CAT 2020 Slot 1 QA

1. **If $\log_4 5 = (\log_4 y)(\log_6 \sqrt{5})$, then y equals __.**

A 40

B **36**

C 30

D 50

Solution

$$\log_4 5 = (\log_4 y)(\log_6 \sqrt{5})$$

$$\Rightarrow \frac{\log 5 \times \log 6}{\frac{1}{2}\log 5} = \log y$$

$$\Rightarrow 2\log 6 = \log y$$

$$\Rightarrow \log 6^2 = \log y$$

$$\Rightarrow y = 36$$

2. **An alloy is prepared by mixing three metals A, B, and C in the proportion 3:4:7 by volume. The weights of the same volume of the metals A, B, and C are in the ratio 5:2:6. In 130 kg of the alloy, the weight (in kg) of the metal C is _.**

A 96 kg

B **84 kg**

C 70 kg

D 48 kg

Solution

Ratio of volumes of A, B, C in the Alloy = 3:4:7

Ratio of weights of the same volumes or density of A, B, and C = 5:2:6

Ratio of weights of A, B, and C in the alloy = $3 \times 5:4 \times 2:7 \times 6 = 15:8:42$

	Volume	**Density**	**Weight**
A	3	5	15
B	4	2	8
C	7	6	42

	Volume	**Density**	**Weight**
Overall	14	$\frac{65}{14}$	65

Total weight of alloy = 130 kg

Weight of metal C $= \frac{42}{15+8+42} \times 130 = 84\ kg$

Typesetting math: 100%

3. **Leaving home at the same time, Amal reaches his office at 10:15 a.m. if he travels at 8 km/h, and at 9:40 am if he travels at 15 km/h. Leaving home at 9:10 a.m., at what speed (in km/h) must he travel so as to reach office exactly at 10 a.m.?**

A 13

B 14

C 11

D **12**

Solution

Let the distance between home and office be d km.

According to the question, we can write the following:

$\frac{d}{8}= \frac{d}{35}$

=> $\frac{7d}{120} =\frac{35}{60}$

=> d = 10 km

Now he should reach the office in 50 min.

d = 10 km

t = 50 min = $\frac{5}{6}$ h

Required speed = $\frac{10}{\frac{5}{6}}=12$ km/hr

4. **Let A, B, and C be three positive integers such that the sum of A and the mean of B and C is 5. In addition, the sum of B and the mean of A and C is 7. Then, the sum of A and B is __.**

A 4

B 5

C 7

D **6**

Solution

The given equations are the following:

$A + \frac{B+C}{2} =5 \ \ => 2A+B+C=10$ ....(1)

$B + \frac{A+C}{2} =7 \ \ => A+2B+C=14$ ....(2)

Subtracting equation (1) from equation (2), we get the following:
B – A = 4 ....(3)

Putting A = 1, we will get B = 5; this is the minimum value of B.
If A = C = 1, then, from (1), we will get B = 6 = max value of B
But for A = 1, we were already getting B = 5.
Hence, A = C = 1 is not possible.
=>2A+B+C=10

From this, by putting A = 1 and B =5, we get C = 3

So, the only possible solution is A = 1, B = 5, and C = 3.

Therefore, A + B = 6

5. **A solid right circular cone of height 27 cm is cut into two pieces parallel to its base at a height of 18 cm from the base. If the difference in volume of the two pieces is 225 cc, the volume, in cc, of the original cone is _.**

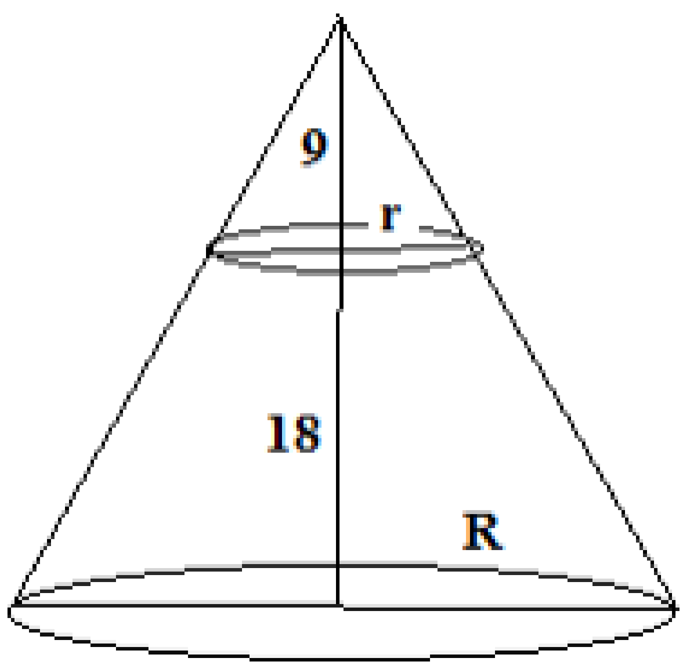
A 256

B 232

C 264

D **243**

Solution



Height of original cone = 27 cm

Height of smaller cone = 27 – 18 = 9 cm

Ratio of height of original cone and smaller cone = 27:9 = 3:1

Using the similarity of triangles, we get the following:

The ratio of the radius of original cone to that of the smaller cone is

$\frac{R}{r}=\frac{3}{1}$

Ratio of volumes = $(\frac{3}{1})^3$=$\frac{27}{1}$=27: 1

Let the volume of the original cone be 27k.

And the volume of the smaller cone = k

Volume of frustum = (27 – 1)k = 26k

Given,

26k – k = 225

25k = 225 = 9 × 25

⇒ k = 9

Volume of original cone = 27k = 27 × 9 = 243 cc

6. **Two persons are walking beside a railway track at respective speeds of 2 and 4 km per hour in the same direction. A train came from behind them and crossed them in 90 and 100 seconds, respectively. The time, in seconds, taken by the train to cross an electric post is nearest to _.**

A 78

B 82

C 87

D 75

Solution

Let the length of the train be x metres and the speed of the train be s km/h.

As they are moving in the same direction, relative speed between the train and the first person = (s – 2) km/h

Similarly, relative speed between the train and the second person = (s – 4) km/h [because they are moving in the same direction]

Since the distance (length of train) is same in both the cases, we can write the following:

$\frac{S_1}{S_2} =\frac{T_2}{T_1}$

Substituting these values, we get:

$=> \frac{S-2}{S-4}=\frac{100}{90}$

=> S= 22 km/hr.

Let the time taken by the train to cross an electric post be ‘t’.
We can write the following:

$=> \frac{S-2}{S}=\frac{t}{90}$

$=> \frac{20}{22}=\frac{t}{90}$

=> $t \approx 82$ sec.

7. **A gentleman decided to treat a few children in the following manner. He gives half of his total stock of toffees and one extra to the first child, and then the half of the remaining stock along with one extra to the second and continues giving away in this fashion. His total stock exhausts after he takes care of 5 children. How many toffees were there in his stock initially?**

A 50

B 62

C 56

D 60

Solution

We can solve this question by reverse calculations as stated in the table below:

After child number	Number of toffees
5	0
4	2(0 + 1) = 2
3	2(2 + 1) = 6
2	2(6 + 1) = 14
1	2(14 + 1) = 30
0	2(30 + 1) = 62

8. **If a, b, and c are positive integers such that ab = 432, bc = 96, and c < 9, then the smallest possible value of a + b + c is __.**

A **46**

B 59

C 49

D 56

Solution

It is given that,

ab = 432, bc = 96

Since a, b, c are positive integers and c < 9, we can get c as 8 or 6 or 4 or 3 or 2 or 1.

We can calculate the possible values of a, b, and c as shown:

Case	A	B	C
1	4.5	96	1
2	9	48	2
3	13.5	32	3

Case	A	B	C
4	18	24	4
5	27	16	6
6	36	12	8

Since all values should be integers, we can say that the cases 1 and 3 are to be ignored.

Hence,

Case	A	B	C	a + b + c
1	9	48	2	59
2	18	24	4	46
3	27	16	6	49
4	36	12	8	56

So, the minimum value of (a + b + c) is 46.

9. **Among 100 students, x1 have birthdays in January, x2 have birthdays in February, and so on. If x0 = max(x1, x2, …, x12), then the smallest possible value of x0 is __.**

A 10

B **9**

C 12

D 8

Solution

For the smallest possible value of x0 ,all (x1, x2, x3,...,x12) should be equal or closest to each other.

If we make all the values to be 8, we will have 4 left. Those 4 can be distributed 1 each.

So, the maximum value will be 9.

10. The mean of all 4-digit even natural numbers of the form 'aabb', where a > 0, is __.

A 4466

B 4864

C 5050

D **5544**

Solution

The number 'aabb' means $1000a + 100a + 10b + b = 11(100a + b)$.

Now, a can be 1,2,3…9 and b can be 0, 2, 4, 6, 8.

The numbers satisfying the given conditions are the following:

1100, 1122, 1144, 1166, 1188

2200, 2222, 2244, 2266, 2288

. . . . . .

. . . . . .

9900, 9922, 9944, 9966, 9988

We can notice that the numbers in each row are in AP.

So, the middle number of each row will be the mean of that row.

So, the mean of the rows are 1144, 2244, 3344, 4444, 5544, 6644, 7744, 8844, and 9944.

Now the mean of these numbers is 5544.

Hence, the mean of all 4-digit even natural numbers of the form ‘aabb’ is 5544.

11. **The number of real valued solutions of the equation $2^x + 2^{-x} = 2 - (x - 2)^2$ is _.**

A 1

B 0

C Infinite

D 2

Solution

Given,

$2^x + 2^{-x} = 2 - (x - 2)^2$

2^x> 0 for all real values of x. We know that, for all positive values 'P', $\square+\frac{1}{\square}\geq2$ and the equality occurs at P = 1.

$2^x + 2^{-x} \geq 2$ ....(1)

Now, since the square of any real quantity can be at least zero, we can write the following:

$(\square - 2)^2 \geq 0$

=>$2 - (x - 2)^2 \leq 2$

Now, the only possible solution is when both the LHS and RHS are equal to 2 simultaneously.

$2^x + 2^{-x} = 2 \ or \ x = 0$

And $2 - (x - 2)^2 = 2 \ or \ x = 2$

So, there is no common value of x for which LHS = RHS.

Hence, the given equation does not have a solution.

12. **How many distinct positive integer valued solutions exist to the equation? $(x^2-7x+11)^{(x^2-13x+42)} =1$**

A 8

B 2

C 4

D **6**

Solution

$(x^2-7x+11)^{(x^2-13x+42)} =1$

in the following cases:

Case	$(x^2-7x+11)$	$(x^2-13x+42)$	Value/s of x
1	1	Immaterial	5 or 2
2	Non zero	0	6 or 7
3	(-1)	Even	3 or 4

So, there are SIX possible values of x.

13. In a group of people, 28% of the members are young, while the rest are old. If 65% of the members are literates, and 25% of the literates are young, then the percentage of old people among the illiterates is nearest to __.

A 66

B 59

C 62

D 55

Solution

From the given data:

	Young	Old	Total
Literates	16.25	48.75	65
Illiterates	11.75	23.25	35
Total	28	72	100

We have filled in the values from the given data.

So, the required percentage = \(\frac{23.25}{35} × 100 \% = 66 \% \)(approx.)

14. **A straight road connects points A and B. Car 1 travels from A to B and Car 2 travels from B to A, both leaving at the same time. After meeting each other, they take 45 minutes and 20 minutes, respectively, to complete their journey. If Car 1 travels at the speed of 60 km/h, then the speed of Car 2, in km/hr, is __.**

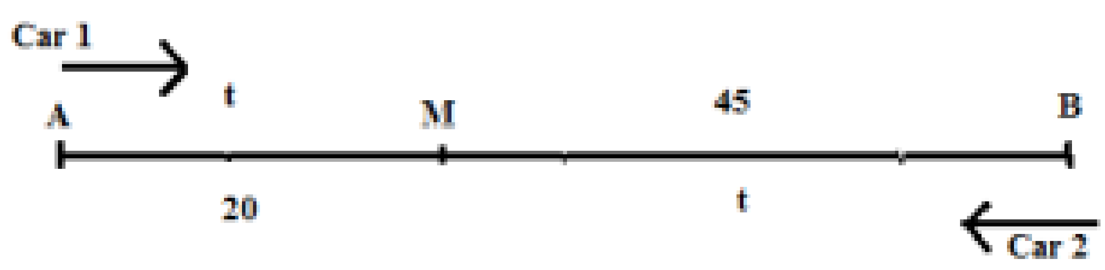
A 100

B 80

C **90**

D 70

Solution



Let the two cars meet at point M and the time they take to meet is ‘t’ min. Also, assume the speed of Car 1 to be S1 and that of Car 2 to be S2. [in suitable units]

Time taken by Car 1 to travel from A to M = t min

Time taken by Car 2 to travel from M to A = 20 min

Since distance is same in both cases, we can write the following:

$\frac{S_1}{S_2}=\frac{20}{t}$ ....(1)

Similarly, we can write the following:

Time taken by Car 1 to travel from M to B = 45 min

Time taken by Car 2 to travel from B to M = t min

Since distance is same in both the cases, we can write the following:

$\frac{S_1}{S_2}=\frac{t}{45}$ ....(2)

From (1) and (2), we get the following:

$\frac{20}{t} = \frac{t}{45}$

=> $t = 30$ min.

Putting $t = 30$ and $S1 = 60$ in the first equation, we get the following:

$\frac{60}{S_2}=\frac{20}{30}$

=> $S_2=90$ km/hr

15. **A person spent Rs. 50000 to purchase a desktop computer and a laptop computer. He sold the desktop at 20% profit and the laptop at 10% loss. If he made a 2% profit overall, then the purchase price, in rupees, of the desktop is __.**

A 10000

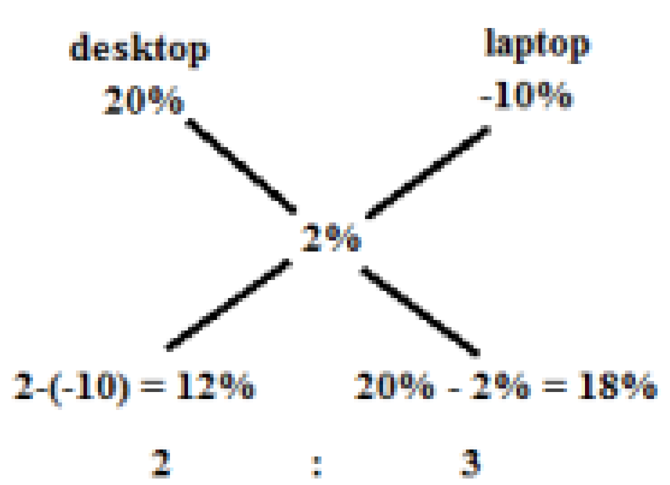
B 20000

C 15000

D 25000

Solution

Using allegations, we can find the ratio of the cost prices of the two articles.



Desktop:Laptop = 2:3

Cost price of desktop = $\frac{2}{2+3} \times 50000 = Rs.\ 20000$

16. If $f(5 + x) = f(5 - x)$ for every real x, and $f(x) = 0$ has four distinct real roots, then the sum of these roots is __.

A **20**

B 0

C 40

D 10

Solution

Given,

$f(5 + x) = f(5 - x)$

Let $x = 5 - y$

Putting this value, we get the following:

$f(10 - y) = f(y)$ ....(1)

Now let a and b be two of the four roots of $f(X) = 0$.

$\Rightarrow f(a) = f(b) = 0$

Substituting $y = a$ in equation (1), we get the following:

$f(10 - a) = f(a) = 0$

This means $10 - a$ is the root of the given equation.

Similarly, if b is a root, then $(10 - b)$ is also a root of the given equation.

Hence, the four roots of the given equation are a, b, $10 - a$, and $10 - b$.

Sum of roots $= a + b + 10 - a + 10 - b = 20$.

17. **Veeru invested Rs. 10000 at 5% simple annual interest, and exactly after two years, Joy invested Rs. 8000 at 10% simple annual interest. How many years after Veeru's investment will their balances, i.e., principal plus accumulated interest, be equal?**

A 10

B 14

C 16

D **12**

Solution

We can draw the following table:

Name	Principal	Rate	Year	Interest	Amount
Veeru	10000	5	T	500T	10000 + 500T
Joy	8000	10	T-2	800(T – 2)	8000 + 800(T – 2)

So, $10000 + 500T = 8000 + 800(T - 2)$

$2000 + 500T = 800T - 1600$

⇨ $3600 = 300T$

⇨ $T = 12$

18. **On a rectangular metal sheet of area 135 sq in, a circle is painted such that the circle touches two possible sides. If the area of the sheet left unpainted is twothirds of the painted area, then the perimeter of the rectangle, in inches, is _.**

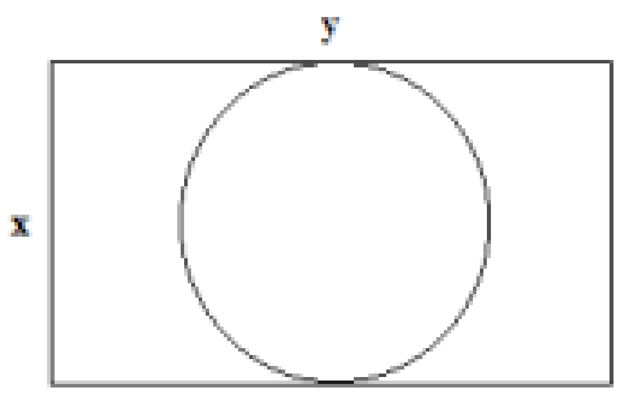
A $5\sqrt{\pi}(3+\frac{9}{\pi})$

B $3\sqrt{\pi}(\frac{5}{2}+\frac{6}{\pi})$

C $4\sqrt{\pi}(3+\frac{9}{\pi})$

D **$3\sqrt{\pi}(5+\frac{12}{\pi})$**

Solution



Let the dimensions of the rectangle be x and y, where x < y.

Then, xy = 135 ……………….(1)

Diameter of the circle = x

⇨ Radius of circle = $\frac{x}{2}$

Painted area of sheet = $\pi (\frac{x}{2})^2$

According to the question, we can write the following:

$135 - \pi (\frac{x}{2})^2 = \frac{2}{3}\ \times \pi (\frac{x}{2})^2$

=> $135 = \frac{5}{3}\ \times \pi (\frac{x}{2})^2$

=> $\frac{81}{\pi} = (\frac{x}{2})^2$

=> $x=\frac{18}{\sqrt{\pi}}$ inches.

Putting this in (1), we get the following:

$y=\frac{135\sqrt{\pi} }{18} = \frac{15\sqrt{\pi}}{2}$ inches.

Perimeter of rectangle = 2x + 2y = $\frac{36}{\sqrt{\pi}} +15\sqrt{\pi} =3\sqrt{\pi} (5+\frac{12}{\pi})$

19. **How many 3-digit numbers are there, for which the product of their digits is more than 2 but less than 7?**

A 21

B 19

C 17

D 15

Solution

Let us assume the three digit number is abc.

Since $2 < a \times b \times c < 7$, the product of the digits can be 3, 4, 5, or 6.

Case	Product	A	B	C	No. of numbers
1	3	1	1	3	3!/2! = 3
2	4	1	1	4	3
3	4	1	2	2	3
4	5	1	1	5	3
5	6	1	2	3	3! = 6
6	6	1	1	6	3
Total					21

20. **The number of distinct real roots of the equation $(\square+\frac{1}{\square})^2-3(\square+\frac{1}{\square})+2=0$ equals _.**

A 0

B **1**

C 10

D 2

Solution

Substituting $(x+\frac{1}{x})$ in the given equation, we get the following:

$y^2- 3y +2 =0$

=> (y 1)(y 2)= 0

=> y = 1 or 2

Case 1: If y = 1

$x + \frac{1}{x }=1$

$x^2- x+ 1= 0$

D = $(–1)^2 – 4 \times 1 \times 1 = -3 < 0$

There is no real root in this case.

Case 2: If $y = 2$

$x + \frac{1}{x} = 2$

$x^2 - 2x + 1 = 0$

D = $(-2)^2 - 4 \times 1 \times 1 = 0$

There is exactly one distinct real root in this case.

Hence, the given equation has only 1 distinct real root

21. **A solution of volume 40 litres has dye and water in the proportion 2:3. Water is added to the solution to change this proportion to 2:5. If one-fourths of this diluted solution is taken out, how many litres of dye must be added to the remaining solution to bring the proportion back to 2:3?**

A 4

B 6

C **8**

D 10

Solution

For the initial solution,

Total volume = 40 litres

Volume of dye = $\frac{2}{5} \ of \ 40 = 16\ litres$ Volume of water = $\frac{3}{5} \ of \ 40 = \ 24 \ litres$

Now we add water to change the proportion to 2:5.

So, the volume of dye will not change.

Volume of dye in the new solution = 16 litres

Since water:dye = 5:2

⇨ Volume of water in new solution = $\frac{5}{2} \ of \ 16 = 40 \ litres$

Total volume of new solution = 56 litres

Now $\frac{1}{4}$th of the solution is removed.

So, $\frac{3}{4}$th of the solution remains.

Total volume left = $\frac{3}{4}$ of 56 = 42 litres

Volume of dye left = $\frac{3}{4}$ of 16 = 12 litres

Volume of water left = $\frac{3}{4}$ of 40 = 30 litres

Volume of dye = $\frac{2}{3}$ of 30 = 20 litres

Volume of dye added = 20 – 12 = 8 litres

22. **The area of the region satisfying the inequalities $|x| - y \leq 1$, $y \geq 0$ and $y \leq 1$ is**

A **3**

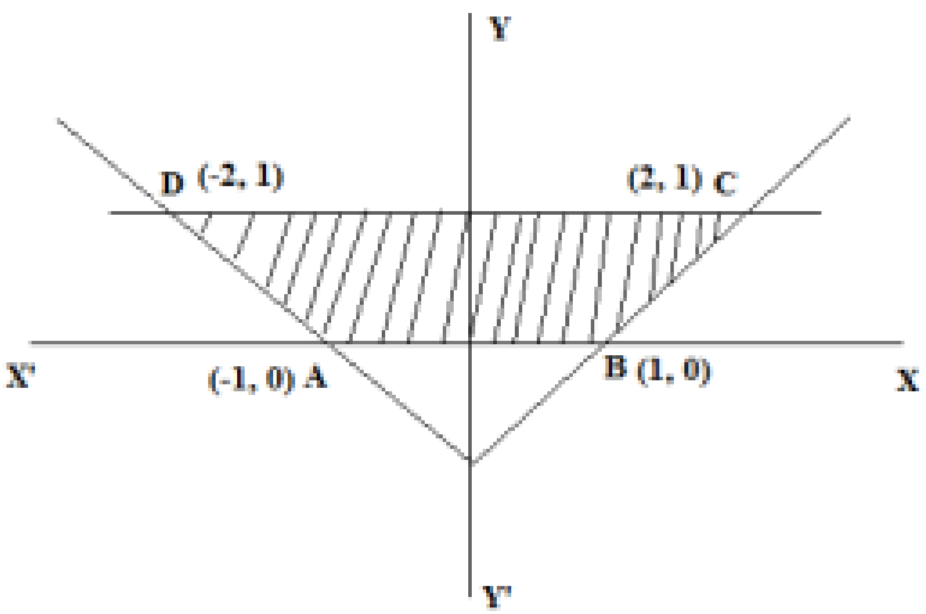
B 5

C 1

D 2

Solution

We can draw the following graph by using the given inequalities.



The required region is the area of the trapezium ABCD shown in the graph.

AB = 2 units

CD = 4 units

$h = 1$ unit

Area of a trapezium is ½ × height × sum of the parallel sides

Area = $\frac{1}{2} (2+4) \times 1 = 3$ sq. unit

23. A circle is inscribed in a rhombus with diagonals 12 cm and 16 cm. The ratio of the area of the circle to the area of the rhombus is _.

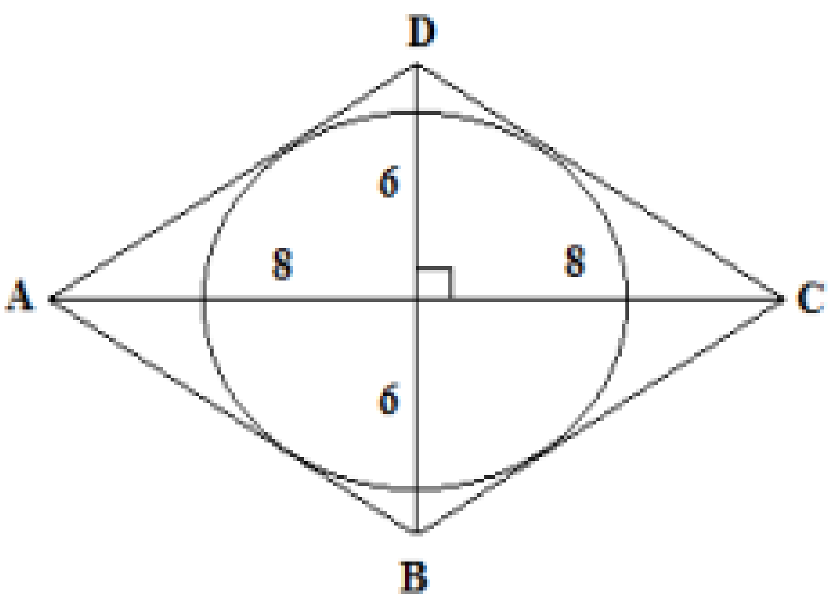
A $\frac{6\pi}{25}$

B $\frac{2\pi}{15}$

C $\frac{3\pi}{25}$

D $\frac{5\pi}{18}$

Solution



We know that diagonals of the rhombus bisect each other at 900.

Therefore, using Pythagoras' theorem, we can write the following:

Side of rhombus $=\sqrt{(\frac{12}{2})^2+(\frac{16}{2})^2} = 10 \ cm$

Area of rhombus = $\frac{1}{2} \times 12 \times 16 = 96 \ cm^2$

Also, area of rhombus = s x r , where ‘s’ is the semi-perimeter and ‘r’ is the radius of the incircle.

96 = 20 x r

r = $\frac{24}{5} \ cm$

$\frac{Area \ of \ circle}{Area \ of \ rhombus} = \frac{\pi (\frac{24}{5})^2}{96} = \frac{6\pi}{25}$

24. **If y is a negative number such that $2^{y^2 \ \log_35}=5 \ \log_23$, then y equals _.**

A $-\log_2(\frac{1}{5})$

B $\log_2(\frac{1}{3})$

C $\log_2(\frac{1}{5})$

D $-\log_2(\frac{1}{3})$

Solution

$2^{y^2 \ \log_35}=5 \ \log_23$

Taking log both sides, we get the following:

$\log 2^{y^2 \ \log_35}=\log 5 \ \log_23$

=>$y^2 \ \log_35 \times \log 2=\log 5 \times \log_23$

=>$y^2 \times \frac{\log 5}{\log 3} \times \log 2=\frac{\log 3}{\log 2} \times \log 5$

Cancelling log 5 from both sides, we get the following:

=>$y^2=(\log_23)^2$

=>$y=\pm \log_23$

Since it is given that ‘y’ is negative, option (B) is the correct answer.

25. **A train travelled at one-thirds of its usual speed, and hence reached the destination 30 minutes after the scheduled time. On its return journey, the train initially travelled at its usual speed for 5 minutes but then stopped for 4 minutes for an emergency. The percentage by which the train must now increase its usual speed so as to reach the destination at the scheduled time, is nearest to _.**

A 67

B 61

C 50

D 58

Solution

Let the scheduled time be T.

If the speed is $\frac{1}{3}$ of the usual speed, then the time should be 3 times the initial time taken.

So, new time = 3T

$3T - T = 30$

$\Rightarrow T = 15$ min

On the return journey, the train travelled at its usual speed for 5 min.

So, the remaining distance should be covered in 10 min at the usual speed.

Since 4 min are wasted, the distance should be covered in 6 min.

The remaining time is $\frac{6}{10} = \frac{3}{5}$ of the usual time.

So, the speed should be $\frac{5}{3} = 166.67 \%$ of the usual speed.

So, the speed should increase by 67% approx.

26. If $x = (4096)^{7+4\sqrt{3}}$, then which of the following equals 64?

A $\frac{x^7}{x^{2\sqrt{3}}}$

B $\frac{x^7}{x^{4\sqrt{3}}}$

C $\frac{x^{\frac{7}{2}}}{x^{2\sqrt{3}}}$

D $\frac{x^{\frac{7}{2}}}{x^{2\sqrt{3}}}$

Solution

$x = (4096)^{7+4\sqrt{3}}$

Taking both sides power, we get the following:

=> $x^{7-4\sqrt{3}} = (4096)^{(7+4\sqrt{3})(7-4\sqrt{3})}$

Simplifying this, we can write the following:

=> $x^{7-4\sqrt{3}} =(64^2)^1$

=> $64=(x^{7-4\sqrt{3}})^{\frac{1}{2}}$

=> $64=(x^{\frac{7}{2}-2\sqrt{3}})$

=> $64=\frac{x^{\frac{7}{2}}}{2\sqrt{3}}$

Hence, option (C) is the correct answer.

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