STATISTICS

Paper – IV

Time Allowed : **Three Hours**

Maximum Marks : **200**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **FOURTEEN** questions divided under **SEVEN** Sections.*

*Candidate has to choose any **TWO** Sections and attempt the questions therein. All the Sections carry equal marks.*

The number of marks carried by a question / part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Diagrams / Figures, wherever required, shall be drawn in the space provided for answering the question itself.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

*Answers must be written in **ENGLISH** only.*

SECTION A
(Operations Research and Reliability)

Q1. (a) (i) A company produces two type of sauces – A and B. Both these sauces are made by blending two ingredients – X and Y. A certain level of flexibility is permitted in the formulae of these products. Indeed, the restrictions are that (i) B must contain no more than 75 percent of X, and (ii) A must contain no less than 25 percent of X, and no less than 50 percent of Y. Up to 400 kg of X and 300 kg of Y could be purchased. The company can sell as much of these sauces as it produces at a price of ₹ 18 for A and ₹ 17 for B. The ingredients X and Y cost ₹ 1·60 and ₹ 2·05 per kg respectively.

The company wishes to maximize its net revenue from the sale of these sauces. Formulate this problem as LP model. *5*

(ii) The production department of a company requires 3600 kg of raw material for manufacturing a particular item per year. It has been estimated that the cost of placing an order is ₹ 36 and cost of carrying inventory is 25 percent of the investment in the inventories. The price is ₹ 10 per kg. Help the Purchase Manager to determine an ordering policy for raw material. *5*

(b) Show that a necessary and sufficient condition for the existence of a feasible solution to the transportation problem is

$$\sum_{i}^{m} a_i = \sum_{j}^{n} b_j .$$

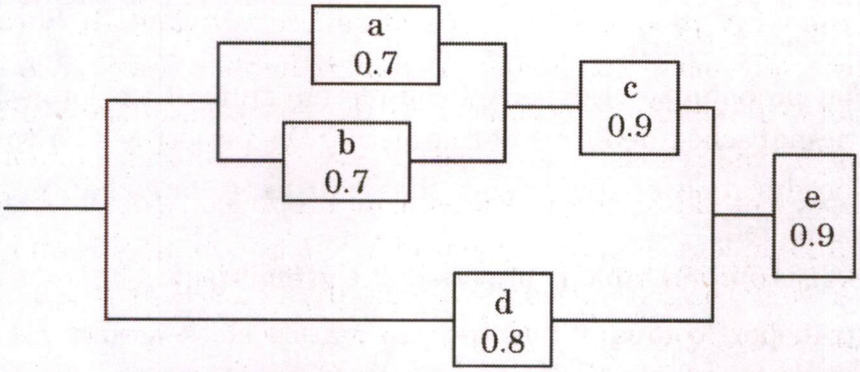
That is, the total capacity (or supply) must equal requirement (or demand). *10*

(c) Solve the following game after reducing it to a 2×2 game. *10*

Player B

		B_1	B_2	B_3
	A_1	1	7	2
Player A	A_2	6	2	7
	A_3	5	1	6

(d) Five elements (a, b, c, d and e) of a system are connected as shown below, which also indicates the reliability of each element. Calculate the system reliability. *10*



(e) If x* and y* be feasible solutions to the primal and dual LP problem, respectively, then a necessary and sufficient condition for x* and y* to be optimal solutions to their respective problems is

$$y_i \cdot x_{n+i} = 0, i = 1, 2 ..., m$$

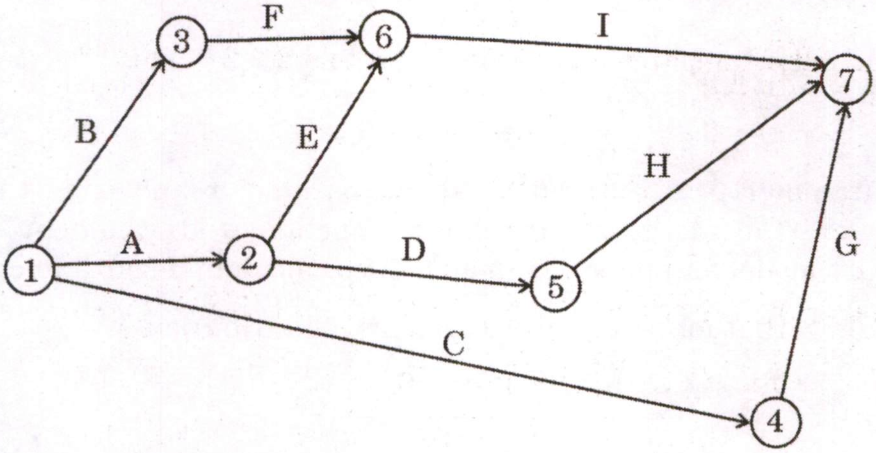
and $$x_j \cdot y_{m+j} = 0, \; j = 1, 2, ..., n$$

where x_{n+i} is the i^{th} slack variable in the primal LP model and y_{m+j} the j^{th} surplus variable for the dual problem. *10*

Q2. Answer any ***two*** questions of the following :

(a) The following network diagram represents activities associated with a project :

Activities :	A	B	C	D	E	F	G	H	I
Optimistic time :	5	18	26	16	15	6	7	7	3
Pessimistic time :	10	22	40	20	25	12	12	9	5
Most likely time :	8	20	33	18	20	9	10	8	4



Determine the following : 25

(i) Expected activity time and variance.

(ii) The earliest and latest expected completion time of each event.

(iii) The critical path.

(iv) The probability of expected completion time of the project if the original scheduled time of completing the project is 41·5 weeks.

(v) The duration of the project that will have 95% chance of being completed.

(Normal distribution table is provided at the end (page 17)).

(b) A road transport company has one reservation clerk on duty at a time. He handles information of bus schedules and makes reservations. Customers arrive at a rate of 8 per hour and the clerk can, on an average, service 12 customers per hour. After stating your assumptions, answer the following :

(i) What is the average number of customers waiting for the service of the clerk ?

(ii) What is the average time a customer has to wait before being served ?

(iii) The management is contemplating to install a computer system for handling information and reservations. This is expected to reduce the service time from 5 to 3 minutes. The additional cost of having the new system works out to ₹ 50 per day. If the cost of goodwill of having to wait is estimated to be 12 paise per minute spent waiting before being served, should the company install the computer system ? 25

Assume an 8-hour working day.

(c) The Weibull probability function for failure is stated as

$$f(t) = \begin{cases} \alpha\, \beta\, t^{\beta - 1} \;\; e^{-\alpha t^{\beta}} & \text{when } t \geq 0 \\ 0 & \text{when } t < 0 \end{cases}$$

The parameter β is referred as shape parameter and α as the scale parameter. Calculate the reliability function and comment on the reliability model for shape parameter. Also calculate hazard function. 25

(d) (i) Show that for a negative exponential distribution

$$f(t) = \lambda\, e^{-\lambda t} \quad ; \quad 0 < \lambda < \infty$$
$$0 \leq t < \infty$$

Intensity function i(t) and hazard rate function h(t) are same. 10

(ii) A city corporation has decided to carry out road repairs on four main arteries of the city. The government has agreed to make a special grant of ₹ 50 lakh towards the cost with a condition that the repairs be done at the lowest cost and quickest time. If the conditions warrant, a supplementary token grant will also be considered favourably. The corporation has floated tenders and five contractors have sent in their bids. In order to expedite work, one road will be awarded to only one contractor.

Cost of repair (₹ in lakh)

	Road →	R_1	R_2	R_3	R_4
	C_1	9	14	19	15
	C_2	7	17	20	19
Contractor	C_3	9	18	21	18
	C_4	10	12	18	19
	C_5	10	15	21	16

I. Find the best way of assigning the repair work to the contractors and the costs.

II. If it is necessary to seek supplementary grants, what should be the amount sought ?

III. Which of the five contractors will be unsuccessful in his bid ? *15*

SECTION B

(Demography and Vital Statistics)

Q3. (a) In the usual notations, prove that *10*

(i) $\frac{dL(x)}{dx} = -d(x)$

(ii) $\frac{d\,\mathring{e}(x)}{dx} = \mu(x)\,\mathring{e}(x) - 1$

where $\mu(x) = \frac{d}{dx} \log_e [l(x)]$

(b) The population of a town in two census years is as follows :

Year	Population
1971	54,892
2001	1,08,431

Calculate the following : *10*

(i) The rate of increase in population per thousand per annum,

(ii) The estimate of the population in 1995.

(c) Show that *10*

(i) $nP_x = P_x \cdot P_{x+1} \ldots\ldots P_{x+n-1}$

(ii) $nq_x = \frac{d_{x+n-1}}{l_x}$

where the notations have their usual meanings.

(d) Fill in the blanks of the following skeleton table which are marked with question marks. *10*

Age x	l_x	d_x	p_x	q_x	L_x	T_x	$\mathring{e}_x$
20	693435	2762	?	?	?	35081126	?
21	620673	–	–	–	–	?	?

(e) (i) Find an expression for $\mu(x)$ assuming $l(x)$ as a fourth degree polynomial. *5*

(ii) Find the differential coefficient of $p(x, t)$ with respect to t and x. *5*

Q4. Answer any ***two*** of the following :

(a) Define vital statistics and write in detail the methods for collecting data for vital statistics. *25*

(b) Compute the crude and standardised death rates of the two populations A and B, regarding A as standard population from the following data : 25

Age Group (years)	A		B	
	Population	Death	Population	Death
Under 10	20,000	600	12,000	372
10 – 20	12,000	240	30,000	660
20 – 40	50,000	1250	62,000	1612
40 – 60	30,000	1050	15,000	525
Above 60	10,000	500	3,000	180

(c) Calculate the general fertility rate, total fertility rate and the gross reproduction rate from the following data, assuming that for every 100 girls, 106 boys are born. 25

Age of Women	Number of Women	Age – SFR (per 1000)
15 – 19	212,619	98·0
20 – 24	198,732	169·6
25 – 29	162,800	158·2
30 – 34	145,362	139·7
35 – 39	128,109	98·6
40 – 44	106,211	42·8
45 – 49	86,753	16·9

(d) What is logistic curve ? Explain the method of three selected points for fitting the logistic curve to a population data. 25

SECTION C
(Survival Analysis and Clinical Trials)

Q5. (a) Obtain the hazard function of Weibull distribution and show that exponential distribution is a particular case of it. *10*

(b) Explain the standard life table method for estimating the distribution of life times. Derive the maximum likelihood estimates of, in the usual notation, q_j s, $j = 1, 2, ..., k$ and find the information matrix. *10*

(c) Construct a test to test the significance of exponential parameter for the case of Type 2 censoring and construct a $(1 - \alpha)$ confidence interval for the parameter. *10*

(d) Discuss the primary and secondary response variables in clinical trials and rules to measure them. *10*

(e) Explain data collection methods and steps to optimize collecting high quality data in clinical trials. *10*

Q6. Answer any ***two*** of the following :

(a) (i) Suppose that a sample of n units is put on test and the test is to be terminated at a preassigned time T or earlier if all units fail before T. Suppose that each unit on test consists of two independent exponential components with means λ_1, λ_2 respectively. Obtain the maximum likelihood estimates of λ_1 and λ_2. *10*

(ii) Obtain approximate maximum likelihood estimates when the number of failures due to either cause is recorded for K specified time intervals. *15*

(b) Estimate the survival probabilities by using an actuarial estimator and find the variances of the estimators from the following data : *25*

Time Interval	Number of Survivors at the beginning of the interval	Observed number of Deaths	Number of Withdrawals
0 – 50	68	16	3
50 – 100	49	11	0
100 – 200	38	4	2
200 – 400	32	5	4
400 – 700	23	2	6
700 – 1000	15	4	3
1000 – 1300	8	1	2
1300 – 1600	5	1	3
1600 +	1	0	1

(c) Compare the empirical distributions of survival times for the groups of patients with dementia at ages of diagnosis 73 and 74 by using log rank test from the following data and comment. 25

Age at Diagnosis	No. of Cases	Length of Survival Time in Years						
73	17	0·58	1·08	1·25	1·67	2·00	2·08	2·17
		3·75	4·25	4·58	4·92	5·25	6·75	7·83
		7·84	9·17	11·50				
74	25	0·50	1·00	1·25	1·41	1·42	1·58	1·66
		1·67	2·25	2·33	2·92	3·08	3·75	3·92
		4·17	4·67	5·00	5·25	5·83	7·25	8·50
		9·33	10·33	11·25	12·50			

(Given χ_1^2 (0·05) = 3·841)

(d) Discuss various designs/approaches used in Phase I to Phase IV studies and their applications. 25

SECTION D
(Quality Control)

Q7. (a) Distinguish between Specification limits and Tolerance limits. *10*

(b) How is the choice between p and np charts made ? Give their applications (three each). *10*

(c) Obtain the factors A_2, D_3 and D_4 used in connection with the computation of control limits for mean and range. *10*

(d) (i) Distinguish between AQL and AOQL.

(ii) Obtain AOQL, if the number of defectives conform to a Poisson distribution with parameter np. *5+5=10*

(e) Discuss Sampling Inspection Plans with reference to statistical quality control. Compare sampling inspection by attributes with inspection by variables. *10*

Q8. Answer any ***two*** of the following :

(a) Write an explanatory note on four important curves which tell the behaviour of a given sampling plan. *25*

(b) (i) Suppose that a process quality characteristic that is normally and independently distributed is plotted in a Shewhart $\overline{X}$ control chart with three sigma control limits $\mu_0 \pm 3\sigma_{\overline{x}}$. Suppose that the process mean μ_0 experiences an upward shift of $1{\cdot}4\sigma$. Determine how long, on an average, will it take to detect this shift if samples of size four are taken every hour. *10*

(Note : Normal distribution table is provided at the end (page 17))

(ii) Explain single sampling plan and obtain its OC and ASN functions. *15*

(c) Derive the ASN for testing $\theta_0 = 0$ against $\theta_1 = 1$ where θ is the normal mean with unit variance keeping $\alpha = 0{\cdot}01$ and $\beta = 0{\cdot}01$ (i.e. keeping both producer's risk and consumer's risk at par at 1% level). *25*

(d) Derive OC function to test $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ where θ is a Bernoulli parameter and write the OC function when $\theta_0 = 0{\cdot}05$, $\theta_1 = 0{\cdot}10$, $\alpha = 0{\cdot}08$ and $\beta = 0{\cdot}03$. *25*

SECTION E
(Multivariate Analysis)

Q9. (a) Let $\mathbf{X} = (X_1, X_2, X_3, X_4)' \sim N_4(\boldsymbol{\mu}, \Sigma)$ with

$\boldsymbol{\mu} = (1, 2, 3, 4)'$ and

$$\Sigma = \begin{pmatrix} 4 & 3 & 2 & 1 \\ 3 & 3 & 2 & 1 \\ 2 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

Then obtain the distribution of

$$\begin{pmatrix} X_1 - X_2 - X_3 + X_4 \\ X_1 + X_2 + X_3 + X_4 \end{pmatrix}.$$ *10*

(b) Let $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_N$ be N independent observations from $N_p(\boldsymbol{\mu}_0, \Sigma)$, where $\boldsymbol{\mu}_0$ is a given vector. Derive the maximum likelihood estimator of Σ. *10*

(c) Let $V = \begin{pmatrix} X & Y \\ Y & Z \end{pmatrix}$ follow Wishart distribution $W_2(n, I)$. Obtain the distribution of $U = \frac{1}{5} X + \frac{4}{5}(Y + Z)$. *10*

(d) Define Hotelling's T^2-statistic and show that it is invariant under a non-singular linear transformation. *10*

(e) Let $\mathbf{X}$ be a 3-dimensional random vector with variance-covariance matrix

$$\Sigma = \begin{pmatrix} 9 & 3 & 3 \\ 3 & 9 & 3 \\ 3 & 3 & 9 \end{pmatrix}.$$

Determine the first principal component and the proportion of the total variability that it explains. *10*

Q10. Answer any ***two*** of the following :

(a) (i) Let

$$\mathbf{X} \sim N_3\left(\begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}11 & -6 & 2\\ -6 & 10 & -4\\ 2 & -4 & 6\end{pmatrix}\right),$$

$Z_1 = X_2 - X_3$

$Z_2 = X_2 + X_3$

$Z_3 \mid Z_1, Z_2 \sim N(Z_1 + Z_2, 10).$

Derive : *15*

I. The joint distribution of Z_1, Z_2 and Z_3

II. The conditional mean of X_3 given X_1 and X_2

III. $\rho^2_{3\cdot 12}$

(ii) Let $\mathbf{X} = (X_1, X_2, X_3)'$ have trivariate normal distribution $N_3(\boldsymbol{\mu}, \Sigma)$, where $\boldsymbol{\mu} = (1, 1, 1)'$ and

$$\Sigma = \begin{pmatrix}1 & 1 & 1\\ 1 & 2 & 2\\ 1 & 2 & 3\end{pmatrix}.$$

Show that $(X_2 - X_1)^2 + (X_3 - X_2)^2$ has a chi-square distribution. State the degrees of freedom. *10*

(b) (i) Two psychological tests were given to 32 men and 32 women. The variables taken were y_1 : tool recognition, y_2 : vocabulary.

Mean vectors and pooled sample covariance matrix of two samples are

$$\overline{\mathbf{y}}_1 = \begin{pmatrix}15\cdot97\\15\cdot91\end{pmatrix}, \quad \overline{\mathbf{y}}_2 = \begin{pmatrix}12\cdot34\\13\cdot91\end{pmatrix}$$

and $$S = \begin{pmatrix}7\cdot164 & 6\cdot047\\ 6\cdot047 & 15\cdot89\end{pmatrix}.$$

Use Hotelling's T^2-statistic to test $H_0 : \mu_1 = \mu_2$ at 5% level of significance. *15*

[You are given : $F_{0\cdot05}(2, 61) = 3\cdot14$, $F_{0\cdot05}(61, 2) = 19\cdot49$]

(ii) Let $\begin{pmatrix} \mathbf{y} \\ \mathbf{x} \end{pmatrix}$ be a partitioned random vector with mean vector $\begin{pmatrix} \boldsymbol{\mu}_y \\ \boldsymbol{\mu}_x \end{pmatrix}$ and covariance matrix $\begin{pmatrix} \Sigma_{yy} & \Sigma_{yx} \\ \Sigma_{xy} & \Sigma_{xx} \end{pmatrix}$. Show that for any matrix A of suitable dimensions

$$E(\mathbf{x}' A \mathbf{y}) = \text{trace}(A \Sigma_{yx}) + \boldsymbol{\mu}_x' A \boldsymbol{\mu}_y .$$ *10*

(c) (i) Suppose $n_1 = 11$ and $n_2 = 12$ observations are made on two random vectors $\mathbf{X}_1$ and $\mathbf{X}_2$ which are assumed to have bivariate normal distribution with a common covariance matrix Σ, but possibly different mean vectors $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$. The sample mean vectors and pooled covariance matrix are

$$\overline{\mathbf{X}}_1 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \ \overline{\mathbf{X}}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \ S_{pooled} = \begin{pmatrix} 7 & -1 \\ -1 & 5 \end{pmatrix}.$$

Obtain Mahalanobis sample distance D^2 and Fisher's linear discriminant function. Assign the observation $\mathbf{X}_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ to either population π_1 or π_2. *15*

(ii) If $A_1 \sim W_p(n_1, \Sigma)$ and $A_2 \sim W_p(n_2, \Sigma)$ which is independent of A_1, then derive the distribution of $A_1 + A_2$. *10*

(d) (i) Define canonical correlations and canonical variates. Prove that canonical correlations are invariant under a non-singular linear transformation. *10*

(ii) For a random sample of size n from

$N_4(\boldsymbol{\mu}, \Sigma)$, we obtain

$\hat{\boldsymbol{\mu}} = (18, 15, 18, 14)'$ and

$$\hat{\Sigma} = \begin{pmatrix} 9{\cdot}5 & 5{\cdot}2 & 6{\cdot}9 & 4{\cdot}6 \\ 5{\cdot}2 & 5{\cdot}4 & 5{\cdot}1 & 3{\cdot}5 \\ 6{\cdot}9 & 5{\cdot}1 & 10{\cdot}0 & 5{\cdot}6 \\ 4{\cdot}6 & 3{\cdot}5 & 5{\cdot}6 & 4{\cdot}5 \end{pmatrix}.$$

I. Find the estimates of the parameters of the conditional distribution of (X_3, X_4) given (X_1, X_2).

II. Find the partial correlation $r_{34 \cdot 12}$. *15*

SECTION F

(Design and Analysis of Experiments)

Q11. (a) (i) What are the main sources of experimental error ?

(ii) How is the efficiency of an experiment increased by increased replications and local control ? *5+5=10*

(b) How is the efficiency of a design measured ? Derive the expressions to measure the efficiency of a Latin Square Design over Randomised Block Design, when : *10*

(i) rows are used as blocks

(ii) columns are used as blocks

(c) Prove that for a 2^n factorial experiment in blocks of 2^k plots, if any two effects are confounded, then their generalised interaction is also confounded. *10*

(d) What do you understand by "Analysis of Covariance" ? Illustrate by giving suitable example. How does the use of concomitant variables help to control the error of an experiment ? *10*

(e) A factorial experiment with seven factors, each at two levels, is to be conducted in blocks of size 8. Write the complete set of interactions to be confounded such that no main effect and two-factor interactions get confounded. *10*

Q12. Answer any ***two*** of the following :

(a) (i) Write down the Yates' algorithm for analysing 3^2 factorial experiments. *10*

(ii) In a 3^3 factorial experiment conducted in two replications in blocks of 3^2 plots, the following information is given :

Treatment Combinations in one of the Blocks

Replication 1	Replication 2
100	001
201	102
112	012
210	110
222	121
002	200
121	020
020	222
011	211

Identify the confounded interaction in each replicate. *15*

(b) (i) State the restrictions imposed on number of treatments along with their replications as we pass from a CRD to an RBD and then to an LSD. *10*

(ii) Consider an RBD with one missing value. Perform the appropriate analysis estimating the missing value. *15*

(c) (i) "Split-Plot Designs are sometimes referred to factorial designs with main effects confounded." Discuss. *10*

(ii) In Split-Plot Designs, if e_1 and e_2 are the main plot and sub-plot errors, both estimated in units of single sub-plot, explain why e_1 is expected to be larger than e_2. *5*

(iii) Complete the following ANOVA table given for a Split-Plot Design : *10*

Source of Variation	d.f.	S.S.	M.S.S.	F
Blocks	3	27	–	3
Varieties	–	–	–	4
Error I	6	–	–	
Manure	–	–	–	4
Variety × Manure	–	–	18	–
Error II	27	243	9	

(d) (i) How do you compare the differences in a set of paired treatment means using : *10*

I. Least significant difference.

II. Duncan's multiple range test.

(ii) Explain the statistical analysis of 'Analysis of Covariance' for a one-way layout with one concomitant variable. *15*

SECTION G
(Computing with C and R)

Q13. (a) Write a C-program to compute the roots of a quadratic equation using switch statement. *10*

(b) Write a C-program to test whether a given string is palindrome defining a function. *10*

(c) Write a C-program to find the median of a given vector of elements. *10*

(d) Write R code for correlation coefficient and correlation test. *10*

(e) Write R code to perform two-sample t-test writing your own function. *10*

Q14. Answer any ***two*** of the following :

(a) Given student records containing name, register number, and average marks, write a C-program using structures to assign grades A, B, C and D.

(i) Arrange student records according to merit.

(ii) Give number of students in each grade and display the results. *25*

(b) Write an interactive C-program to fit a Poisson distribution for a given observed frequency distribution and test for its goodness of fit and store the results in a file. *25*

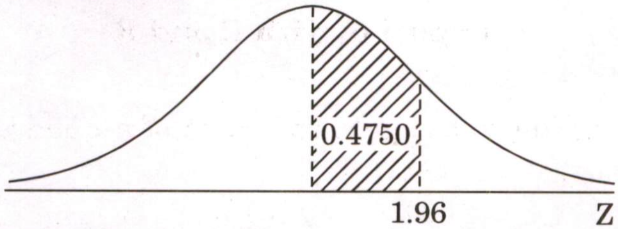
(c) Given a two-way ANOVA table with 5 treatments, 8 blocks and 3 observations per cell, write R code to (i) test the normality of the observations (ii) perform two-way ANOVA and post hoc test, and (iii) display the results. *25*

(d) In a study sample, males of various ages were tested for blindness and the results recorded as given below :

Age :	25	35	45	55	75
Number tested :	60	60	60	60	60
Number blind :	4	15	25	29	34

Write a R code to fit a logistic model and display the result. *25*

P(0 < Z < 1.96) = 0.4750



The Normal Distribution

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000