dividing $2x^3 + x^2 - 2x - 1$ by x−1 we get the other roots x = -1/2 , x = -1

Hence $\alpha^2+\beta^2+\gamma^2 = 9/4$

81. The domain of the function f defined by $f(x) = \frac{\sqrt{x}}{(x-3)(x-5)}$

A. (−∞,−5)∪(−5,3)∪(3,∞)
B. (−∞,5)∪(3,∞]
C. (−∞,−5)∪(−5,0]
D. (−∞,3)∪(3,∞)

Solution

f(x) = **by** $f(x) = \frac{\sqrt{x}}{(x-3)(x-5)}$ is not defined when x>0,x=3,x=−5

Hence the domain = (−∞,−5)∪(−5,0](−∞,−5)∪(−5,0]

82. Which of the following sets of functions is countable ?

1) { f | f : N →→ {0,1}}

2) { f | f : {0,1} →→ N}

3) { f | f : N →→ {0,1} , f(1)≤f(2)

4){ f | f : {0,1} →→ N , f(0)≤f(1)

A. 1 and 3
B. **2 and 4**
C. 1 only
D. 2 only

Solution

$|N| = X_0$ countable.

$|\{0,1\}| = 2$

Number of functions from f: A→B=$|B|^{|A|}$

1)Number of functions = 2^X uncountable.

2) $x^2{}_0$ is countable

3) Uncountable

4) Subset of a countable set is countable

83.The equation of the plane that passes through (1,2,3) and parallel to the plane 4x+5y−3z =7 is

A. 3x+4y−3z=7
B. 4x+5y−3z=5
C. 5x−4y+z=3
D. 4x+5y−3z+7=0

Solution

The equation of the plane parallel to 4x+5y−3z= 7 is given by 4x+5y−3z+b=0

It passes through (1,2,3)

⇒4.1+5.2−3.3+b=0

⇒−b= 4+10−9

b=−5

Hence the equation of the plane is = 4x+5y−3z−5=

84. For what value of k is the the function $f(x)=\frac{1-cos2x}{2x.x}$ **when x not equal to zero and f(x) = k when x = 0 continuous at x =0 ?**

A. 0
B. 12
C. **1**
D. 2

Solution

Continuous at x =0

$\Rightarrow \lim_{x\to0} \frac{1-cos2x}{2x.x} = k$

$\Rightarrow \lim_{x\to 0} \frac{2sin2x}{2x.x} = k$

$\Rightarrow \lim_{x\to 0} \frac{4cos2x}{4x} = k$

$\Rightarrow \lim_{x\to 0} \cos 2x = k$

Hence k=1

85. The value of the integral $\int \frac{\cos(\frac{1}{x})}{x\text{^}2}$ dx over the range $1/\Pi$ to 2/▯ ?

A. **-1**
B. 0
C. 1
D. ▯/2

Solution

Let u = 1/x

Then du = $-1/x^2$ dx

Hence on integrating over the range $1/\Pi$ to 2/▯

=[sinu]

= 0 -1 = -1

86. The number of different symmetric square matrices of order n with each element being either 0 or 1 is

A. 2^n
B. $(2^n)^2$
C. $2^{((n(n+1))/2})$
D. 2n + 1

Solution

Number of free elements = 1+2+3+4+.....n= n(n+1)/2

Each free variable has two choices { 0 , 1}

Total number of matrices = $2^{((n(n+1))/2})$

87. $\lim_{n\to\infty} (1/(n^2 + 1)) + (2/(n^2 + 1)) \ldots\ldots\ldots\ldots (n/(n^2 + 1))$ is

A. **0**
B. 1
C. 2
D. e

Solution

$\lim_{n\to\infty} (1/(n^2 + 1)) + (2/(n^2 + 1)) \ldots\ldots\ldots\ldots (n/(n^2 + 1))$

$\lim_{n\to\infty} (1/(n^2 + 1)) + \lim_{n\to\infty} (2/(n^2 + 1)) \ldots\ldots\ldots\ldots \lim_{n\to\infty} (n/(n^2 + 1)) = 0$

88. Let $\sum X_n$ be a series of real numbers. Which of the following is true ?

A. If $\sum X_n$ is convergent then $\sum_{n=1} x_n$ is absolutely convergent.
B. If $\sum X_n$ is divergent then $\{X_n\}$ does not converge to 0
C. If $x_n \to 0$ then , $\sum_{n=1} x_n$ is convergent.
D. **If $\sum_{n=1} x_n$ is convergent. then X^2_n as $n\to\infty$**

Solution

a) False

b) False. Ex: $\{1/n\} \to 0$ but $\sum 1/n$ diverges

C) False : ex: $1/n \to 0$ but $\sum 1/n$ is divergent

d) True,

$\sum x_n$ convergent $\Rightarrow \lim x_n = 0$

$\Rightarrow \lim X_n \ \lim X_n = 0$

$\Rightarrow \lim X^2_n = 0$

89. The value of $\sqrt{i} + \sqrt{(-i)}$ is

A. 0
B. 1
C. i
D. $\sqrt{2}$

Solution

$\sqrt{I} + \sqrt{(-i)} = x$

Then $X^2 = (\sqrt{i} + \sqrt{(-i)})^2$

$X^2 = 2$

$x=\sqrt{2}$

90. The function $f(z) = (e^z + 1)/(e^z - 1)$ has

A. A removable singularity at z =0
B. A simple pole at z = 0 with residue 1
C. **A simple pole at z = 0 with residue 2**
D. An essential singularity at z =0

Solution

$f(z) = (e^z + 1)/(e^z - 1)$

$\lim_{z\to 0}(z-0)f(z)=\infty \Rightarrow$ pole at z = 0 (simple pole)

$\lim_{z\to 0}(z-0)f(z)=\lim_{z\to 0}\ (e^z + 1)/(e^z - 1)$

$=\lim_{z\to 0}\ (z - 0)f(z) = \lim_{z\to 0}\ (ze^z + e^z + 1)/(e^z)$

$=0+1+1 = 2$

91. The bilinear transformation which maps the points $z=1,-i,1$ into the points $w=i,0,-I$ is

A. i(1-z)/(1+z)
B. i(1+z)/(1-z)
C. (z-i)/(1 + iz)
D. (z+i)/(z-i)

Solution

Consider the option D.

$W = (z+i)/(z-i)$

$Z=1,\ w = (1+i)/(1-i) = i$

$z=-i,\ w=0$

$z=-1, w = (-1 + i)/(-1-i) = -i$

Hence the option D is correct.

92. The value of the integral $\int(e^{-z}/(z+1)\,dz$ where c is the circle $|z| = ½$ is

A. $2\pi i$
B. $2\pi ie$
C. **0**
D. $4\pi i$

Solution

$\int (e^{-z}/(z+1)) dz$ where $c:|z| = 1/2$

$z+1= 0 \Rightarrow z=-1$ is the singular point.

But $z=-1$ lies outside c

$\int (e^{-z}/(z+1)) dz = 0$

93. Let F be a field of order 256. Then

A. F has a subfield of order 8
B. **F has a subfield of order 16**
C. F has a subfield of order 32
D. F has a subfield of order 64

Solution

$|F|=256=2^8$

Subfield of a field of order p^n is given by = { Fp^m: m is a divisor of n}

Subfield of { F_{2^3}= {{ $F_2{}^3$: m is a divisor of 8 }

= { $F_2{}^1$, $F_2{}^2$, $F_2{}^4$, $F_2{}^8$ }

= { F_2, F_4, F_{16}, F_{256}

94. Which of the following is not true ?

A. Every cyclic group is abelian
B. **Every group of odd oder is cyclic**
C. The order of a cyclic group and that of its generating element are same
D. Every subgroup of a cyclic group is cyclic

Solution

These are some of the results related to the cyclic groups.

Every cyclic group is abelian

Every subgroup of a cyclic group is cyclic

The order of a cyclic group and that of its generating element are same

Group of odd order need not be cyclic

95. Which of the following is countable ?

A. Set of all functions from R→{ 0,1}
B. Set of all functions from N→�→ { 0,1}
C. **Set of all finite subsets of N**
D. Set of all subsets of N

Solution

1)Set of all functions from R→ { 0,1} is uncountable. Since any function from R→ { 0,1} is a sequence whose elements are either 0 or 1 . Hence Uncountable.

2)Set of all functions from N→{ 0,1} is uncountable. Since any function from N→{ 0,1} is a sequence whose elements are either 0 or 1 . Hence Uncountable.

4)$|P(N)|\geq|N|$. Hence uncountable

96. The order of the element (1,2) in $Z_5 \times Z_{10}$ is

A. **5**
B. 10
C. 15
D. 20

Solution

Order of (1,2) in $(Z_5 \times Z_{10})$= lcm { order of 1 in Z_5, order of 2 in Z_{10}}

=lcm(5,5)=5

97. The splitting field of the set of polynomials { $x^2 - 2$, $x^2 - 3$ } over Q is

A. $Q(\sqrt{2})$
B. $Q(\sqrt{3})$
C. $Q(\sqrt[3]{2})$
D. $Q(\sqrt{2},\sqrt{3})$

Solution

$X^2 - 2 = 0$ hence $x = \sqrt{2}$

$X^2 - 3 = 0$ hence $x = \sqrt{3}$

Splitting field = $Q(\sqrt{2},\sqrt{3})$

98. The gcd of $3+4i$ and $-4+3i$ in the integral domain $(Z[i],+,.)$ is

A. $3+4i$
B. $-4+3i$
C. **Both A and B**
D. Neither A nor B

Solution

(3+4i)/(-4+3i) =((3+4i)(-4-3i))/((-4+3i)(-4-3i))

= (-2-9i-16i+12)/(16+9)

= -25i/25 = - i ∈Z[i]

Hence (3+4i)/(-4+3i) , (-4+3i)/(3+4i) ∈Z[i]

Both true.

99. Which of the following is not true ?

A. If A is a $m \times n$ matrix and B is an $n \times p$ matrix, then $\text{rank}(AB) \le \min(\text{rank}(A), \text{rank}(B))$
B. If A is a $m \times n$ matrix and B is a non singular matrix, then $\text{rank}(AB) = \text{rank}(A)$
C. If A is a $m \times n$ matrix and B is an $n \times p$ matrix, then $\text{rank}(AB) \le \text{rank}(A)$
D. **If A is a** $m \times n$ **matrix and B is an** $n \times p$ **matrix, then** $\text{rank}(AB) = \min(\text{rank}(A), \text{rank}(B))$

Solution

If A is a $m \times n$ matrix and B is an $n \times p$ matrix, then $\text{rank}(AB) \le \min(\text{rank}(A), \text{rank}(B))$

If A is a $m \times n$ matrix and B is a non singular matrix, then $\text{rank}(AB) = \text{rank}(A)$

If A is a $m \times n$ matrix and B is an $n \times p$ matrix, then $\text{rank}(AB) \le \text{rank}(A)$

$\text{rank}(AB) \le \min(\text{rank}(A), \text{rank}(B))$

100. Let W be the solution space of the system of homogeneous equations $2x+2y+z=0, 3x+3y-2z=0, x+y-3z=$**. The** $\dim W$ **is**

A. 0
B. 1
C. **2**
D. 3

Solution

Corresponding matrix is given by

$$[w] = \begin{bmatrix} 2 & 2 & 1 \\ 3 & 3 & -2 \\ 1 & 1 & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & -3 \\ 0 & 0 & 7 \\ 0 & 0 & 7 \end{bmatrix}$$

$$\approx \begin{bmatrix} 1 & 1 & -3 \\ 0 & 0 & 7 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence rank = 2

THANK YOU