1. Which of the following properties of R is known as Archimedean property?

   A) Every non-empty set of reals which is bounded above has the least upper bound.

   B) For any real x, there exists a unique integer m such that $m \leq x < m+1$.

   C) If $x \in R$, then there exists a natural number n (depending on x) such that $x < n$.

   D) Between any two distinct real numbers, there are infinitely many rational numbers.

2. Given that N is the set of natural numbers, Q is the set of rationals, Z is the set of all integers and $[0,1]$ is the set of reals between 0 and 1. Then state which of the above sets is a compact set?

   A) N B) Q C) Z D) $[0,1]$

3. The value of $\lim\limits_{n\to\infty} n^{1/n}$ equals

   A) 0 B) 1 C) e D) $\sqrt{2}$

4. Let f and g be real valued functions defined on an interval I. If f and g are continuous at $c \in I$, then which of the statements is not true?

   A) $f+g$ is continuous at c B) $f-g$ is not continuous at c

   C) fg is continuous at c D) $\max\{f,g\}$ is continuous at c

5. Let $\sum U_n$ be a series of positive terms. Consider the following statements:

   1. If $\sum U_n$ is convergent, then $\sum U_n^2$ is also convergent.
   2. If $\sum U_n^2$ is convergent, then $\sum U_n$ is also convergent.

   Which of the above statements is/are true?

   A) 1 only B) 2 only C) Both 1 and 2 D) None of these

6. Let $f(x) = \begin{cases} \dfrac{|x|}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ be a real valued function. Consider the following statements:

   1. f is right continuous at $x = 0$.
   2. f has a discontinuity of the first kind from the left at $x = 0$.

   Which of these statements is/are true?

   A) 1 only B) 2 only C) Both 1 and 2 D) None of these

7. Regarding continuity and differentiability of a real valued function f, consider the following statements:

   1. If f is continuous at a point a, then it is not necessary that f is differentiable at a.
   2. If f is not continuous at a, then it cannot be differentiable at a.

   Which of these statements is/are true?

   A) 1 only B) 2 only C) Both 1 and 2 D) None of these

8. The value of the integral, $\int_{-1}^{1} |x|\, dx$ is

   A) 0 B) $\frac{1}{2}$ C) 1 D) $\frac{3}{5}$

9. The residue of $\dfrac{z+1}{z^2-2z}$ at $z=0$ is

   A) $\frac{1}{2}$ B) 1 C) -1 D) $-\frac{1}{2}$

10. The value of $\int_C \dfrac{dz}{z-3}$, where C is the circle $|z-2|=5$, equals

    A) $\dfrac{1}{2}\pi i$ B) πi C) $2\pi i$ D) $\dfrac{3}{2}\pi i$

11. The characteristic roots of a 3-square matrix A are in A.P. Given that $\operatorname{tr}(A)=15$ and $|A|=80$. Then characteristic roots of A are

    A) 2, 5, 8 B) 1, 4, 6 C) 1, 3, 4 D) 3, 4, 6

12. Let S_1 and S_2 be two subspaces of a fixed vector space. Consider the statements,

    1. $S_1 \cup S_2$ is a subspace. 2. $S_1 \cap S_2$ is a subspace.

    Which of these statements is/are true ?

    A) 1 only B) 2 only C) Both 1 and 2 D) None of these

13. Given $S_1 = \{(1,0,2),(0,1,2),(4,-2,4)\}$ and $S_2 = \{(1,2,3),(0.5,1,1.5),(1.5,3,4.5)\}$ two sets of vectors from R^3. Then which of the following is correct?

    A) Both S_1 and S_2 are linearly independent set of vectors

    B) Both S_1 and S_2 are linearly dependent set of vectors

    C) Vectors in S_1 are linearly independent but those in S_2 are not

    D) Vectors in S_2 are linearly independent but those in S_1 are not

14. Let $\lambda_1, \lambda_2, \lambda_3$ be the eigen values of a matrix A of order 3. Consider the statements:

    1. $\lambda_1 + \lambda_2 + \lambda_3 = \operatorname{tr}(A)$

    2. $\lambda_1 \times \lambda_2 \times \lambda_3 = |A|$

    Which of these statements is/are true?

    A) 1 only B) 2 only C) Both 1 and 2 D) None of these

15. The matrix of the quadratic form, $x^2 + z^2 + 2xy + 5yz$ is

    A) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & \frac{5}{2} \\ 0 & \frac{5}{2} & 0 \end{bmatrix}$ D) $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & \frac{5}{2} \\ 0 & \frac{5}{2} & 0 \end{bmatrix}$

16. If A and B are symmetric matrices of the same order, then which one of the following is not correct?

    A) $A+B$ is a symmetric matrix B) $AB+BA$ is a symmetric matrix

    C) AB is a symmetric matrix D) $A+A^T$ and $B+B^T$ are symmetric matrices

17. Which one of the following statements about the Borel field $\mathcal{B}$ of subsets of R is not true?

A) $\mathcal{B}$ is a σ-field generated by the class $\{(-\infty, x), x \in R\}$

B) $\mathcal{B}$ is a minimal σ-field containing the class $\{(-\infty, x), x \in R\}$

C) $\mathcal{B}$ is a class of subsets of R

D) $\mathcal{B}$ is a σ-field generated by the class $\{(-\infty, x], x \in R\}$

18. The Lebesgue outer measure of an interval is its

A) initial point B) midpoint C) final point D) length

19. Let f be an extended real valued measurable function defined on a measurable set. Consider the following statements for any $a \in R$:

1. The set $\{x : f(x) \leq a\}$ is measurable. 2. The set $\{x : f(x) > a\}$ is measurable.

Which of these statements is/are true?

A) 1 only B) 2 only C) both 1 and 2 D) None of these

20. If $\{f_n\}$ is a sequence of measurable functions then the inequality

$\int \limsup_{n\to\infty} f_n \, d\mu \geq \limsup_{n\to\infty} \int f_n \, d\mu$

is satisfied only if

A) $\{f_n\}$ is monotonic decreasing

B) $\{f_n\}$ is monotonic increasing

C) $\{f_n\}$ is bounded below by an integrable function

D) $\{f_n\}$ is bounded above by an integrable function

21. If $(\Omega, \mathcal{F}, \mu)$ is a σ-finite measure space and ν is a σ-finite measure on $\mathcal{F}$, then there exists a finite valued non-negative measurable function f on Ω such that for each $E \in \mathcal{F}$, $\nu(E) = \int_E f \, d\mu$, provided

A) μ is absolutely continuous w.r.t. ν

B) ν is absolutely continuous w.r.t. μ

C) μ is a σ-finite measure

D) No additional condition is required

22. A fair coin is tossed 5 times. If A denotes the event that at least one head showing up, then $P(A)$ equals

A) $\dfrac{27}{32}$ B) $\dfrac{5}{32}$ C) $\dfrac{31}{32}$ D) $\dfrac{3}{32}$

23. If A and B are independent events with $P(A) = 0.3$ and $P(B) = 0.4$, then probability that A does not occur but B occurs is

A) 0.70 B) 0.28 C) 0.21 D) 0.18

24. If an event A is independent of itself, then the value of $P(A)$ is

A) 0 B) 1/2 C) 1 D) 0 or 1

25. Let $X : \Omega \to R$ be a random variable. Let $\mathcal{A}$ be a σ-field of subsets of Ω and $\mathcal{B}$ be the Borel field of subsets of R. Which of the following statements is not true?

A) $X^{-1}(B) \in \mathcal{A}$ for some $B \in \mathcal{B}$ B) $X^{-1}(B) \in \mathcal{A}, \forall B \in \mathcal{B}$

C) $X^{-1}(-\infty, x] \in \mathcal{A}, \forall x \in R$ D) $X^{-1}[x, \infty) \in \mathcal{A}, \forall x \in R$

26. If a random variable X can take positive integer values n with respective probabilities $P(X = n)$, where $n = 1, 2, \ldots,$ then $E(X)$ equals

A) $\sum_{k=n}^{\infty} P(X \geq k)$ B) $\sum_{k=n}^{\infty} P(X < k)$ C) $\sum_{k=1}^{\infty} P(X \geq k)$ D) $\sum_{k=1}^{\infty} P(X < k)$

27. Chebychev's inequality is a special case of the inequality

A) Liapunouv B) Kolmogorov C) Jensen D) Markov

28. The characteristic function of a random variable which is degenerate at 0 is

A) e^{-t} B) e^{it} C) e^{t} D) 1

29. Consider the following real valued function of two variables:

$$F(x, y) = \begin{cases} 0, \text{ if } x < 0 \text{ or } x + y < 1 \text{ or } y < 0 \\ 1, \text{ otherwise} \end{cases}$$

Which one of the following is true ?

A) F is a distribution function

B) F is a step function

C) F is an absolutely continuous distribution function

D) F is not a distribution function

30. The distribution corresponding to the characteristic function $e^{-|t|}$, $t \in R$ is

A) standard Cauchy B) double exponential C) gamma D) exponential

31. Let $\{X_n\}$ be a sequence of random variables defined on a probability space and X, a random variable defined on the same probability space. Consider the following modes of convergence:

1. $X_n \xrightarrow{d} X$ 2. $X_n \xrightarrow{P} X$ 3. $X_n \xrightarrow{\text{a.s.}} X$

Which one of the following relations holds good?

A) $1 \Rightarrow 2$ B) $1 \Rightarrow 3$ C) $2 \Rightarrow 3$ D) $3 \Rightarrow 2$

32. Let $\{X_n\}$ be a sequence of i.i.d. random variables with $E(X_n^2) < \infty, \forall n$ and let Z be the standard normal variate. Write $S_n = \sum_{i=1}^{n} X_i$, $n \geq 1$. If $\sigma^2 = V(X_n) > 0$, then which one of the following is true ?

A) $\dfrac{S_n - E(S_n)}{\sqrt{n}\sigma} \xrightarrow{d} Z$ B) $\dfrac{S_n - E(S_n)}{\sqrt{n\sigma^2}} \xrightarrow{d} Z$

C) $\dfrac{S_n - E(S_n)}{n\sigma^2} \xrightarrow{d} Z$ D) $\dfrac{S_n - E(S_n)}{n\sigma} \xrightarrow{d} Z$

33. If $X \sim$ binomial(n, p), what is the distribution of $Y = n - X$?

A) binomial(n, p) B) negative binomial(n, p)

C) binomial$(n, 1-p)$ D) negative binomial$(n, 1-p)$

34. Let X and Y be random variables such that $Cov(X, Y) = 0$. Then we may conclude that

A) X and Y are independent B) $Cov(X^2, Y^2) = 0$

C) $Cov(X^3, Y^3) = 0$ D) $V(X+Y) = V(X) + V(Y)$

35. A man throws an unbiased die till either 1 or 2 appears. What is the probability that he succeeds at the 4th draw ?

A) $\frac{8}{81}$ *B)* $\frac{4}{81}$ *C)* $\frac{16}{243}$ *D)* $\frac{4}{243}$

36. If X is exponentially distributed with mean 10, then $P(X > 30)$ equals

A) e^{-1} *B)* e^{-2} *C)* e^{-3} *D)* e^{-4}

37. If the variance of a normal distribution is 2, what is its fourth central moment?

A) 12 B) 6 C) 8 D) 10

38. If X has the standard exponential distribution, then the distribution of $1 - e^{-X}$ is

A) standard normal B) standard Cauchy

C) uniform over $(0, 1)$ D) standard double exponential

39. If X_1, X_2, X_3, X_4 are independent standard normal variates, then the distribution of $Y = \dfrac{X_3 - X_4}{\sqrt{X_1^2 + X_2^2}}$ is

A) $\chi^2_{(1)}$ B) $\chi^2_{(2)}$ C) Student's $t_{(1)}$ D) Student's $t_{(2)}$

40. The distribution of the square of Student's $t_{(n)}$ variate is

A) $F_{(n,1)}$ B) $F_{(1,n)}$ C) Cauchy D) standard normal

41. If X has the standard Cauchy distribution, then the distribution of $\frac{1}{X}$ is

A) standard normal B) standard Cauchy C) exponential D) double exponential

42. Let $X \sim N(\mu, \sigma^2)$. If $P(X < -0.5) = 0.05$ and $P(X < 1.14) = 0.95$, then the values of μ and σ^2 are respectively

A) 0.32 and 1/4 B) 0.32 and 4 C) 0.64 and 2 D) 0 and 1

43. If X and Y are independent random variables with density functions f_1 and f_2 respectively, then the pdf of $W = \frac{X}{Y}$ is given by

A) $\int_{-\infty}^{\infty} f_1(xw) f_2(x) |x| dx$ B) $\int_{-\infty}^{\infty} f_1(x) f_2(xw) |x| dx$

C) $\int_{-\infty}^{\infty} f_1(xw) f_2(x) \frac{1}{|x|} dx$ D) $\int_{-\infty}^{\infty} f_1(x) f_2(xw) \frac{1}{|x|} dx$

44. If $X_1, X_2, \ldots, X_5$ is a random sample from the standard exponential population, then pdf of the sample median with value y is

A) $30\,(1-e^{-y})^2 e^{-2y},\ y>0$ B) $30\,(1-e^{-y})^2 e^{-3y},\ y>0$

C) $30\,(1-e^{-y}) e^{-3y},\ y>0$ D) $30\,(1-e^{-y})^2 e^{-y},\ y>0$

45. In random samples of odd size from $U(0,1)$ population, what is the mean of the distribution of sample median ?

A) 0 B) 1/4 C) 1/2 D) 1

46. If (X,Y) has a bivariate normal distribution with parameters μ_1, μ_2, σ_1^2, σ_2^2 and ρ, what is the variance of the conditional distribution of $X/Y=y$?

A) $\sigma_2^2(1-\rho^2)$ B) $\sigma_1^2(1-\rho^2)$ C) $\sigma_2^2\rho^2$ D) $\sigma_1^2\rho^2$

47. In a trivariate distribution, if $R_{3.12}$ denotes the multiple correlation coefficient of X_3 on X_1 and X_2, which one of the following inequalities is satisfied by $R_{3.12}$?

A) $-1 \leq R_{3.12} \leq 1$ B) $0 \leq R_{3.12} \leq 1$ C) $-1 < R_{3.12} < 1$ D) $0 < R_{3.12} < 1$

48. In a trivariate distribution, values of the simple correlation coefficients are given by $r_{12}=0.7$, $r_{13}=0.5$, $r_{23}=0.5$. Then value of the partial correlation $r_{12.3}$ is

A) 0.8 B) 0.7 C) 0.6 D) 0.4

49. Let $X_1, X_2, \ldots, X_n$ be iid $P(\lambda)$ random variables, where λ is unknown.

Write $T_1 = \frac{1}{n}\sum_{i=1}^n X_i$ and $T_2 = \frac{1}{n}\sum_{i=1}^n (X_i-\lambda)^2$. Then

A) T_1 and T_2 are statistics

B) T_1 is not a statistic but T_2 is a statistic

C) T_1 is a statistic but T_2 is not a statistic

D) T_1 and T_2 are not statistics

50. A random sample $(X_1, X_2, \ldots, X_n)$ is drawn from $N(\mu, 1)$.

Which one of the following is an unbiased estimator of μ^2+1 ?

A) $\bar{X}^2+1$ B) $\bar{X}^2$ C) $\dfrac{1}{n}\sum_{i=1}^n X_i^2+1$ D) $\dfrac{1}{n}\sum_{i=1}^n X_i^2$

51. The condition that $V(t)\to 0$ as $n\to\infty$ for an unbiased estimator t to be a consistent estimator is

A) necessary and sufficient B) necessary only

C) sufficient only D) neither necessary nor sufficient

52. Let (X_1, X_2, X_3) be a random sample from $N(\mu, \sigma^2)$, where σ^2 is known. Consider the estimators $\bar{X}$ and $T=\frac{1}{4}(X_1+2X_2+X_3)$ for μ. Then the efficiency of T relative to $\bar{X}$ is

A) 1 B) $\frac{8}{9}$ C) $\frac{7}{9}$ D) $\frac{2}{9}$

53. If a statistic $T = T(X_1, X_2, \ldots, X_n)$ provides as much information about the parameter as the random sample $(X_1, X_2, \ldots, X_n)$ does, then T is

A) an unbiased estimator B) a consistent estimator

C) a sufficient estimator D) an efficient estimator

54. Let $X_1, X_2, \ldots, X_n$ be iid random variables with pdf $f(x) = e^{-(x-\theta)}$, $x > \theta$. Which of the following is a sufficient statistic for θ ?

A) $\sum_{i=1}^{n} X_i$ B) $\bar{X}$ C) $X_{n:n}$ D) $X_{1:n}$

55. Let T_1 and T_2 be two unbiased estimators for θ. Suppose T_1 is the MVUE. If e is the efficiency of T_2 w.r.t. T_1, then the correlation coefficient between T_1 and T_2 is

A) $e^{1/2}$ B) e C) $e^{-1/2}$ D) e^{-1}

56. Let $(X_1, X_2, \ldots, X_n)$ be a random sample drawn from a population with pmf,

$P_N(x) = \frac{1}{N}$, $x = 1, 2, \ldots, N$ and equal to 0, otherwise.

Consider the following statements:

1. The family $\{P_N,\ N \geq 1\}$ is complete. 2. The family $\{P_N,\ N > 1\}$ is not complete.

Which one of the above statements is/are true ?

A) 1 only B) 2 only C) both 1 and 2 D) none of these

57. The following is a random sample of size 5 taken from a uniform distribution over $(0, \theta)$: 1.1, 2.7, 5, 3.1, 4.3

The mle of θ^2 is

A) 1.1 B) 5 C) 25 D) 1.21

58. Let $f(x) = \theta e^{-\theta x}$, $0 < x < \infty$. What is the estimator of θ obtained by the method of moments ?

A) $\bar{X}$ B) $\frac{1}{\bar{X}^2}$ C) $\bar{X}^2$ D) $\frac{1}{\bar{X}}$

59. The mean of 9 observations drawn at random from a normal population with mean μ and standard deviation 9 is 18. What is the shortest 95% confidence interval for μ ?

A) (13.05, 22.95) B) (9, 27) C) (12.12, 23.88) D) (15, 21)

60. For testing $H_0 : \theta = 1$ against $H_1 : \theta = 1.2$ based on a single observation x from a population with pdf $f(x; \theta) = \theta e^{-\theta x}$ if $x \geq 0; \theta > 0$ and equal to 0, otherwise, let the critical region be $x > 2$. Then size of the test is

A) e^{-2} B) e^{2} C) $e^{-1.2}$ D) e^{-1}

61. Let α and β be the probabilities of type I and type II errors in testing a simple hypothesis against a simple alternative. Under which of the following conditions, a best critical region can be chosen using Neyman-Pearson lemma?

A) For fixed α, β is maximum B) For fixed α, $1 - \beta$ is maximum

C) For fixed β, α is maximum D) For fixed β, $1 - \alpha$ is maximum

62. For the SPRT of strength (α, β), the stopping bounds A and B $(A > B)$ satisfy

A) $A \leq \dfrac{1-\beta}{\alpha}$, $B \geq \dfrac{\beta}{1-\alpha}$ B) $A \geq \dfrac{1-\beta}{\alpha}$, $B \leq \dfrac{\beta}{1-\alpha}$

C) $A \leq \dfrac{1-\alpha}{\beta}$, $B \geq \dfrac{\alpha}{1-\beta}$ D) $A \geq \dfrac{1-\alpha}{\beta}$, $B \leq \dfrac{\alpha}{1-\beta}$

63. For testing a hypothesis that a sample comes from a specified distribution against the alternative that it is from some other distribution, which one of the following non-parametric tests is used ?

A) Wilcoxon B) Median C) Kolmogorov-Smirnov D) None of these

64. Match List-I with List-II and choose the correct answer using the codes given below the lists.

List-I	List-II
a. Wilcoxon test	1. goodness of fit
b. Run test	2. ranks
c. Kolmogorov-Smirnov test	3. randomness

Codes:

A) a-1, b-3, c-2 B) a-2, b-3, c-1 C) a-2, b-1,c-3 D) a-1, b-2, c-3

65. The method of complete enumeration is not feasible for determining

A) proportion of females in a country

B) average weight of students of a college

C) total area under paddy cultivation in a state

D) average length of life of electric bulbs manufactured by a company

66. From a population of size 10, a sample of size 2 is drawn. The total number of possible samples to be drawn from the population using SRSWR exceeds the total number of samples using SRSWOR by the number

A) 100 B) 45 C) 55 D) 50

67. Let a population of 5 units have its mean 12 and its mean square 100. A sample of size 2 is drawn without replacement. If the sample mean is denoted by $\bar{x}$, then $E(\bar{x}^2)$ is

A) 30 B) 144 C) 174 D) 150

68. A finite population is divided into three strata of sizes 20, 40 and x respectively. A stratified random sample is drawn from the population using proportional allocation. If total sample size is 30 and the number of units included in the sample from the first stratum is 6, then x equals

A) 40 B) 50 C) 60 D) 30

69. From a population with size 23, a sample of 4 units is to be selected by systematic sampling. If the 10th unit is selected first, what are the other units to be included in the sample?

A) 14, 20, 3 B) 16, 22, 5 C) 12, 18, 24 D) 15, 21, 4

70. In populations with linear trend, consider the following variances:

1. V_{sy}, variance under systematic sampling
2. V_{st}, variance under stratified sampling
3. V_{ran}, variance under simple random sampling

The correct sequence of the above variances in the increasing order is

A) 1, 2, 3 B) 2, 1, 3 C) 1, 3, 2 D) 3, 1, 2

71. Given that $N = 800$, $n = 80$, $\bar{x} = 200$, $\bar{y} = 400$, $\bar{X} = 220$. Then the ratio estimator of $\bar{Y}$ is

A) 110 B) 220 C) 540 D) 440

72. In SRSWOR, with usual notations, an approximation to the sampling variance of the linear regression estimator, $\bar{y}_{lr} = \bar{y} + b(\bar{X} - \bar{x})$, is given by

A) $\dfrac{(1-f)S_y^2}{n}$ B) $\dfrac{(1-f)S_y^2}{N}$ C) $\dfrac{(1-f)S_y^2(1-\rho^2)}{n}$ D) $\dfrac{(1-f)S_y^2(1-\rho^2)}{N}$

73. In one-way ANOVA with t classes each having r values such that all values are equal to a constant β, the correction factor is given by

A) βrt B) β^2 C) $\beta^2 rt$ D) 1

74. Consider the following linear forms in y_1, y_2, y_3 :

1. $3y_1 + 4y_2 - 7y_3$ 2. $y_1 - 2y_2 + y_3$ 3. $y_1 - 2y_2 + 2y_3$

Which of these forms is/are linear contrasts ?

A) Only 1 B) 2 and 3

C) Only 2 D) 1 and 2

75. For CRD, which one of the following is correct ?

A) It uses the principle of local control only.

B) It uses the principles of replication and randomization.

C) It uses the principles of randomization and local control.

D) It is based on the analysis of two-way classified data.

76. The following is the layout of an experimental design with four treatments A, B, C, D applied to 4 blocks, each having 4 plots:

Block I: A B C D Block II: D A B C

Block III: B C A D Block IV: C A D B

The above design is a

A) Split-plot design B) Completely randomized design

C) Randomized block design D) Latin square design

77. In a RBD with r blocks, t treatments and one missing observation, degrees of freedom (d.f.) for the error sum of squares is

A) $rt - r - 1$ B) $rt - r - t$ C) $rt - r - t + 1$ D) $rt - r$

78. While analysing the data of $m \times m$ LSD, the error d.f. in the analysis of variance is equal to

A) $(m-1)(m-2)$ B) $m(m-1)(m-2)$

C) $(m^2 - 2)$ D) $m^2 - m - 2$

79. Consider a BIBD in which v treatments are arranged in b blocks each with k plots, each treatment occurs once and only once in r plots and any two treatments occur together in λ blocks. Which one of the following inequalities is satisfied by the parameters of the BIBD ?

A) $\lambda \geq r$ *B)* $b \geq v - r + 1$ *C)* $r \geq k$ *D)* $v \geq b$

80. In a 2^5-factorial experiment with factors A, B, C, D, E, the interactions ABCD and CDE are confounded. Which of the following interactions gets confounded automatically?

A) CD B) AB C) BC D) ABE