

# GATE Civil



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1. A completely mixed dilute suspension of sand particles having diameters 0.25, 0.35, 0.40, 0.45 and 0.50mm are filled in a transparent glass column of diameter 10cm and 2.50m. The suspension is allowed to settle without any disturbance. It is observed that all particles of diameter 0.35 mm settle to the bottom of the column in 30s. For the same period of 30s, the percentage removal (round off to integer value) of particles of diameters 0.45 and 0.50 mm from the suspension is

100

Answer range : 95.0-105.0

### Solution

- As we know that settling velocity for discrete particles is given by stokes law as  $V_T = \frac{(G-1)\gamma d^2}{18\mu} \Rightarrow V_T \propto d^2$ .
- For 30 second duration if 0.35 mm particle size settles completely then % removal of particle size 0.45 mm and 0.50 mm will be 100% respectively for each. As settling velocity of particle size 0.45 mm and 0.50 mm will be greater than settling velocity of size 0.35 mm ( $V_T \propto d^2$ ).

2. "I have not yet decided what I will do this evening; I \_\_\_\_\_ visit a friend."

A Mite

B Would

C Might

D Didn't

### Solution

- The complete sentence is:
- "I have not yet decided what I will do this evening; I might visit a friend."
- We do not know what the person or subject wants to do in the evening.
- So we are using the verb "might" in order to emphasize the mood of the speaker.
- Here we are using the past tense of the verb 'may'.
- We have to use the verb 'ma'y which indicates he may do something just he is not quite sure.

3. **Eject : Insert :: Advance : \_\_\_\_\_ ?**

**A** Advent

**B** Progress

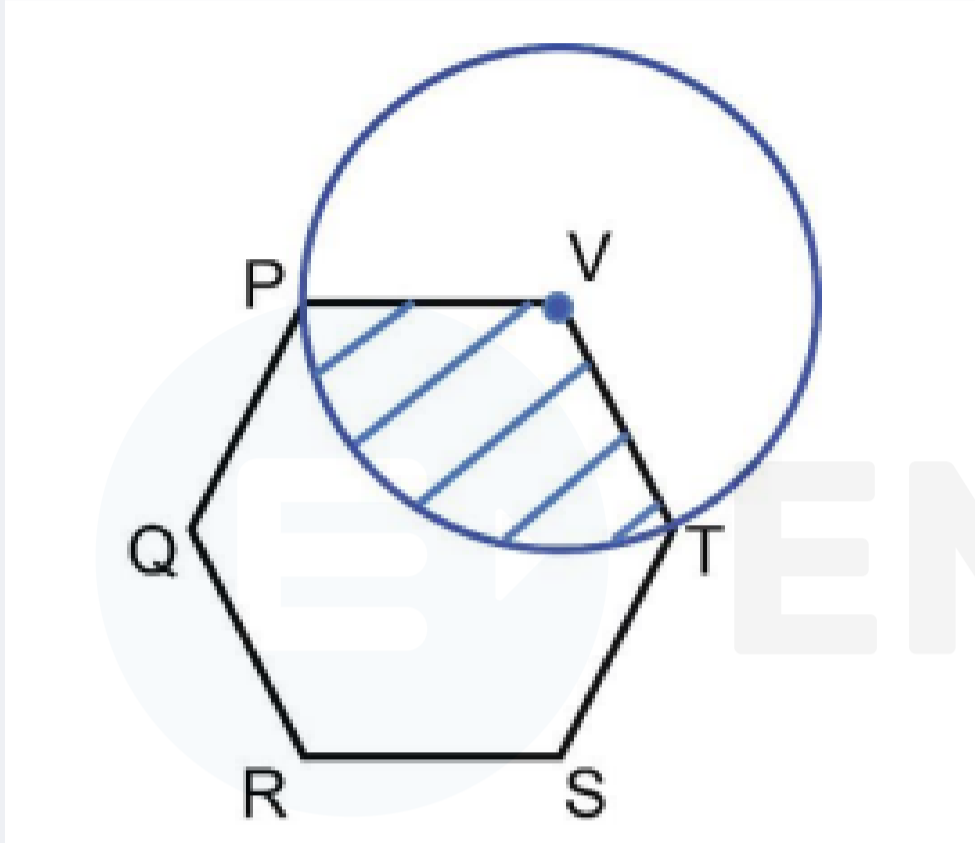
**C** Retreat

**D** Loan

### **Solution**

- Meaning of eject is to throw out especially by physical force, authority or influence.
- Insert is the opposite of eject.
- Meaning of insert is to put or thrust in.
- Retreat and advance are opposite terms where retreat means an act of moving away especially from something difficult, dangerous or disagreeable.
- Advance means to move forward along a course.
- Thus option C is the correct answer.

4. In the given figure, PQRSTV is a regular hexagon with each side of length 5 cm. A circle is drawn with its centre at V such that it passes through P. What is the area(in  $cm^2$ ) of the shaded region? (The diagram is representative).



**A**  $\frac{25\pi}{3}$

**B**  $\frac{20\pi}{3}$

**C**  $6\pi$

**D**  $7\pi$

### Solution

- Sum of interior angle =  $(n - 2) \times 180^0$
- Each angle of regular hexagon  $\Rightarrow$

- $\theta = \frac{(n-2) \times 180^0}{n} = 120^0$

- Required area under shaded region =  $\frac{\theta}{360^0 \times \pi \times 25} = \frac{120^0}{360^0 \times \pi \times 25} = \frac{25\pi}{3}$

5. A duck named Donald Duck says “All ducks always lie.”

Based only on the information above, which one of the following statements can be logically inferred with certainty:

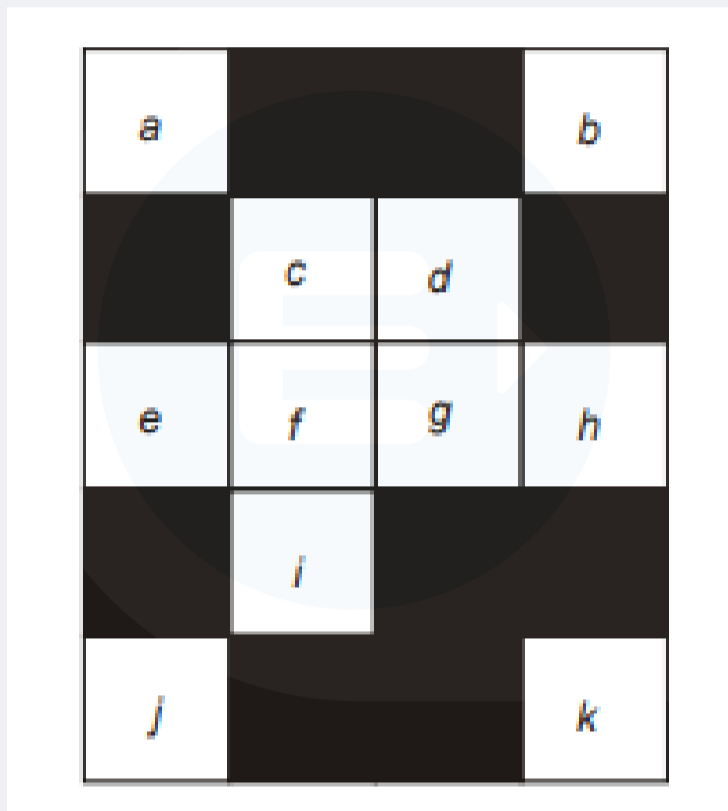
- A** Donald Duck always lies
- B** Donald Duck always tells the truth
- C** Donald Duck’s statement is true

**D** Donald Duck’s statement is false

### Solution

- The statement that "all ducks always lie" cannot be logically inferred, as the information provided does not indicate that ducks always lie.
- The statement cannot be logically inferred, as it is possible that the duck can say truth also.
- Since the statement use the word 'all', the statement is false.

6. A line of symmetry is defined as a line that divides a figure into two parts in a way such that each part is a mirror image of the other part about that line. The figure below consists of 20 unit squares arranged as shown. In addition to the given black squares, upto 5 more may be coloured black. Which one among the following options depicts the minimum number of boxes that must be coloured black to achieve two lines of symmetry? (The figure is representative)



A d

B c, d, i

C c, i

D c, d, i, f, g

**Solution**



- If box d or i is coloured block we get one line of symmetry.
- i.e. horizontal or vertical.
- But for two line of symmetry, we need to atleast colour 3 box.
- i.e. a, d and i will only give two line of symmetry.

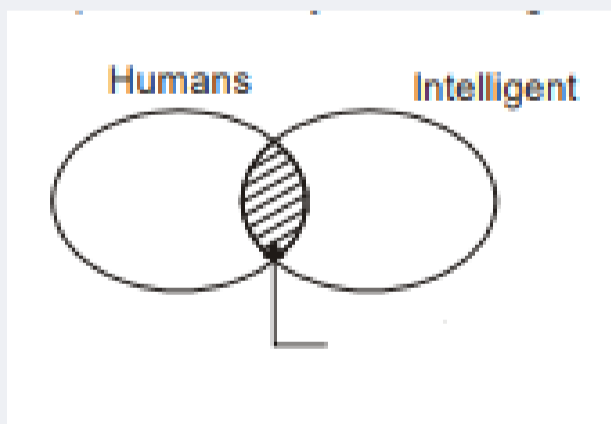
7. Based only on the truth of the statement ‘Some humans are intelligent’, which one of the following options can be logically inferred with certainty?

- A** No human is intelligent
- B** All humans are intelligent
- C** Some non-humans are intelligent

**D** Some intelligent beings are humans

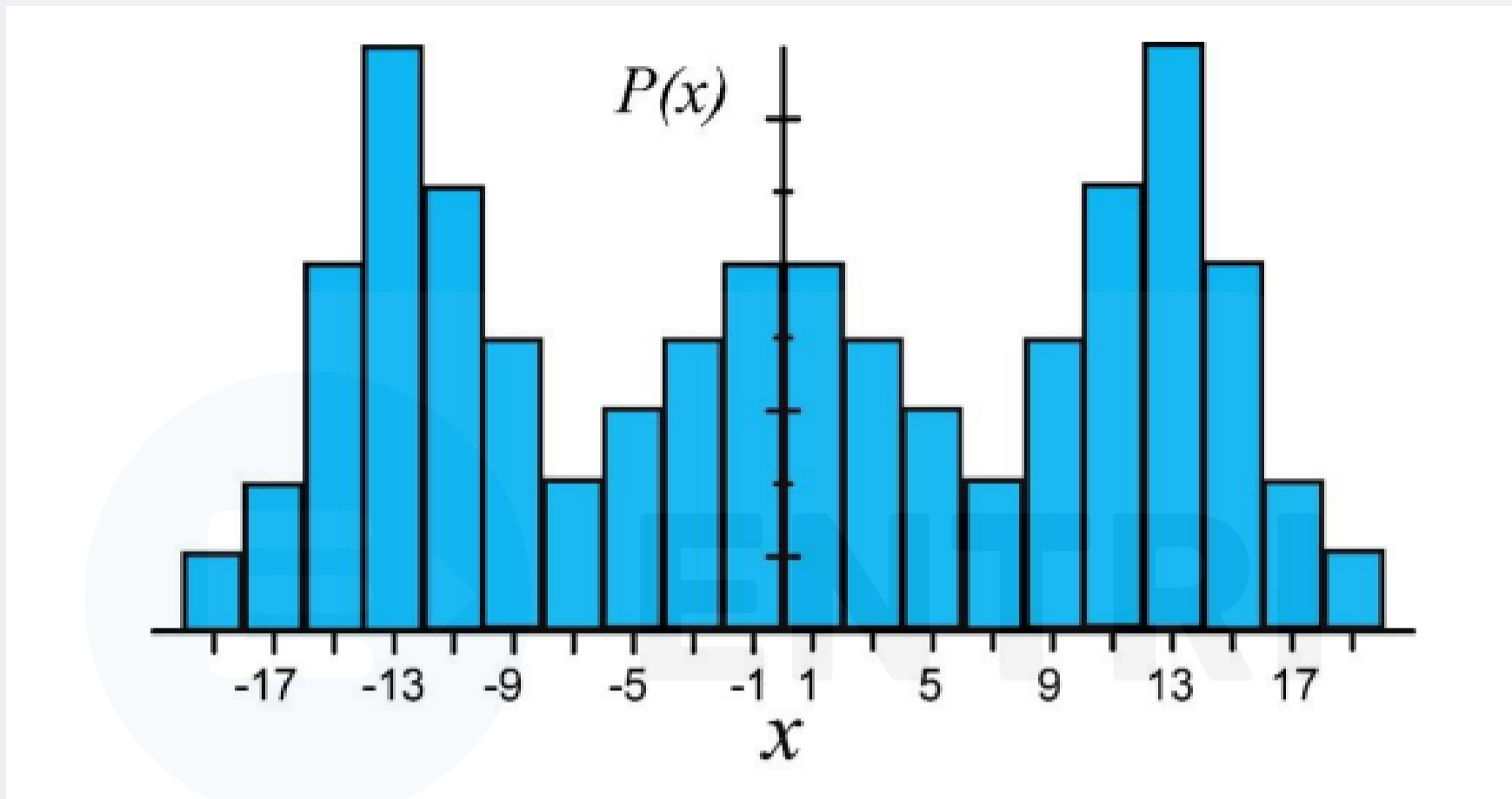
### Solution

- “Some humans are intelligent” can be represented by Venn diagram.



- From the venn diagram the shaded region represents the intelligent humans.
- Thus the option D "some intelligent beings are humans" is true.

8. Which one of the options can be inferred about the mean, median, and mode for the given probability distribution (i.e. probability mass function),  $P(x)$ , of a variable  $x$ ?



**A** mean = median  $\neq$  mode

**B** mean = median = mode

**C** mean  $\neq$  median = mode

**D** mean  $\neq$  mode = median

### Solution

- Considering the Y axis,
- Median =  $\frac{-1+1}{0} = 0$

- Mode = -13, 13 (Value with high probability)
- Since the given distribution contains bi - model data, median  $\neq$  mode.
- Thus mean = median  $\neq$  mode is the correct possible answer.

9. **The James Webb telescope, recently launched in space, is giving humankind unprecedented access to the depths of time by imaging very old stars formed almost 13 billion years ago. Astrophysicists and cosmologists believe that this odyssey in space may even shed light on the existence of dark matter. Dark matter is supposed to interact only via the gravitational interaction and not through the electromagnetic-, the weak- or the strong-interaction. This may justify the epithet “dark” in dark matter.**

**Based on the above paragraph, which one of the following statements is FALSE?**

**A**

No other telescope has captured images of stars older than those captured by the James Webb telescope

**B**

People other than astrophysicists and cosmologists may also believe in the existence of dark matter

**C**

The James Webb telescope could be of use in the research on dark matter

**D**

**If dark matter was known to interact via the strong-interaction, then the epithet “dark” would be justified**

### **Solution**

- In the first sentence, it is said that the approach is unprecedented.

- Meaning of unprecedented is never done or known before. i.e. it was first approach by James Webb telescope to access the depths of time by imaging very old stars formed almost 13 billion years ago.
- Thus first statement is true that no other telescope has captured images of stars older than those captured by the James Webb telescope.
- The second sentence mention that astrophysicists and cosmologists believe in the existence of dark matter.
- It doesn't mention about others.
- So, people other than astrophysicists and cosmologists may also believe in the existence of dark matter.
- Since dark matter is supposed to interact only via the gravitational interaction, it is possible to use James Webb telescope in the research on dark matter.
- Thus options A, B and C are correct and only option D is wrong.

10. Let  $a = 30!$ ,  $b = 50!$ , and  $c = 100!$ . Consider the following numbers:

- $\log_a c$
- $\log_c a$
- $\log_b a$
- $\log_a b$

Which one of the following inequalities is CORRECT?

**A**  $\log_c a < \log_b a < \log_a b < \log_a c$

**B**  $\log_c a < \log_a b < \log_b a < \log_b c$

**C**  $\log_c a < \log_b a < \log_a c < \log_a b$

**D**  $\log_b a < \log_c a < \log_a b < \log_a c$

### Solution

$$\bullet \log_a c = \frac{\log c}{\log a} = \frac{\log 100!}{\log 30!}$$

$$\bullet \log_c a = \frac{\log a}{\log c} = \frac{\log 30!}{\log 100!}$$

$$\bullet \log_b a = \frac{\log a}{\log b} = \frac{\log 30!}{\log 50!}$$

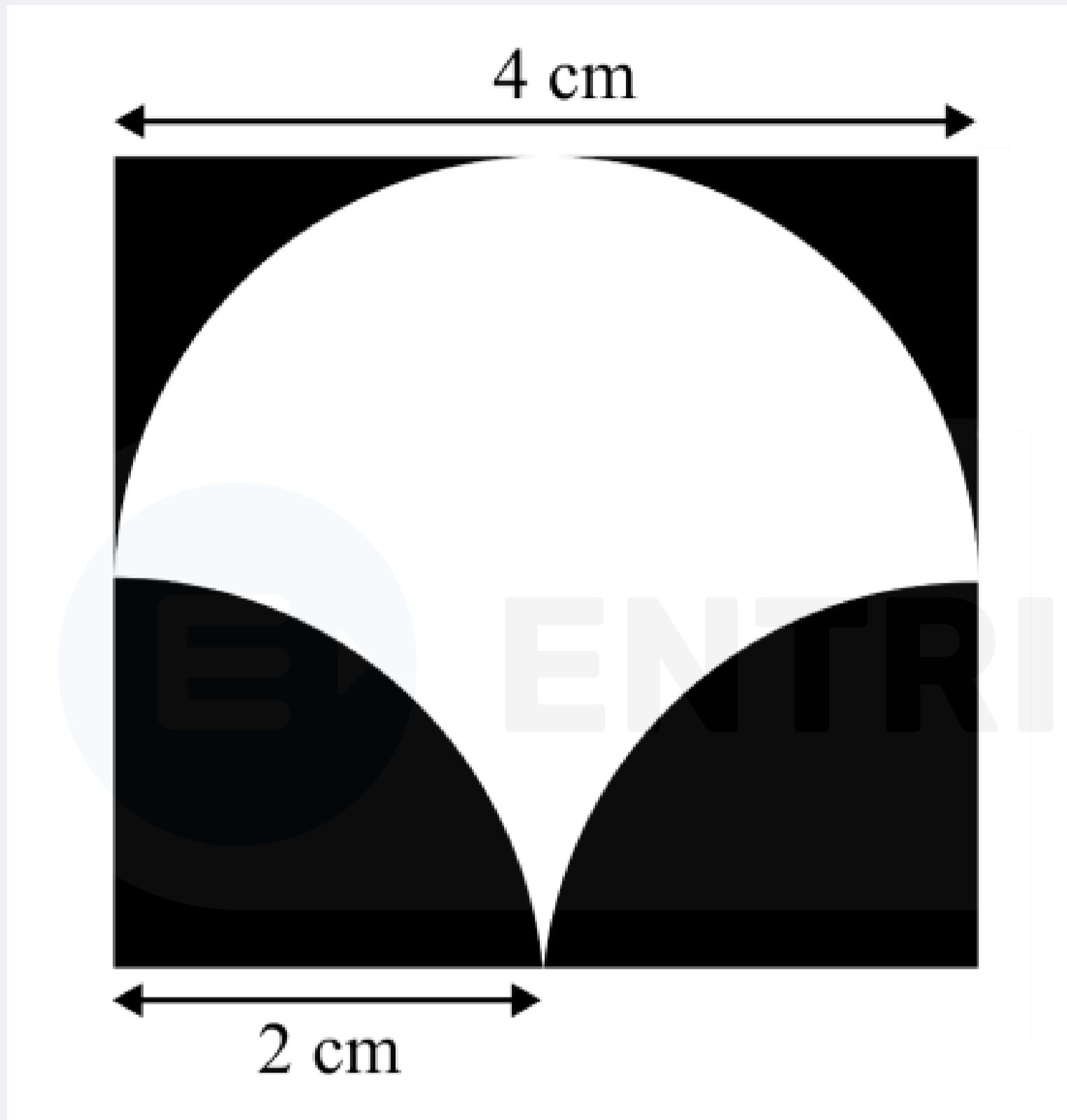
$$\bullet \log_a b = \frac{\log b}{\log a} = \frac{\log 50!}{\log 30!}$$

- From the above calculations it is clear that  $\log_a c$  is the greatest one and  $\log_c a$  is the smallest one.

- It is also clear that  $\log_b a$  is greater than  $\log_a b$ .
- Thus the correct order is:
- $\log_c a < \log_b a < \log_a b < \log_a c$



11. A square of side length 4 cm is given. The boundary of the shaded region is defined by one semi-circle on the top and two circular arcs at the bottom, each of radius 2 cm, as shown.



The area of the shaded region is \_\_\_\_\_  $cm^2$ .

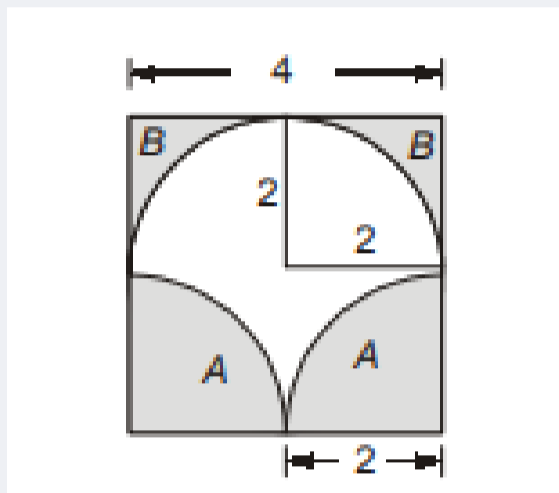
**A** 8

**B** 4

**C** 12

**D** 10

### Solution



- Top shaded area =  $4 \times 2 - \frac{\pi 2^2}{2} = 8 - 2\pi$
- Bottom shaded area =  $\frac{\pi 2^2}{4} \times 2 = 2\pi$
- Total shaded area =  $8 - 2\pi + 2\pi = 8 \text{ cm}^2$

12. For the integral  $I = \int_{-1}^1 \frac{1}{x^2} dx$ , which of the following statement is true?

A  $I = 0$

B  $I = 2$

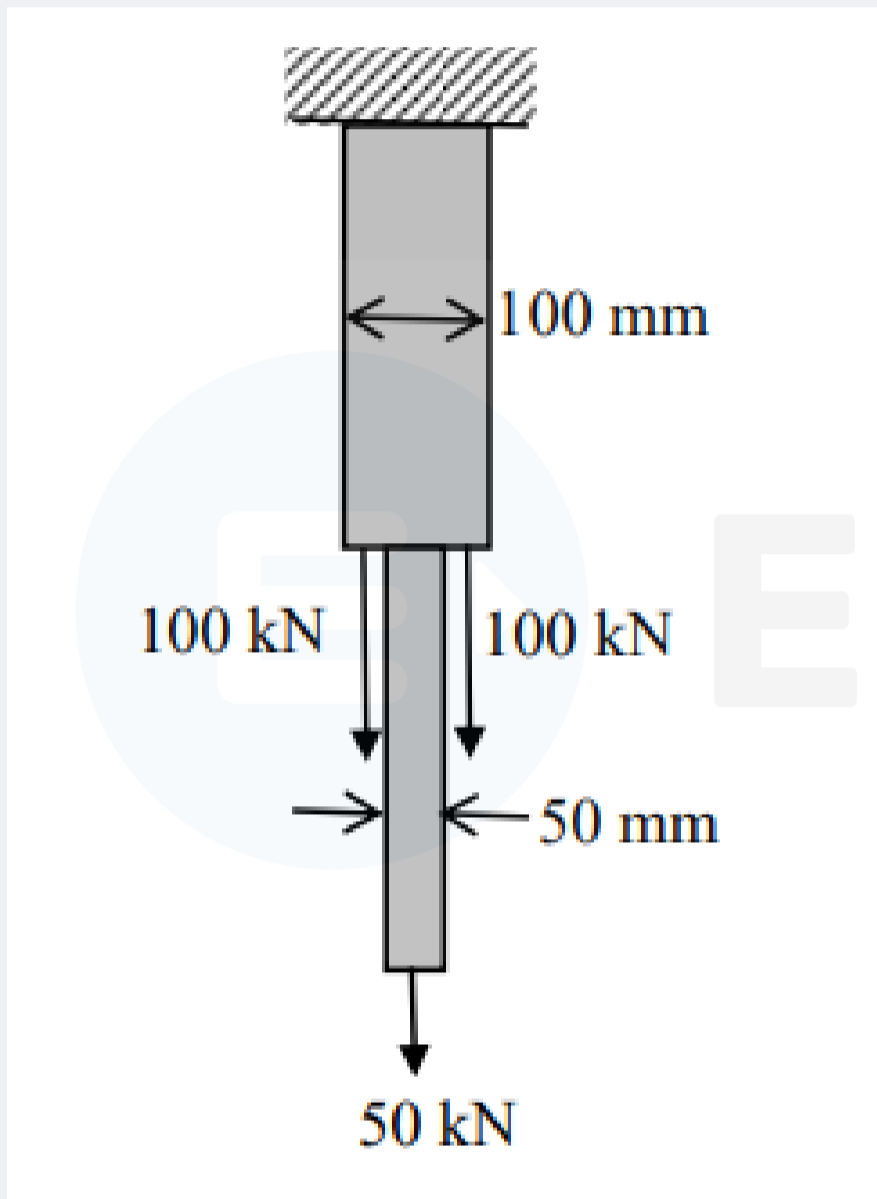
C  $I = -2$

D The integral does not converge

### Solution

- $f(x) = \frac{1}{x^2}$  is not defined at  $x = 0$
- So,  $I = \int_{-1}^0 \frac{1}{x^2} dx + \int_0^1 \frac{1}{x^2} dx = \left(-\frac{1}{x}\right)_{-1}^0 + \left(-\frac{1}{x}\right)_0^1$
- $\frac{1}{x}$  is not defined at  $x = 0$ .
- Therefore, the integral does not converge.

13. A hanger is made of two bars of different sizes. Each bar has a square cross-section. The hanger is loaded by three-point loads in the mid vertical plane as shown in the figure. Ignore the self-weight of the hanger. What is the maximum tensile stress in  $N/mm^2$  anywhere in the hanger without considering stress concentration effects?



A 15.0

**B 25.0**

C 35.0

D 45.0

## Solution

- Let us consider the top member as member 1 and bottom member as member 2.
- Stress in member (1),  $\sigma_1 = \frac{P_1}{A_1} = \frac{250 \times 10^3}{100 \times 100} = 25 \text{ N/mm}^2$
- Stress in member (2),  $\sigma_2 = \frac{P_2}{A_2} = \frac{50 \times 10^3}{50 \times 50} = 20 \text{ N/mm}^2$
- Therefore, maximum tensile stress anywhere in the hanger without considering stress concentration effects is:  $\sigma_{max} = \sigma_1 = 25 \text{ N/mm}^2$

14. Creep of concrete under compression is defined as the:

**A** Increase in the magnitude of strain under constant stress

**B** Increase in the magnitude of stress under constant strain

**C** Decrease in the magnitude of strain under constant stress

**D** Decrease in the magnitude of stress under constant strain

### Solution

- Under sustained compressive loading, deformation in concrete increases with time even though the applied stress is not changed.
- Basically, long term pressure or stress on concrete can make it change shape.
- This deformation usually occurs in the direction the force is being applied.
- Like a concrete column getting more compressed, or a beam bending.
- The time dependent component of strain is called creep.
- Creep under compression is defined as increase in the magnitude of strain under constant stress.

15. A singly reinforced concrete beam of balanced section is made of M20 grade concrete and Fe415 grade steel bars. The magnitudes of the maximum compressive strain in concrete and the tensile strain in the bars at ultimate state under flexure, as per IS 456: 2000 are \_\_\_\_\_ respectively. (round off to four decimal places)

**A** 0.0035 and 0.0038

**B** 0.0020 and 0.0018

**C** 0.0035 and 0.0041

**D** 0.0020 and 0.0031

### Solution

- Given data,
- Grade of concrete M-20
- Grade of steel Fe-415
- As per Clause No. 38.1, IS 456 : 2000,
- Maximum strain in concrete at the outermost compression fibre = 0.0035
- strain in the tension reinforcement for balanced section at ultimate state under flexure (Maximum compressive strain in steel)
- $\Rightarrow 0.002 + \frac{f_y}{1.15E_s} = 0.002 + \frac{415}{1.15 \times 2 \times 10^5} = 0.0038$
- Hence correct answer is option A.





16. In cement concrete mix design, with the increase in water-cement ratio, which one of the following statements is TRUE?

**A** Compressive strength decreases but workability increases

**B** Compressive strength increases but workability decreases

**C** Both compressive strength and workability decrease

**D** Both compressive strength and workability increase

### Solution

- As the water-cement ratio increases, the porosity in the hardened concrete increases.
- Due the increase in porosity, the strength of concrete decreases.
- Increase in water-cement ratio causes higher water availability in the concrete.
- High amount of water content increases the workability.
- Hence option A is the correct answer.

17. The specific gravity of a soil is 2.60. The soil is at 50% degree of saturation with a water content of 15%. The void ratio of the soil is:

A 0.35

B 0.78

C 0.87

D 1.28

### Solution

- Given:
- Specific gravity,  $G = 2.6$
- Saturation,  $S = 0.5$
- Water content,  $w = 0.15$
- As we know  $\Rightarrow$
- $Se = Gw$
- $0.5 \times e = 2.6 \times 0.15$
- $e = \frac{2.6 \times 0.15}{0.5} = 0.78$

18. A group of 9 friction piles are arranged in a square grid maintaining equal spacing in all directions. Each pile is of diameter 300 mm and length 7 m. Assume that the soil is cohesionless with effective friction angle  $\phi' = 32^\circ$ . What is the center-to-center spacing of the piles (in m) for the pile group efficiency of 60%?

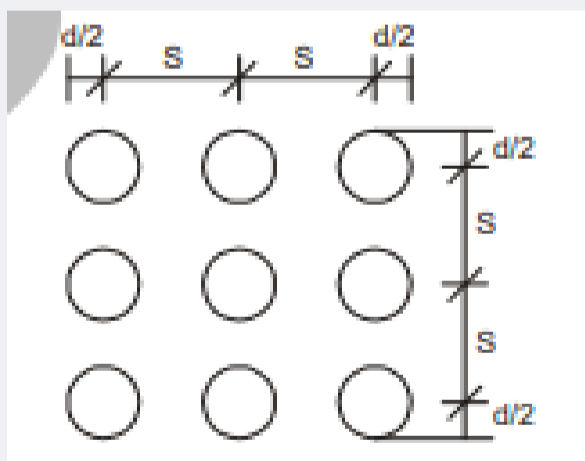
A 0.582

B 0.486

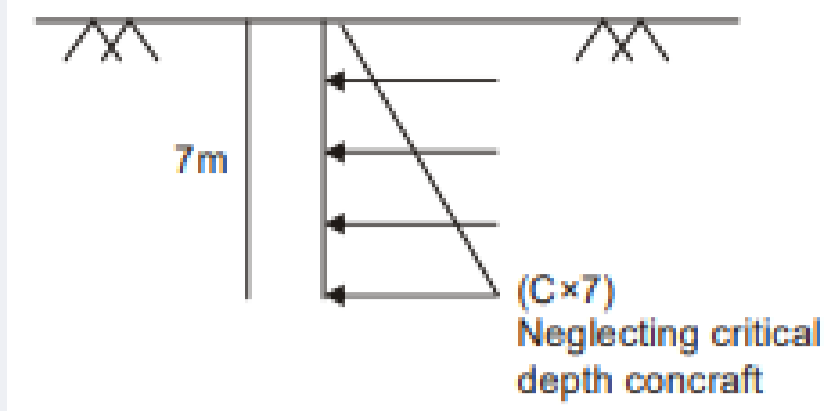
C 0.391

D 0.677

### Solution



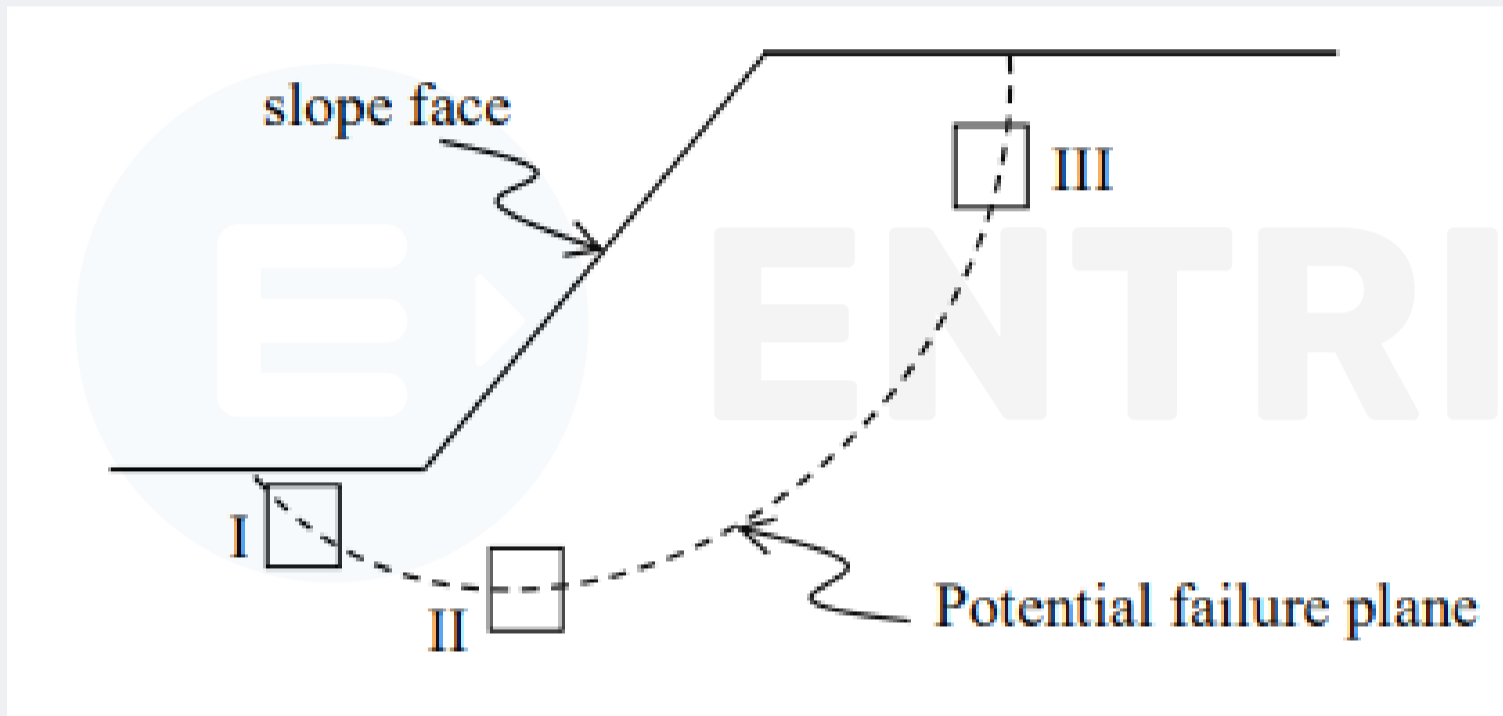
- $d = 300 \text{ mm}$
- $l = 7 \text{ m}$
- $C^I = 0$
- $\phi^I = 32^\circ$
- $\eta_{group} = 60\%$



- As the  $N_q$  value is not given, we can neglect the point bearing resistance and the pile capacity can be calculated using only the frictional capacity.
- $\eta_g = 0.6 = \frac{\text{Block failure strength}}{\text{individual pile strength}}$
- $0.6 = \frac{K\bar{\sigma}\tan\delta \times 4(2s+d) \times 7m}{9 \times (\bar{\sigma}\tan\delta)k \times (\pi d) \times 7}$
- $0.6 = \frac{4(2s+d)}{\pi d \times 9} = \frac{\frac{0.6 \times \pi d \times 9}{4} - d}{2}$
- $S = 0.486 \text{ m}$

19. A possible slope failure is shown in the figure. Three soil samples are taken from different locations (I, II and III) of the potential failure plane. Which is the most appropriate shear strength test for each of the sample to identify the failure mechanism? Identify the correct combination from the following options:

**P: Triaxial compression test Q: Triaxial extension test R: Direct shear or shear box test S: Vane shear test**



**A I-Q, II-R, III-P**

**B I-R, II-P, III-Q**

**C I-S, II-Q, III-R**

**D I-P, II-R, III-Q**

**Solution**

- I.  $\sigma_H > \sigma_V \rightarrow$  Triaxial extension (As  $\sigma_V$  decreases depth from ground level is less)
- II. Failure plane is same as direct shear box test i.e perpendicular to vertical plane.
- III.  $\sigma_V > \sigma_H \rightarrow$  Triaxial compression test.

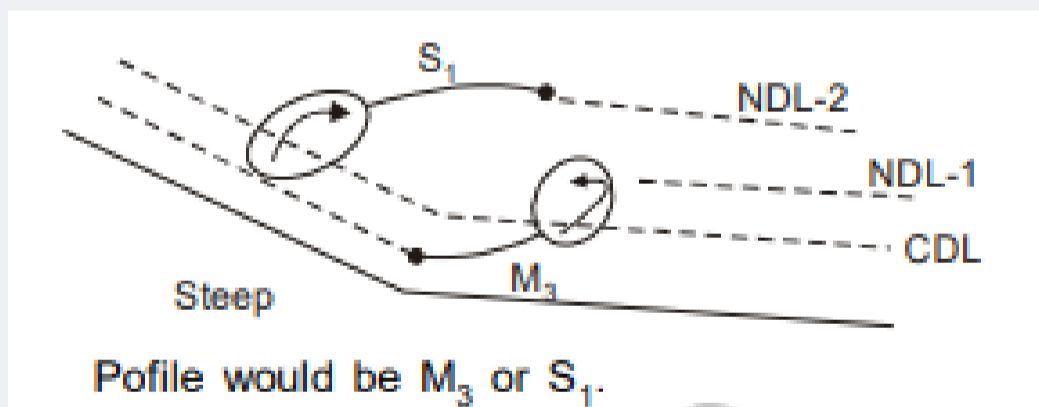
20. When a supercritical stream enters a mild-sloped (M) channel section, the type of flow profile would become:

- A  $M_1$
- B  $M_2$
- C  $M_3$
- D  $M_1$  and  $M_2$

### Solution

When super critical stream enters a mild (m) channel section, the type of

flow.



- Profile would be  $M_3$  or  $S_1$  .

21. Which one of the following statements is TRUE for Greenhouse Gas (GHG) in the atmosphere?

A

GHG absorbs the incoming short wavelength solar radiation to the earth surface, and allows the long wavelength radiation coming from the earth surface to pass through

B

GHG allows the incoming long wavelength solar radiation to pass through to the earth surface, and absorbs the short wavelength radiation coming from the earth surface

C

GHG allows the incoming long wavelength solar radiation to pass through to the earth surface, and absorbs the short wavelength radiation coming from the earth surface

D

**GHG allows the incoming short wavelength solar radiation to pass through to the earth surface, and absorbs the long wavelength radiation coming from the earth surface**

### Solution

- Greenhouse gases (also known as GHGs) are gases in the earth's atmosphere that trap heat.
- Greenhouse gases in the atmosphere (such as water vapor and carbon dioxide) absorb most of the Earth's emitted longwave infrared radiation, which heats the lower atmosphere.



- Greenhouse gases let short-wavelength radiation come into the Earth's atmosphere from the sun.
- However, they absorb and re-radiate Earth's long-wavelength radiation back towards to Earth's surface keeping the temperature on Earth warm enough to inhabit.
- Thus option D is the correct answer.

22.  $G_1$  and  $G_2$  are the slopes of the approach and departure grades of a vertical curve, respectively.

Given  $|G_1| < |G_2|$  and  $|G_1| \neq |G_2| \neq 0$

- **Statement 1:**  $+G_1$  followed by  $+G_2$  results in a sag vertical curve.
- **Statement 2:**  $-G_1$  followed by  $-G_2$  results in a sag vertical curve.
- **Statement 3:**  $+G_1$  followed by  $-G_2$  results in a crest vertical curve.

Which option amongst the following is true?

**A** Statement 1 and Statement 3 are correct; Statement 2 is wrong

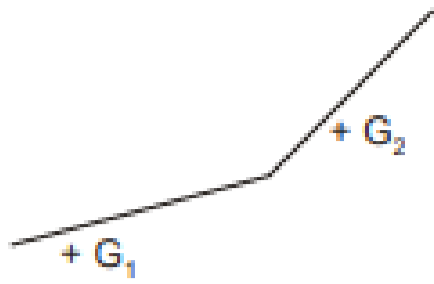
**B** Statement 1 and Statement 2 are correct; Statement 3 is wrong

**C** Statement 1 is correct; Statement 2 and Statement 3 are wrong

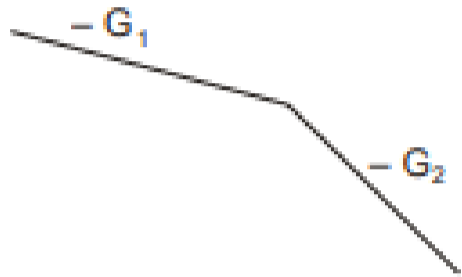
**D** Statement 2 is correct; Statement 1 and Statement 3 are wrong

### Solution

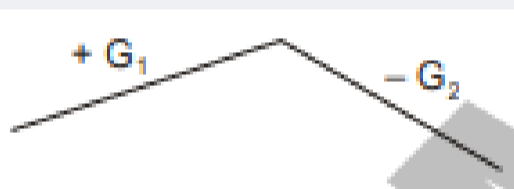
- Given that  $|G_1| < |G_2|$  and  $|G_1| \neq |G_2| \neq 0$
- Statement 1:



- $|G_1| < G_2|$
- This results in a vertical sag curve.
- Statement 2:



- $|G_1| < G_2|$
- This results in a vertical crest curve
- Statement 3:



- $|G_1| < G_2|$
- This results in a vertical crest curve.
- Since statements 1 and 3 are correct and statement 2 is wrong, option A is the correct answer.

23. The direct and reversed zenith angles observed by a theodolite are  $56^{\circ}00' 00''$  and  $303^{\circ}00' 00''$ , respectively. What is the vertical collimation correction?

A  $+1^{\circ}00' 00''$

B  $-1^{\circ}00' 00''$

C  $-0^{\circ}30' 00''$

D  $+0^{\circ}30' 00''$

### Solution

- Vertical collimation error: A vertical collimation error exists on a total station if the  $0^{\circ}$  to  $180^{\circ}$  line in the vertical circle does not coincide with its vertical axis.
- Direct zenith angle =  $56^{\circ}00' 00''$
- Reversed zenith angle =  $303^{\circ}00' 00''$
- Vertical collimation Error =  $\frac{360^{\circ} - \text{sum of direct and reversed zenith angle}}{2}$
- Error =  $\frac{360^{\circ} - (56^{\circ}00'00'' + 303^{\circ}00'00'')}{2} = 0.5^{\circ} = 30'$
- Thus correct answer is option D =  $+0^{\circ}30' 00''$

24. A student is scanning his 10 inch  $\times$  10 inch certificate at 600 dots per inch (dpi) to convert it to raster. What is the percentage reduction in number of pixels if the same certificate is scanned at 300 dpi?

A 62

B 88

C 75

D 50

### Solution

- Number of dots as per 600 dpi =  $(10 \times 600)^2 = 36000000$
- Number of dots as per 300 dpi =  $(10 \times 300)^2 = 9000000$
- Percentage reduction in number of pixels =  $\frac{36000000 - 9000000}{36000000} \times 100$   
 $= \frac{27}{36} \times 100 = 75\%$

25. The following function is defined over the interval  $[-L, L]$ :

- $f(x) = px^4 + qx^5$

If it is expressed as a Fourier series,

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left[ a_n \sin\left(\frac{\pi x}{L}\right) + b_n \cos\left(\frac{\pi x}{L}\right) \right]$$

which options amongst the following are true?

A  $a_n, n = 1, 2, \dots, \infty$  depend on p

B  $a_n, n = 1, 2, \dots, \infty$  depend on q

C  $b_n, n = 1, 2, \dots, \infty$  depend on p

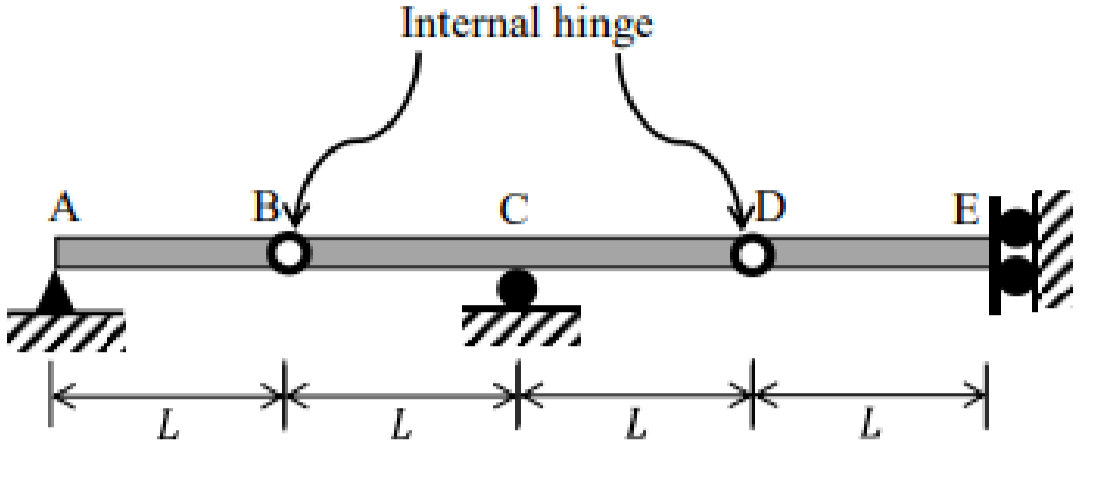
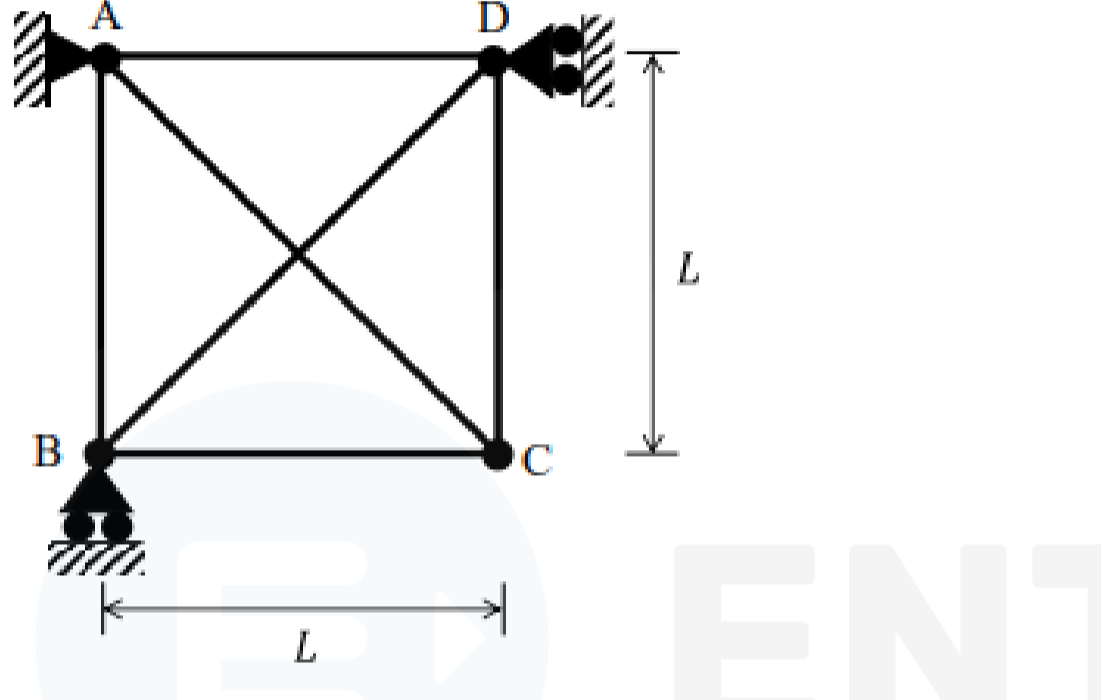
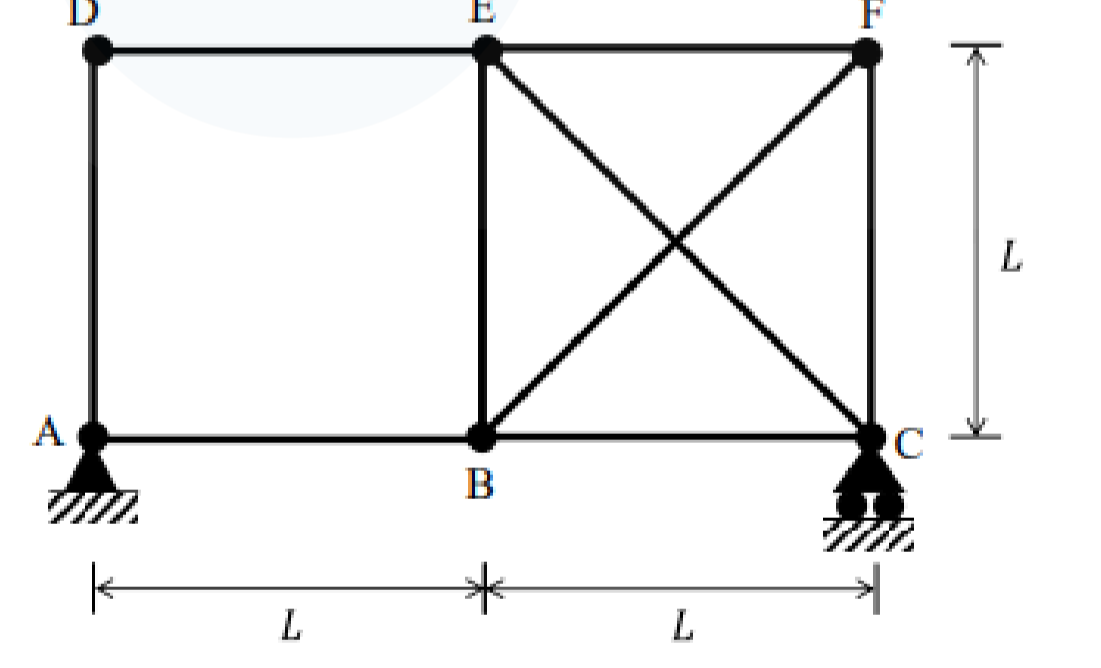
D  $b_n, n = 1, 2, \dots, \infty$  depend on p

### Solution

- $f(x) = px^4 + qx^5$
- $b_n = \frac{1}{l} \int_{-l}^l f(x) \cos\left(\frac{n\pi x}{l}\right) dx$
- $b_n = \frac{1}{l} \int_{-l}^l (px^4 + qx^5) \cos\left(\frac{n\pi x}{l}\right) dx$
- $b_n = \frac{1}{l} \int_{-l}^l (px^4) \cos\left(\frac{n\pi x}{l}\right) dx + 0$
- $\because qx^5 \cos\left(\frac{n\pi x}{l}\right)$  is an odd function.
- So,  $b_n$  depends on p.
- $a_n = \frac{1}{l} \int_{-l}^l f(x) \sin\left(\frac{n\pi x}{l}\right) dx$

- $a_n = \frac{1}{l} \int_{-l}^l (px^4 + qx^5) \sin\left(\frac{n\pi x}{l}\right) dx$
- $a_n = 0 + \frac{1}{l} \int_{-l}^l (qx^5) \sin\left(\frac{n\pi x}{l}\right) dx$
- $\because px^4 \sin\frac{n\pi x}{l}$  is an odd function.
- So,  $a_n$  depends on  $q$ .
  
- Since  $b_n$  depends on  $p$  and  $a_n$  depends on  $q$ , both options B and C are correct.

26. Consider the following three structures:

 <p>Internal hinge</p>	<p>Structure I: Beam with hinge support at A, roller at C, guided roller at E, and internal hinges at B and D</p>
	<p>Structure II: Pin-jointed truss, with hinge support at A, and rollers at B and D</p>
	<p>Structure III: Pin-jointed truss, with hinge support at A and roller at C</p>

Which of the following statements is/are TRUE?

**A** Structure I is unstable



**B** Structure II is unstable

**C** Structure III is unstable

**D** All three structures are stable

### Solution

- **Structure I** → Unstable
- Internal hinge at D can cause rigid body rotation.
- **Structure II** → **Unstable**
- Since all reactions are concurrent at point A, rigid body rotation can take place.
- **Structure III** → **Unstable**
- Even if  $m = 2j - 3$  criteria is satisfied, there is an improper arrangement of members in 1st panel.
- Hence, shear in 1st panel is not resisted by any member. So, it is unstable.
- Thus options A, B and C are correct

27. Identify the waterborne diseases caused by viral pathogens:

A Acute anterior poliomyelitis

B Cholera

C Infectious hepatitis

**D Both A and C**

### Solution

- Waterborne diseases are illnesses caused by microscopic organisms, like viruses and bacteria, that are ingested through contaminated water or by coming in contact with feces.
- Waterborne diseases caused by Viral pathogens are:
  - Acute anterior poliomyelitis
  - Infectious hepatitis
- Waterborne diseases caused by Bacterial pathogens are:
  - Cholera
  - Typhoid fever

28. Which of the following statements is/are TRUE for the Refuse-Derived Fuel (RDF) in the context of Municipal Solid Waste (MSW) management?

A

Higher Heating Value (HHV) of the unprocessed MSW is higher than the HHV of RDF processed from the same MSW

B

RDF can be made in the powdered form

C

Inorganic fraction of MSW is mostly converted to RDF

D

RDF cannot be used in conjunction with oil

### Solution

- **(A) is false.**
- The Higher Heating Value (HHV) of Refuse-Derived Fuel (RDF) processed from Municipal Solid Waste (MSW) is typically higher than the HHV of unprocessed MSW.
- This is because RDF processing removes non-combustible materials from MSW and concentrates the combustible fraction.
- **(B) is true.**
- RDF can be produced in various forms including pellets, fluff, and powdered form.
- The form of RDF depends on the specific process and intended end use.

- **(C) is false.**
- We can also said that its partially true.
- RDF processing typically removes a significant portion of the inorganic fraction of MSW, but not all of it.
- The remaining inorganic fraction is typically mixed with the RDF product.
- **(D) is false.**
- RDF can be used as a fuel in a variety of combustion systems, including those that burn oil.
- In fact, RDF is often used as a supplemental fuel in cement kilns and other industrial processes that also use oil as a fuel.

29. The probabilities of occurrences of two independent events A and B are 0.5 and 0.8, respectively. What is the probability of occurrence of at least A or B (rounded off to one decimal place)?

0.9

Answer range : 0.85-0.95

### Solution

- As it is independent events:
- P (at least one of A or B)
- $P(A \cup B) = p(A) + p(B) - p(A \cap B)$
- $P(A \cup B) = p(A) + p(B) - p(A) p(B)$
- $P(A \cup B) = 0.5 + 0.8 - 0.5 \times 0.8$
- $P(A \cup B) = 0.9$

30. In the differential equation  $\frac{dy}{dx} + \alpha xy = 0$ ,  $\alpha$  is a positive constant. If  $y = 1.0$  at  $x = 0.0$ , and  $y = 0.8$  at  $x = 1.0$ , the value of  $\alpha$  is: (rounded off to three decimal places).

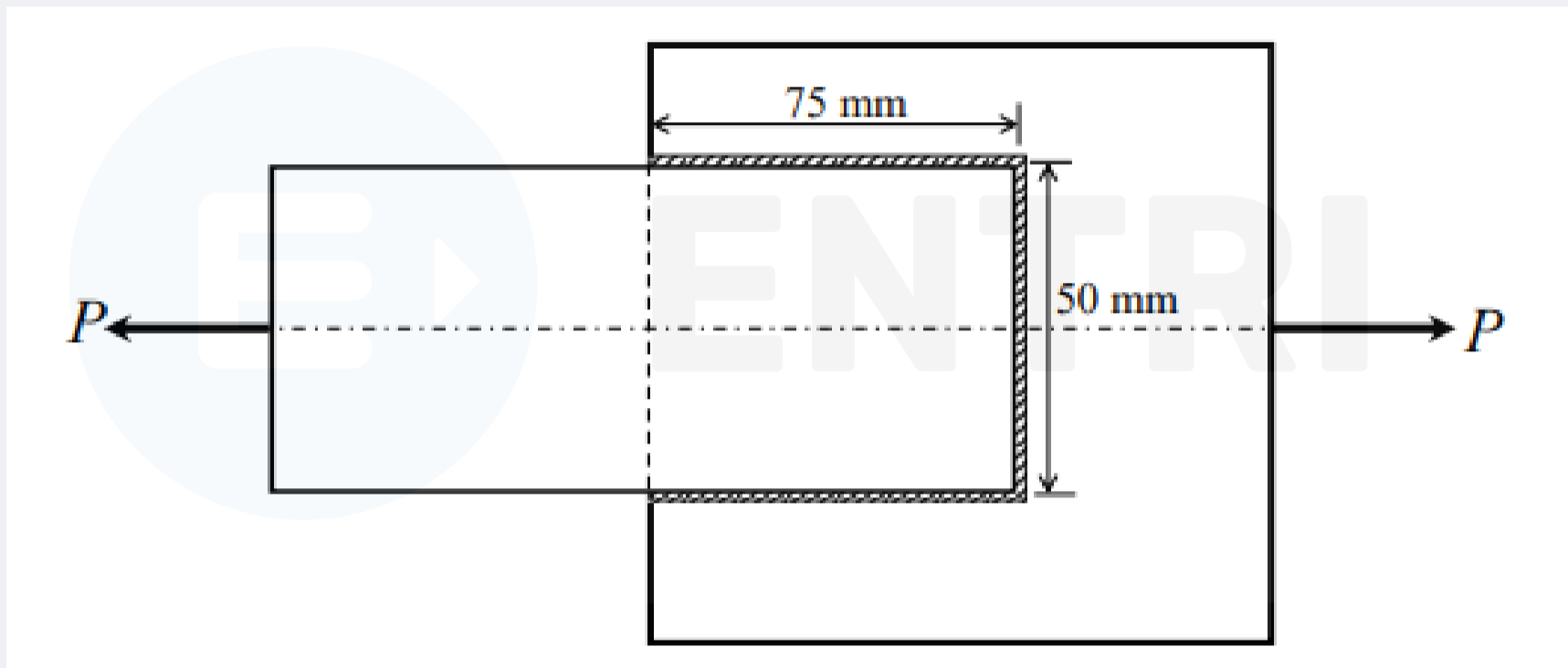
0.446

Answer range : 0.443-0.449

### Solution

- $\frac{dy}{dx} + \alpha xy = 0$
- $\frac{dy}{dx} = -\alpha xy$
- $\frac{dy}{y} = -\alpha x dx$
- $\int \frac{dy}{y} = -\alpha \int x dx + C$
- $\ln y = -\frac{\alpha x^2}{2} + C$
- Put  $y = 1$  at  $x = 0 \Rightarrow$
- $\ln 1 = 0 + C$
- $C = 0$
- $\ln y = -\frac{\alpha x^2}{2}$
- Put  $y = 0.8$  at  $x = 1 \Rightarrow$
- $\ln (0.8) = -\frac{\alpha}{2}$
- $\alpha = -2 \ln (0.8)$
- $\alpha = 0.4462$
- i.e the value of  $\alpha$  is 0.446

31. Consider the fillet-welded lap joint shown in the figure (not to scale). The length of the weld shown is the effective length. The welded surfaces meet at right angle. The weld size is 8 mm, and the permissible stress in the weld is 120 MPa. What is the safe load  $P$  (in kN, rounded off to one decimal place) that can be transmitted by this welded joint?



134.4

Answer range : 133.0-136.0

### Solution

- Given,
- Size of weld =  $S = 8$  mm
- Permissible stress = 120 MPa
- Effective throat thickness =  $0.7 S = 0.7 \times 8 = 5.6$  mm
- Effective length =  $l_{eff.} = 75 + 50 + 75 = 200$  mm
- Safe load =  $P = \text{Permissible stress} \times \text{Effective throat thickness} \times \text{Effective length of weld}$
- $P = 120 \times 5.6 \times 200 = 134400$  N = 134.4 kN

32. A drained direct shear test was carried out on a sandy soil. Under a normal stress of 50 kPa, the test specimen failed at a shear stress of 35 kPa. The angle of internal friction of the sample is: (in degrees, round off to the nearest integer)

35

Answer range : 34.0-36.0

### Solution

- Given,  $\sigma_n = 50$  kPa
- $\tau_f = 35$  kPa
- For sandy soil,  $C = 0$
- Shear stress,  $\tau_f = C + \sigma_n \tan \phi_f$
- $35 = 50 \tan \phi_f$
- $\tan \phi_f = \frac{35}{50}$
- The angle of internal friction of the sample,  $\tan \phi_f = 35^\circ$



33. A canal supplies water to an area growing wheat over 100 hectares. The duration between the first and last watering is 120 days, and the total depth of water required by the crop is 35 cm. The most intense watering is required over a period of 30 days and requires a total depth of water equal to 12 cm. Assuming precipitation to be negligible and neglecting all losses, the minimum discharge (in  $m^3/s$ , rounded off to three decimal places) in the canal to satisfy the crop requirement is:

0.046

Answer range : 0.043-0.049

### Solution

- Area =  $A = 100$  Ha
- Base period =  $B = 120$  days
- Delta =  $\Delta = 35$  cm
- More intense watering is required for = 30 days and the depth of water required (Delta for 30 days) = 12 cm
- So, 12 cm is required in the first 30 days.
- Hence,  $(35 - 12) = 23$  cm will be required for  $(120 - 30) = 90$  days (next 90 days)
- So, Discharge for the first 30 days =  $\frac{A}{\Delta} = \frac{A}{864 \frac{B}{\Delta}} = \frac{100}{864 \times \frac{30}{12}} = 0.046$  cumecs
- Discharge for the next 90 days =  $\frac{A}{\Delta} = \frac{100}{864 \times \frac{90}{23}} = 0.029$  cumecs
- Hence, minimum design discharge required = Max.  $(0.046, 0.029) = 0.046$  cumecs

34. The ordinates of a one-hour unit hydrograph for a catchment are given below:

$t$ (hour)	0	1	2	3	4	5	6	7
$Q$ ( $m^3/s$ )	0	9	21	18	12	5	2	0

Using the principle of superposition, a  $D$ -hour unit hydrograph for the catchment was derived from this one-hour unit hydrograph. The ordinates of the  $D$ -hour unit hydrograph were obtained as  $3 m^3/s$  at  $t = 1$  hour and  $10 m^3/s$  at  $t = 2$  hour. The value of  $D$  (in integer) is:

3

Answer range : 2.5-3.5

### Solution

Time	1hr-UH	1hr UH lagged by 1 hr	1hr-UH lagged by 1 more hr	3 hr DRH	Ordinate of 3hr UH = $\frac{\text{Ordinate of 3hr DRH}}{3cm}$
(1)	(2)	(3)	(4)	(5)	(6)
0	0	-	-	0	0
1	9	0	-	9	3
2	21	9	0	30	10
3	18	21	9	48	16
4	12	18	21	51	17
5	5	12	18	35	11.67
6	2	5	12	19	6.33
7	0	2	5	7	2.33
		0	2	2	0.67
			0	0	0

- Firstly lag the UH by 1 hr.
- By lagging 1 hr, We obtained a 2 hr UH of ordinate  $4.5 m^3/s$  at 1 hr and  $15 m^3/s$  at 2 hr.
- So, further lag the UH by 1 more hr we obtained a 3 hr, UH of ordinates  $3 m^3/s$  at 1 hr and  $10 m^3/s$  at 2 hr.

- Therefore  $D = 3 \text{ hr}$

35. For a horizontal curve, the radius of a circular curve is obtained as 300 m with the design speed as 15 m/s. If the allowable jerk is  $0.75 \text{ m/s}^3$ , what is the minimum length (in m, in integer) of the transition curve?

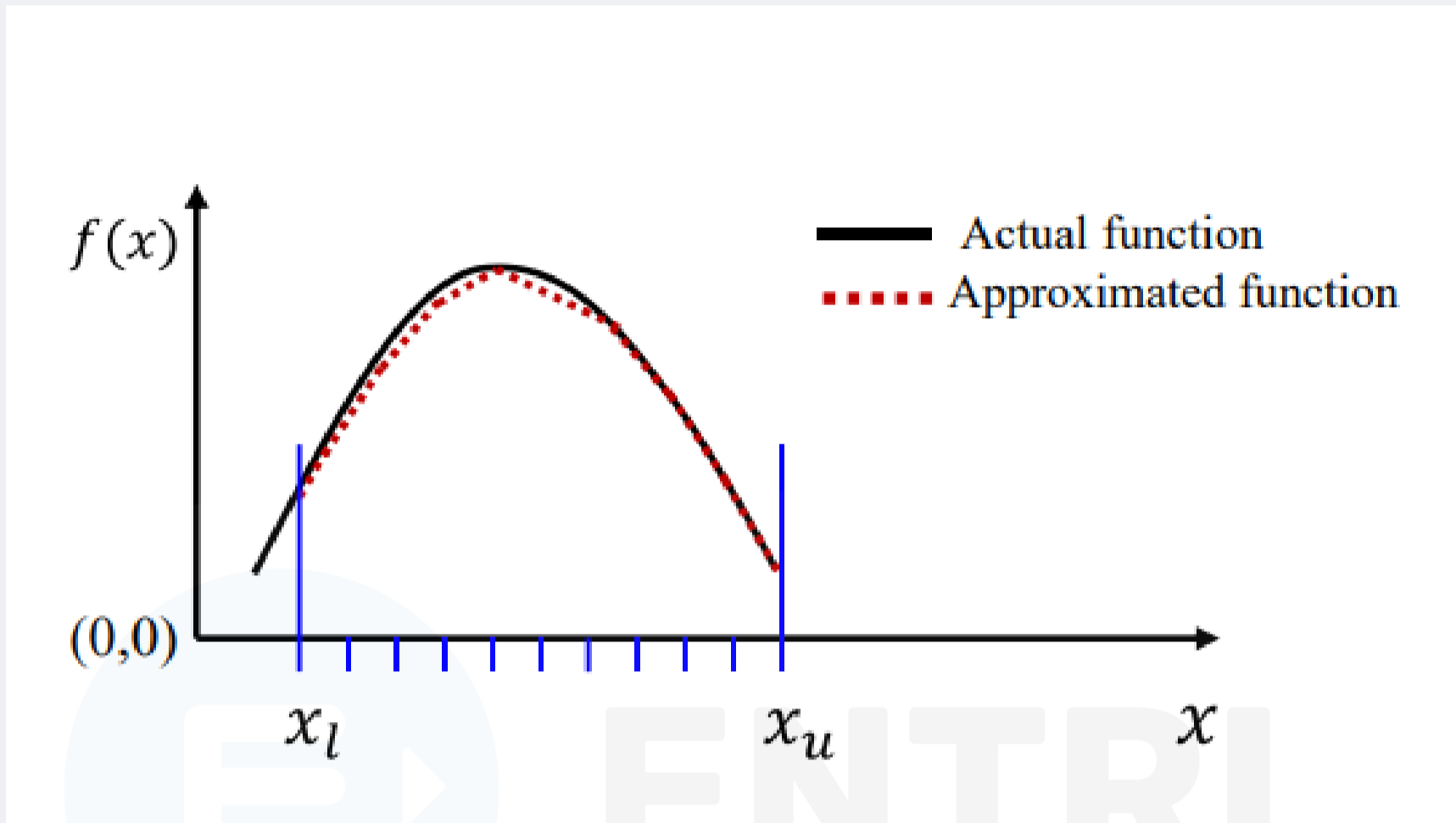
15

Answer range : 14.5-15.5

### Solution

- Radius of circular curve,  $R = 300 \text{ m}$
- $V_d = 15 \text{ m/s}$
- Allowable jerk (C) =  $0.75 \text{ 3/sec}^3$
- $L_{min} = \frac{m^3}{CR}$
- $L_{min} = \frac{15^3}{0.75 \times 300} = 15 \text{ m}$

36. A function  $f(x)$ , that is smooth and convex-shaped between interval  $(x_l, x_u)$  is shown in the figure. This function is observed at odd number of regularly spaced points. If the area under the function is computed numerically, then:



**A**

The numerical value of the area obtained using the trapezoidal rule will be less than the actual

**B**

The numerical value of the area obtained using the trapezoidal rule will be more than the actual

**C**

The numerical value of the area obtained using the trapezoidal rule will be exactly equal to the actual

**D**

With the given details, the numerical value of area cannot be obtained using trapezoidal rule

### Solution

- Trapezoidal Rule is a rule that evaluates the area under the curves by dividing the total area into smaller trapezoids rather than using rectangles.
- This integration works by approximating the region under the graph of a function as a trapezoid, and it calculates the area.
- This rule takes the average of the left and the right sum.
- Approximated function has under estimation so numerical value of the area obtained using trapezoidal rule will be less than the actual.
- The Trapezoidal Rule does not give accurate value as Simpson's Rule when the underlying function is smooth.
- Both Simpson's Rule and Trapezoidal Rule give the approximation value, but Simpson's Rule results in even more accurate approximation value of the integrals.
- Thus option A is correct.

37. Consider a doubly reinforced RCC beam with the option of using either Fe250 plain bars or Fe500 deformed bars in the compression zone. The modulus of elasticity of steel is  $2 \times 10^5 \text{ N/mm}^2$ . As per IS456:2000, in which type(s) of the bars, the stress in the compression steel ( $f_{sc}$ ) can reach the design strength ( $0.87 f_y$ ) at the limit state of collapse?

**A** Fe250 plain bars only

**B** Fe500 deformed bars only

**C** Both Fe250 plain bars and Fe500 deformed bars

**D** Neither Fe250 plain bars nor Fe500 deformed bars

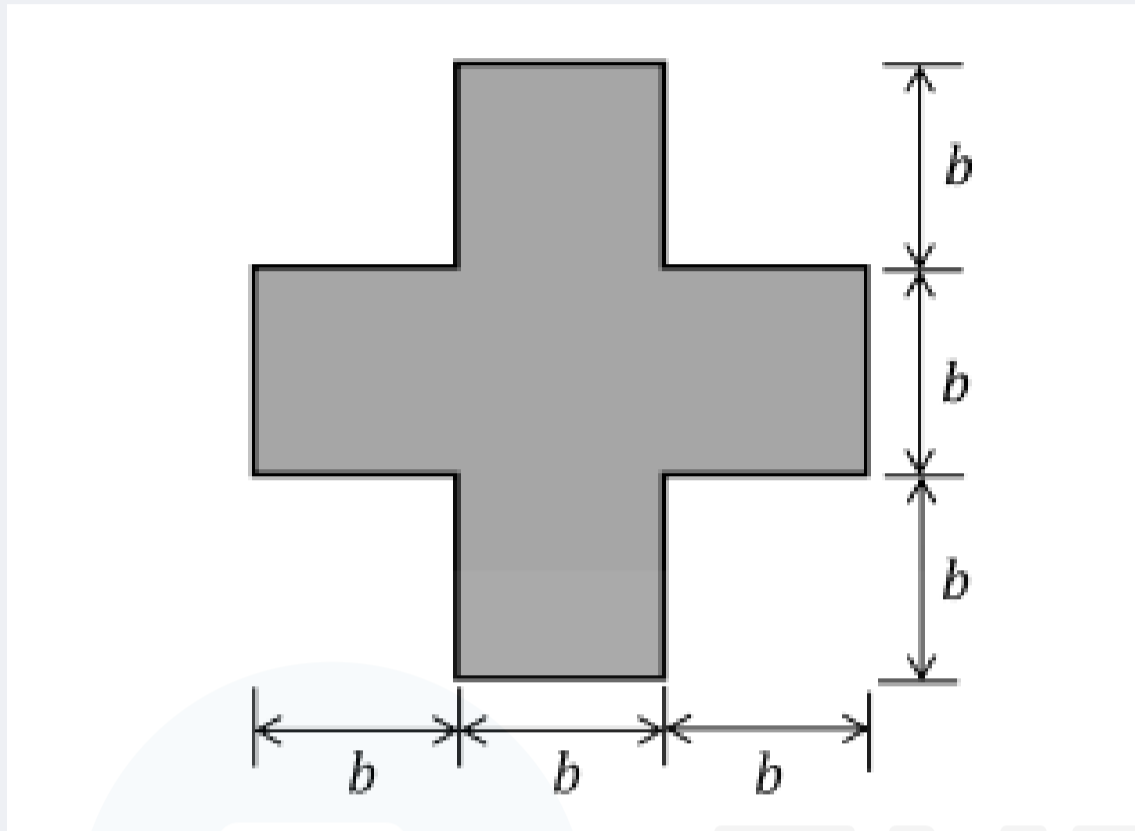
### Solution

- In a doubly reinforced beam, at the limit state of collapse, the strain in the extreme compression fibre will be 0.0035.
- As the compression reinforcement will be below this, the strain in it will be always less than 0.0035.
- For Fe250, the yield strain =  $\frac{0.87 \times 250}{2 \times 10^5} = 0.0010875$
- For Fe500, the yield strain =  $0.002 + \frac{0.87 f_y}{E_s} = 0.002 + \frac{0.87 \times 500}{2 \times 10^5} = 0.004175 > 0.0010875$

- So, it is possible that Fe250 can reach upto its yield strain and hence can reach the design strength of  $(0.87 f_y)$ .
- Hence option A is correct.



38. Consider the horizontal axis passing through the centroid of the steel beam cross - section shown in the figure. What is the shape factor (rounded off to one decimal place) for the cross-section?



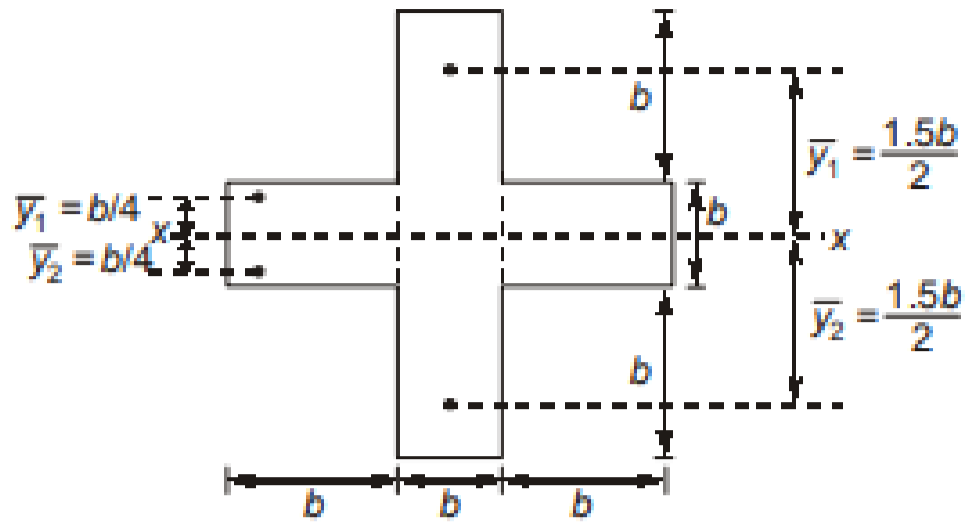
A 1.5

**B 1.7**

C 1.3

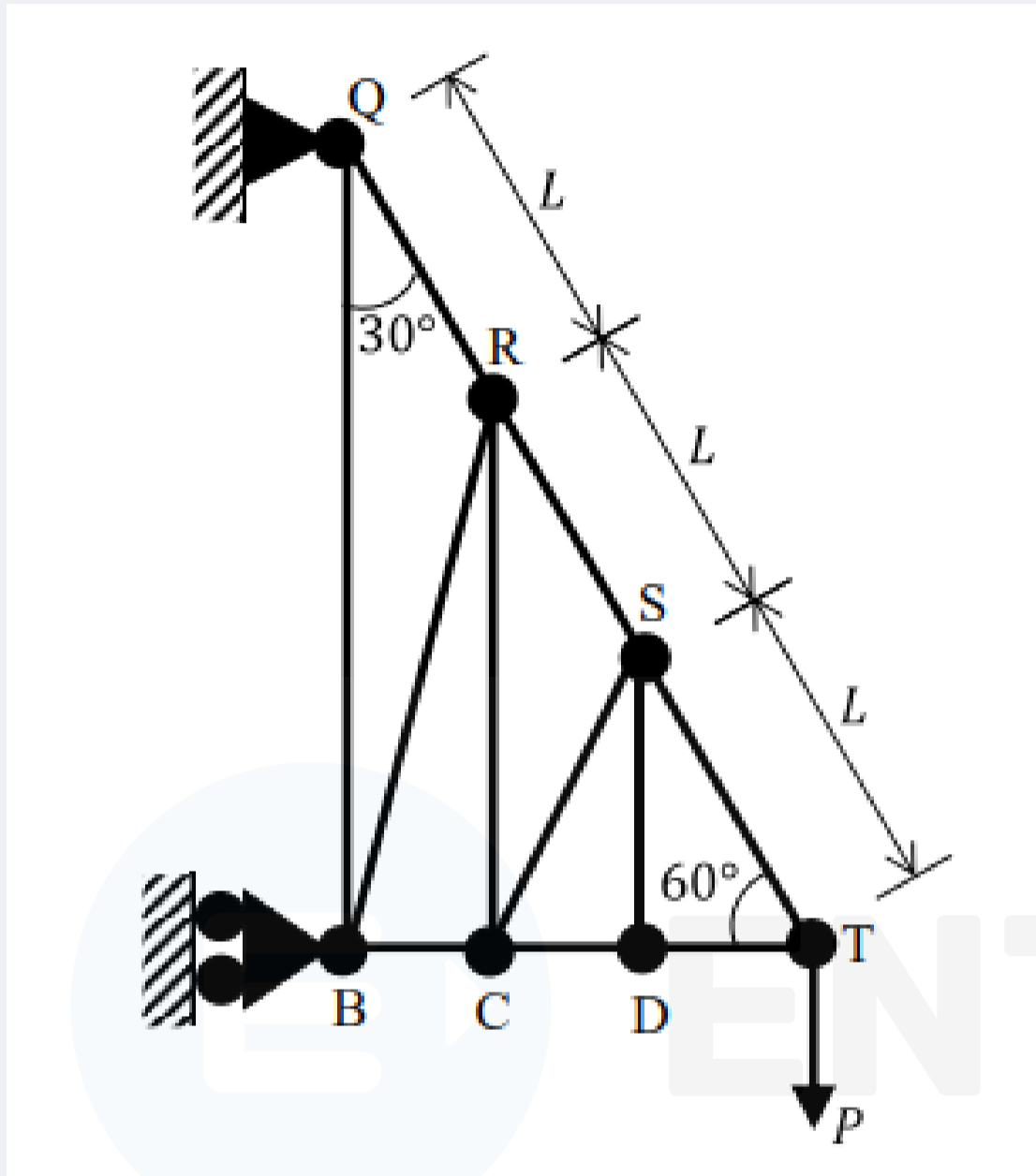
D 2.0

**Solution**



- Plastic Section modulus:
- $Z_p = \frac{A}{2} [\bar{y}_1 + \bar{y}_2]$
- $Z_p = (b \times 1.5b) \left[ \frac{1.5b}{2} + \frac{1.5b}{2} \right] + \left[ 2b \times \frac{b}{2} \right] \times \left[ \frac{b}{4} + \frac{b}{4} \right] = 2.75 b^3$
- Elastic section modulus,  $Z = \frac{I_{xx}}{y} = \frac{\left[ \frac{b \times (3b)^3}{12} + \frac{2b \times b^3}{12} \right]}{1.5b} =$
- $Z = \frac{29}{18} b^3$
- Shape factor, S.F. =  $\frac{Z_p}{Z} = \frac{2.75b^3}{\frac{29}{18} b^3} = 1.7$

39. Consider the pin-jointed truss shown in the figure (not to scale). All members have the same axial rigidity,  $AE$ . Members  $QR$ ,  $RS$ , and  $ST$  have the same length  $L$ . Angles  $QBT$ ,  $RCT$ ,  $SDT$  are all  $90^\circ$ . Angles  $BQT$ ,  $CRT$ ,  $DST$  are all  $30^\circ$ . The joint  $T$  carries a vertical load  $P$ . The vertical deflection of joint  $T$  is  $k \frac{PL}{AE}$ . What is the value of  $k$ ?



A 1.5

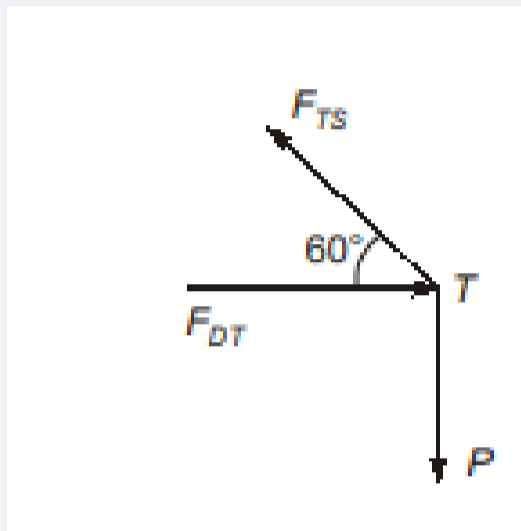
B 4.5

C 3.0

**D** 9.0

## Solution

- At joint T,



- $\Sigma F_y = 0 \Rightarrow$
- $F_{TS} \sin 60^\circ = P$
- $F_{TS} = \frac{2P}{\sqrt{3}} (T)$
- $\Sigma F_x = 0 \Rightarrow$
- $F_{TS} \cos 60^\circ = F_{DT}$
- $F_{DT} = \frac{2P}{\sqrt{3}} \left(\frac{1}{2}\right) = \frac{P}{\sqrt{3}} (T)$
- $F_{TS} = F_{SR} = F_{RQ} = \frac{2P}{\sqrt{3}} (T)$
- $F_{DT} = F_{CD} = F_{BL} = \frac{P}{\sqrt{3}} (T)$
- Similarly,
- $K_{ST} = K_{SR} = K_{RQ} = \frac{2}{\sqrt{3}} (T)$
- $K_{DT} = K_{CD} = K_{BC} = \frac{1}{\sqrt{3}} (T)$
- Deflection,  $\delta_{VT} = \Sigma \frac{PkL}{AE}$
- $\delta_{VT} = \frac{\left(\frac{2P}{\sqrt{3}}\right)\left(\frac{2}{\sqrt{3}}\right)L \times 3}{AE} + \frac{\left(\frac{P}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)\frac{L}{2} \times 3}{AE}$
- $\delta_{VT} = \frac{4PL}{AE} + \frac{PL}{AE} = \frac{4.5PL}{AE}$

- $\therefore k = 4.5$

40. With reference to the compaction test conducted on soils, which of the following is **INCORRECT**?

**A**

Peak point of the compaction curve gives the maximum dry unit weight and optimum moisture content

**B**

With increase in the compaction effort, the maximum dry unit weight increases

**C**

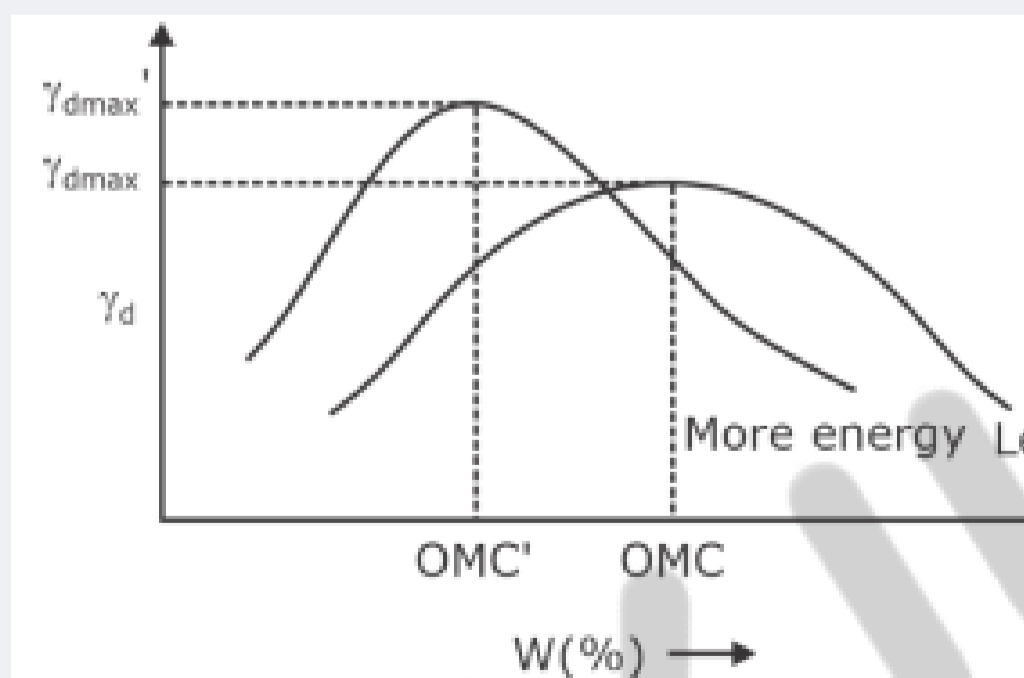
With increase in the compaction effort, the optimum moisture content decreases

**D**

**With increase in the compaction effort, the optimum moisture content decreases**

### Solution

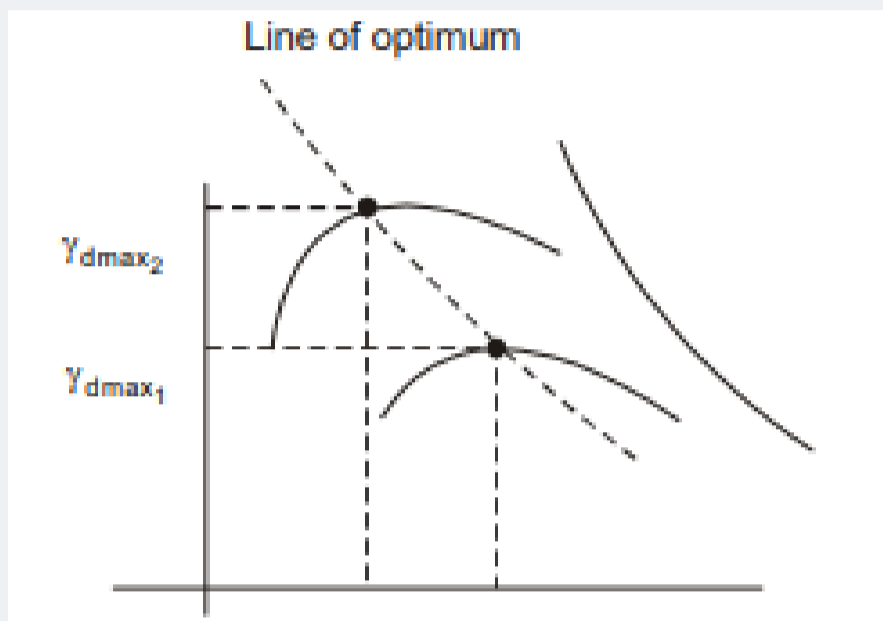
- First three points are correct.



The line with max values is

more energy line and other one is less energy line.

- Peak point of the compaction curve gives the maximum dry unit weight and optimum moisture content.
- With increase in the compaction effort, the maximum dry unit weight increases.
- With increase in the compaction effort, the optimum moisture content decreases.



- Compaction curve do not cross the zero-air-voids line.
- Thus option D is the correct answer.

41. Consider that a force  $P$  is acting on the surface of a half-space (Boussinesq's problem). The expression for the vertical stress ( $\sigma_z$ ) at any point  $(r, z)$ , within the half-space is given as,

$$\sigma_z = \frac{3P}{2\pi} \frac{z^3}{(r^2+z^2)^{\frac{5}{2}}}$$

where,  $r$  is the radial distance, and  $z$  is the depth with downward direction taken as positive. At any given  $r$ , there is a variation of ( $\sigma_z$ ) along  $z$ , and at a specific  $z$ , the value of ( $\sigma_z$ ) will be maximum. What is the locus of the maximum ( $\sigma_z$ )?

**A**  $z^2 = \frac{3}{2}r^2$

**B**  $z^3 = \frac{3}{2}r^2$

**C**  $z^2 = \frac{5}{2}r^2$

**D**  $z^3 = \frac{5}{2}r^2$

### Solution

- The expression for the vertical stress ( $\sigma_z$ ) at any point  $(r, z)$  with the half-space is given as:

- $\sigma_z = \frac{3P}{2\pi} \frac{z^3}{(r^2+z^2)^{\frac{5}{2}}}$

- For  $\sigma_z$  to be maximum:

- $\frac{d\sigma_z}{dz} = 0 \Rightarrow$



$$\frac{3P}{2\pi} \left[ \frac{(r^2+z^2)^{\frac{5}{2}} 3z^2 - z^3 \frac{5}{2} (r^2+z^2)^{\frac{3}{2}} 2z}{(r^2+z^2)^5} \right] = 0$$

- $3r^2 + 3z^2 - 5z^2 = 0$

- $z^2 = \frac{3}{2}r^2$

42. A square footing of size  $2.5 \text{ m} \times 2.5 \text{ m}$  is placed  $1.0 \text{ m}$  below the ground surface on a cohesionless homogeneous soil stratum. Considering that the groundwater table is located at the base of the footing, the unit weights of soil above and below the groundwater table are  $18 \text{ kN/m}^3$  and  $20 \text{ kN/m}^3$ , respectively, and the bearing capacity factor  $N_q$  is  $58$ , the net ultimate bearing capacity of the soil is estimated as  $1706 \text{ kPa}$  (unit weight of water =  $10 \text{ kN/m}^3$ ).

Earlier, a plate load test was carried out with a circular plate of  $30 \text{ cm}$  diameter in the same foundation pit during a dry season, when the water table was located beyond the plate influence zone. Using Terzaghi's bearing capacity formulation, what is the ultimate bearing capacity (in kPa) of the plate?

**A** 110.16

**B** 61.20

**C** 204.00

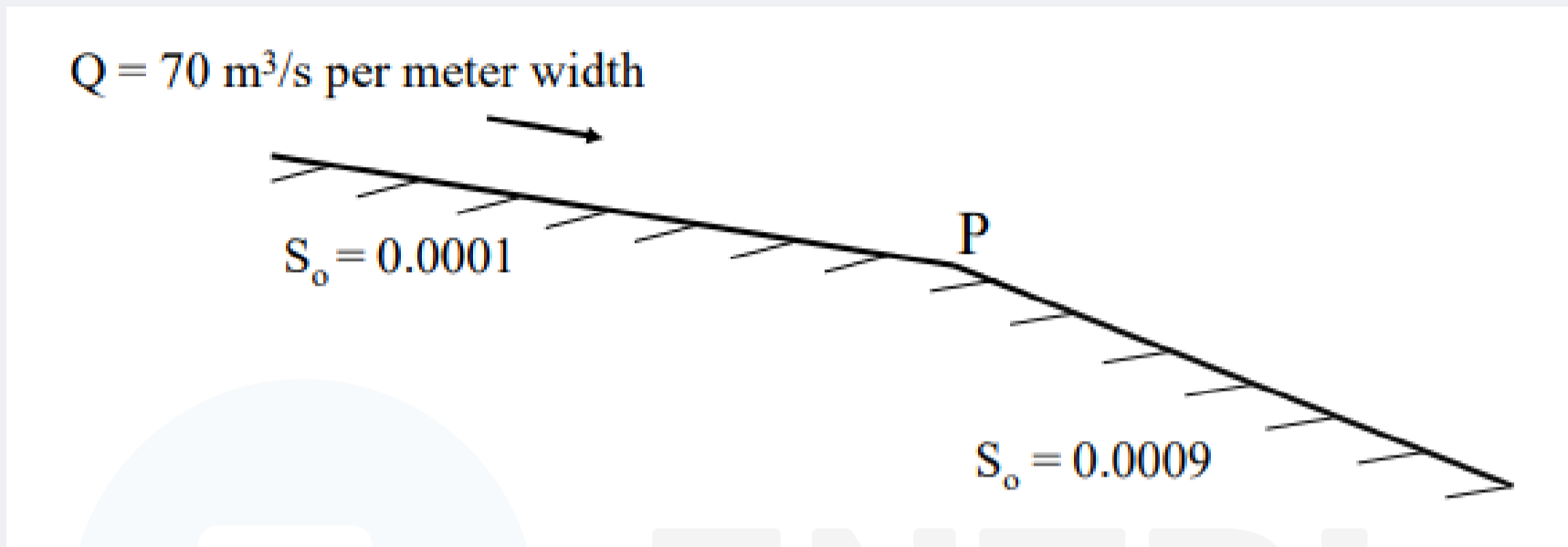
**D** 163.20

### Solution

- $\gamma_1 = 18 \text{ kN/m}^3$
- $\gamma_{sat} = 20 \text{ kN/m}^3$
- $\gamma_w = 10 \text{ kN/m}^3$
- $q_{nu} = 1706 \text{ kPa}$

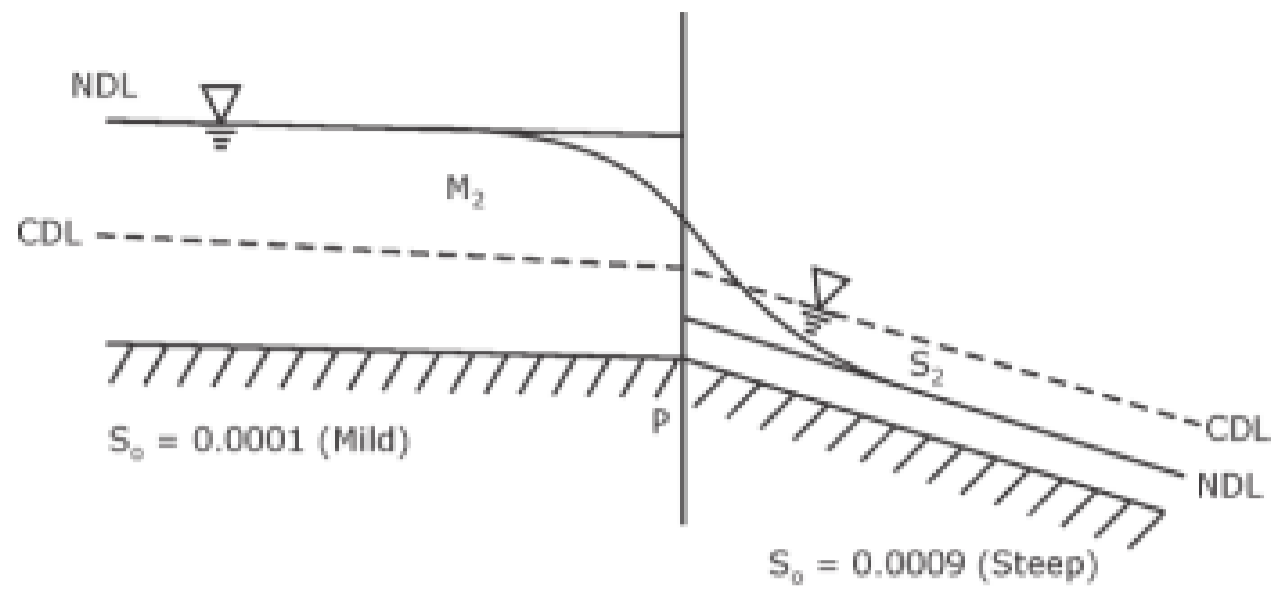
- Plate load test data:
- Dia,  $d = 30$  cm with no water table effect
- $q_{nu} = q_u - \bar{\sigma} = q_u - \gamma D_f$
- $1706 = [1.3CN_c + \gamma D_f N_q + 0.4B\gamma N_\gamma]$
- $1706 = 18 \times 1 \times 58 + 0.4 \times 2.5 \times 10 \times N_f - 18\gamma_f$
- $N_f = 68$
- For Plate: [Surcharge at the plate level is zero]
- $q_u = 1.3CN_c + \gamma D_f N_q + 0.3B\gamma N_\gamma$
- $q_u = 0 + 0.3 \times 0.3 \times 18 \times 68$
- $q_u = 110.16$  kPa

43. A very wide rectangular channel carries a discharge ( $Q$ ) of  $70 \text{ m}^3/\text{s}$  per meter width. Its bed slope changes from 0.0001 to 0.0009 at a point P, as shown in the figure (not to scale). The Manning's roughness coefficient of the channel is 0.01. What water surface profile(s) exist(s) near the point P?



- A**  $M_2$  and  $S_2$
- B**  $M_2$  only
- C**  $S_2$  only
- D**  $S_2$  and hydraulic jump

### Solution



- CDL will remain same.
- But since, NDL varies inversely with slope, the NDL in mild slope is above the CDL, but in steep slope, NDL is below the CDL.
- So, as clear from the diagram, flow profile will be  $M_2$  and  $S_2$ .

44. A jet of water having a velocity of 20 m/s strikes a series of plates fixed radially on a wheel revolving in the same direction as the jet at 15 m/s. What is the percentage efficiency of the plates? (round off to one decimal place)

**A** 37.5

**B** 66.7

**C** 50.5

**D** 88.9

### Solution

- Given water jet velocity ( $V$ ) = 20 m/s
- Wheel velocity ( $u$ ) = 15 m/s
- Efficiency,  $\eta = \frac{2u(V-u)}{V^2} \times 100$
- $\eta = \frac{2 \times 15 \times (20 - 15)}{20^2} \times 100 = 37.5 \%$

45. In the following table, identify the correct set of associations between the entries in Column-1 and Column-2.

Column-1	Column-2
P: Reverse Osmosis	I: Ponding
Q: Trickling Filter	II: Freundlich Isotherm
R: Coagulation	III: Concentration Polarization
S: Adsorption	IV: Charge Neutralization

A P-II, Q-I, S-III

B Q-III, R-II, S-IV

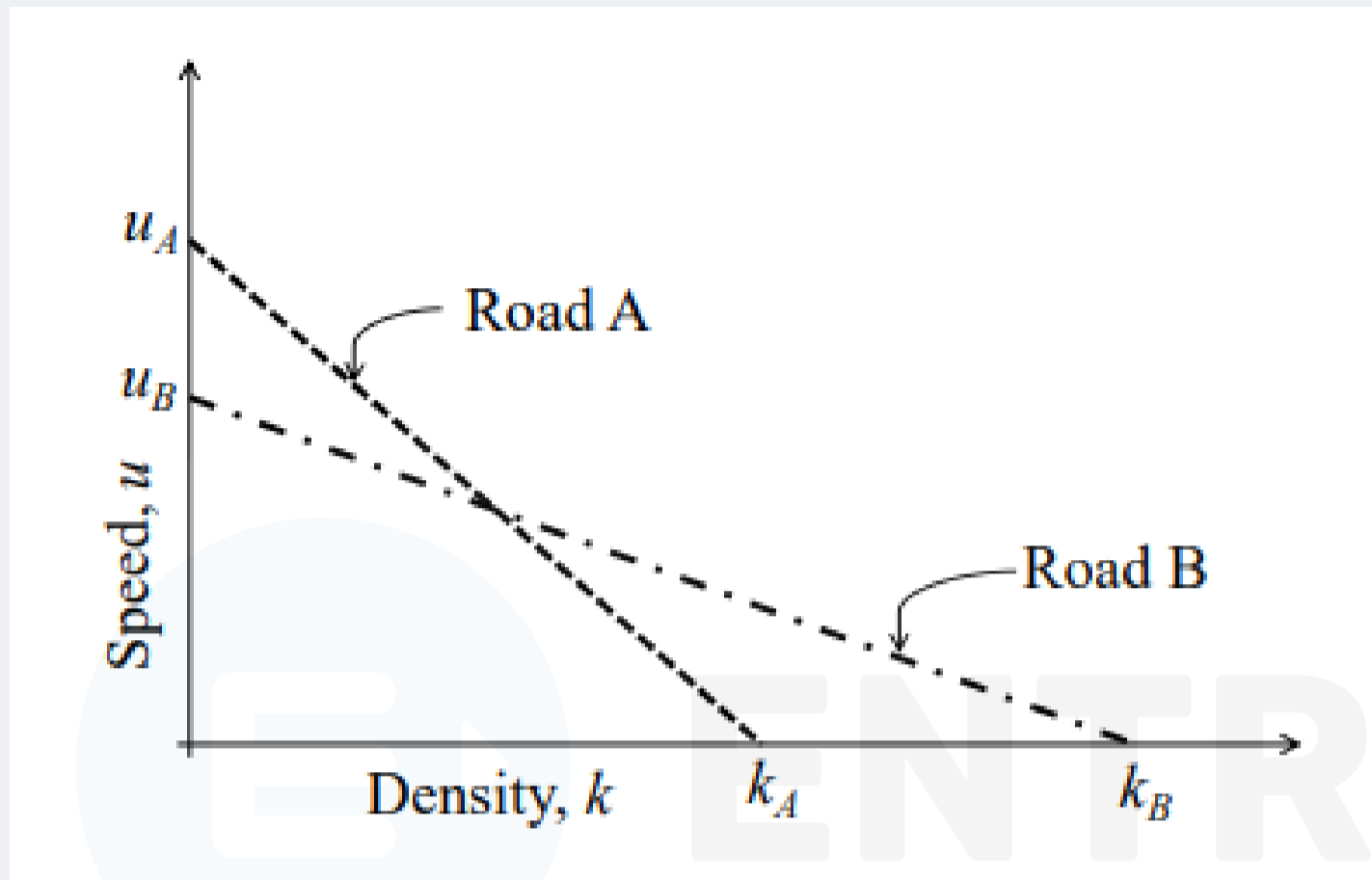
C P-IV, R-I, S-II

**D P-III, Q-I, R-IV**

### Solution

- Reverse Osmosis – Concentration Polarization
- Trickling Filter – Ponding
- Coagulation– Charge Neutralization
- Adsorption- Freundlich Isotherm
- Thus correct answer is option D

46. A plot of speed-density relationship (linear) of two roads (Road A and Road B) is shown in the figure.



If the capacity of Road A is  $C_A$  and the capacity of Road B is  $C_B$ , what is  $\frac{C_A}{C_B}$  ?

A  $\frac{K_A}{K_B}$

B  $\frac{U_A}{U_B}$

C  $\frac{K_A U_A}{K_B U_B}$

D  $\frac{K_A U_B}{K_B U_A}$

### Solution

- As per Greenshield model, we know.



- $q_{max} = \frac{V_{sf}k_j}{4}$

$$\frac{\text{Capacity } A}{\text{Capacity } B} = \frac{\frac{(k_J)_A \times (V_{SF})_A}{4}}{\frac{(k_J)_B \times (V_{SF})_B}{4}}$$

- $\frac{\text{Capacity } A}{\text{Capacity } B} = \frac{K_A \times U_A}{K_B \times U_B}$

47. For the matrix

$[A] =$

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

which of the following statements is/are TRUE?

A The eigenvalues of  $[A]^T$  are same as the eigenvalues of  $[A]$

B The eigenvalues of  $[A]^{-1}$  are the reciprocals of the eigenvalues of  $[A]$

C The eigenvectors of  $[A]^T$  are same as the eigenvalues of  $[A]$

D The eigenvectors of  $[A]^{-1}$  are same as the eigenvalues of  $[A]$

### Solution

- By standard properties:
- Eigen values of  $[A]^T$  and  $[A]$  are same.
- Eigen vectors of  $A$  and  $[A]^T$  are same.
- Eigen values of  $[A]^{-1}$  is reciprocal of Eigen value of  $[A]$
- So, statements A,B and D are true.

48. For the function  $f(x) = e^x |\sin x|$ ;  $x \in \mathbb{R}$ , which of the following statements is/are TRUE?

**A** The function is continuous at all  $x$

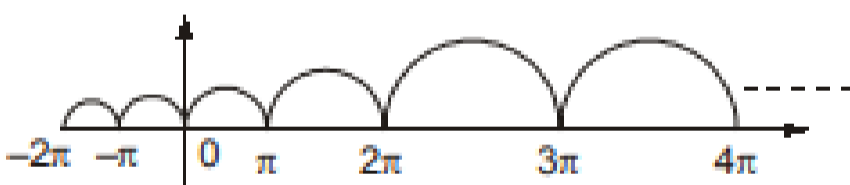
**B** The function is differentiable at all  $x$

**C** The function is periodic

**D** The function is bounded

### Solution

- Given that  $f(x) = e^x |\sin x|$

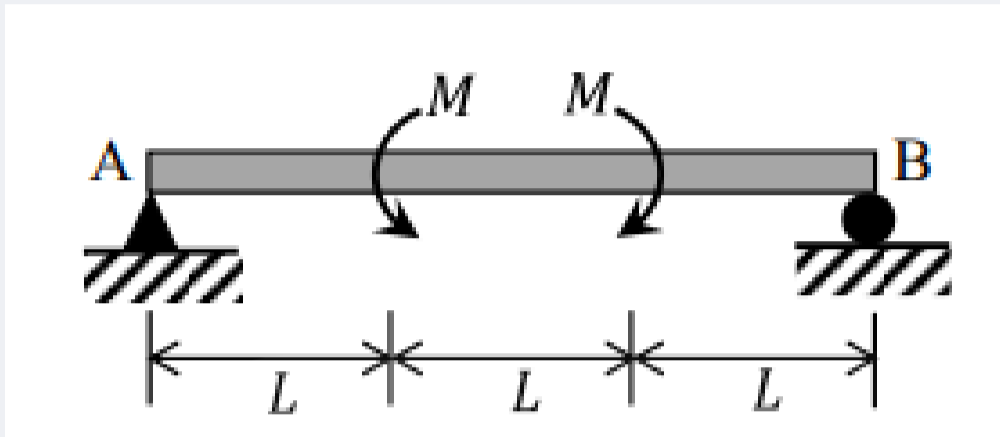


- By observing graph we can easily say given function is not periodic and not bounded.
- In the graph there are corner points so function is not differentiable
- From above graph its clear that for every  $x \in \mathbb{R}$

$$\lim_{h \rightarrow 0} f(x-h) = \lim_{h \rightarrow 0} f(x+h) = f(x)$$

- So, function is always continuous.

49. Consider the beam shown in the figure (not to scale), on a hinge support at end A and a roller support at end B. The beam has a constant flexural rigidity, and is subjected to the external moments of magnitude  $M$  at one-third spans, as shown in the figure. Which of the following statements is/are TRUE?



A Support reactions are zero

B Shear force is zero everywhere

C Bending moment is zero everywhere

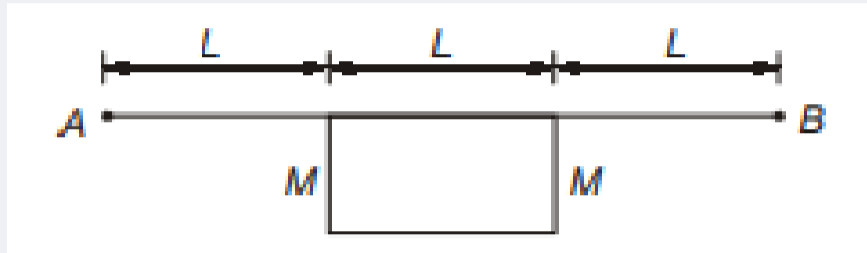
D Deflection is zero everywhere

### Solution

- $\Sigma F_y = 0$
- $V_A + V_B = 0 \rightarrow (1)$
- $\Sigma M_A = 0$

$\Rightarrow$

- $V_B \times L - M = 0$
- i.e.  $V_B = 0 \rightarrow (2)$
- Putting (2) in (1)  $\Rightarrow$
- $V_A = 0$
- $\therefore$  Shear force is zero everywhere in the beam.
- Thus reactions are also zero.
- Bending moment diagram is shown below.



- So, bending moment is not zero everywhere.
- Also deflection is not zero everywhere.
- Thus statements A and B are true.

50. Which of the following statements is/are TRUE in relation to the Maximum Mixing Depth (or Height) ' $D'_{max}$  in the atmosphere?

A  $D_{max}$  is always equal to the height of the layer of unstable air

B Ventilation coefficient depends on  $D_{max}$

C

A smaller  $D_{max}$  will have a smaller air pollution potential if other meteorological conditions remain same

D Vertical dispersion of pollutants occurs up to  $D_{max}$

### Solution

- The ventilation coefficient depends on the maximum mixing depth.
- The ventilation coefficient is a measure of the rate of air exchange between indoor and outdoor environments.
- It represents the fraction of outdoor air that is brought into a building through natural or mechanical ventilation systems.
- The more the maximum mixing depth, the more complete the air exchange between indoor and outdoor air will be.
- Vertical dispersion refers to the vertical mixing of pollutants in the atmosphere. A smaller vertical dispersion means that pollutants are less likely to spread out vertically and become diluted in the air, leading to a higher concentration of pollutants at the ground level.

- This can lead to a higher potential for air pollution and negative impacts on human health and the environment.
- Therefore, smaller vertical dispersion is generally considered an indicator of a higher air pollution potential.
- The maximum mixing depth is not always equal to half of the layer of unstable air.
- The maximum mixing depth refers to the height at which air is well mixed between the indoor and outdoor environments and is determined by several factors, including atmospheric stability, wind speed, and building design.
- The layer of unstable air is determined by the temperature and moisture profile of the atmosphere and refers to the height at which vertical air mixing occurs.
- In certain atmospheric conditions, the maximum mixing depth may be equal to half of the layer of unstable air, but this is not always.

51. Which of the following options match the test reporting conventions with the given material tests in the table?

Test reporting convention	Material test
(P) Reported as ratio	(I) Solubility of bitumen
(Q) Reported as percentage	(II) Softening point of bitumen
(R) Reported in temperature	(III) Los Angeles abrasion test
(S) Reported in length	(IV) Flash point of bitumen
	(V) Ductility of bitumen
	(VI) Specific gravity of bitumen
	(VII) Thin film oven test

A (P) - (VI); (Q) - (I); (R) - (II); (S) - (VII)

B (P) - (VI); (Q) - (III); (R) - (IV); (S) - (V)

C (P) - (VI); (Q) - (I); (R) - (II); (S) - (V)



**D** (P) - (VI); (Q) - (III); (R) - (IV); (S) - (VII)

### Solution

- Specific gravity of bitumen is the ratio of mass of given volume of substance to the mass of equal volume of water, the temperature of both being specified.
1. Solubility of bitumen: It is defined as weight of bitumen soluble in 100 ml of carbon disulphide.
  2. Softening point of bitumen: It is the temperature at which bitumen attains a particular degree of softness.
  3. Los angles abrasion test: It is reported as percentage of material finer than 1.7 mm with respect to total weight.
  4. Flash point of bitumen: It is the lowest temperature at which bitumen catch fire momentarily.
  5. Ductility of bitumen: Ductility is measured by stretching a standard briquette of bitumen. The distance in cm that the briquette can be stretched before breaking is the ductility.
  6. Specific gravity of bitumen: It is the ratio of unit weight of bitumen to unit weight of water.
  7. Thin film oven test: It measure mass change a sample as a present initial mass.
- Thus statements B and C are correct.

52. The differential equation,

$$\frac{du}{dt} = 2tu^2 = 1$$

is solved by employing a backward difference scheme within the finite difference framework. The value of  $u$  at the  $(n - 1)^{th}$  time-step, for some  $n$ , is 1.75. The corresponding time ( $t$ ) is 3.14 s. Each time step is 0.01 s long. Then, the value of  $(u_n - u_{n-1})$  is: (round off to three decimal places)

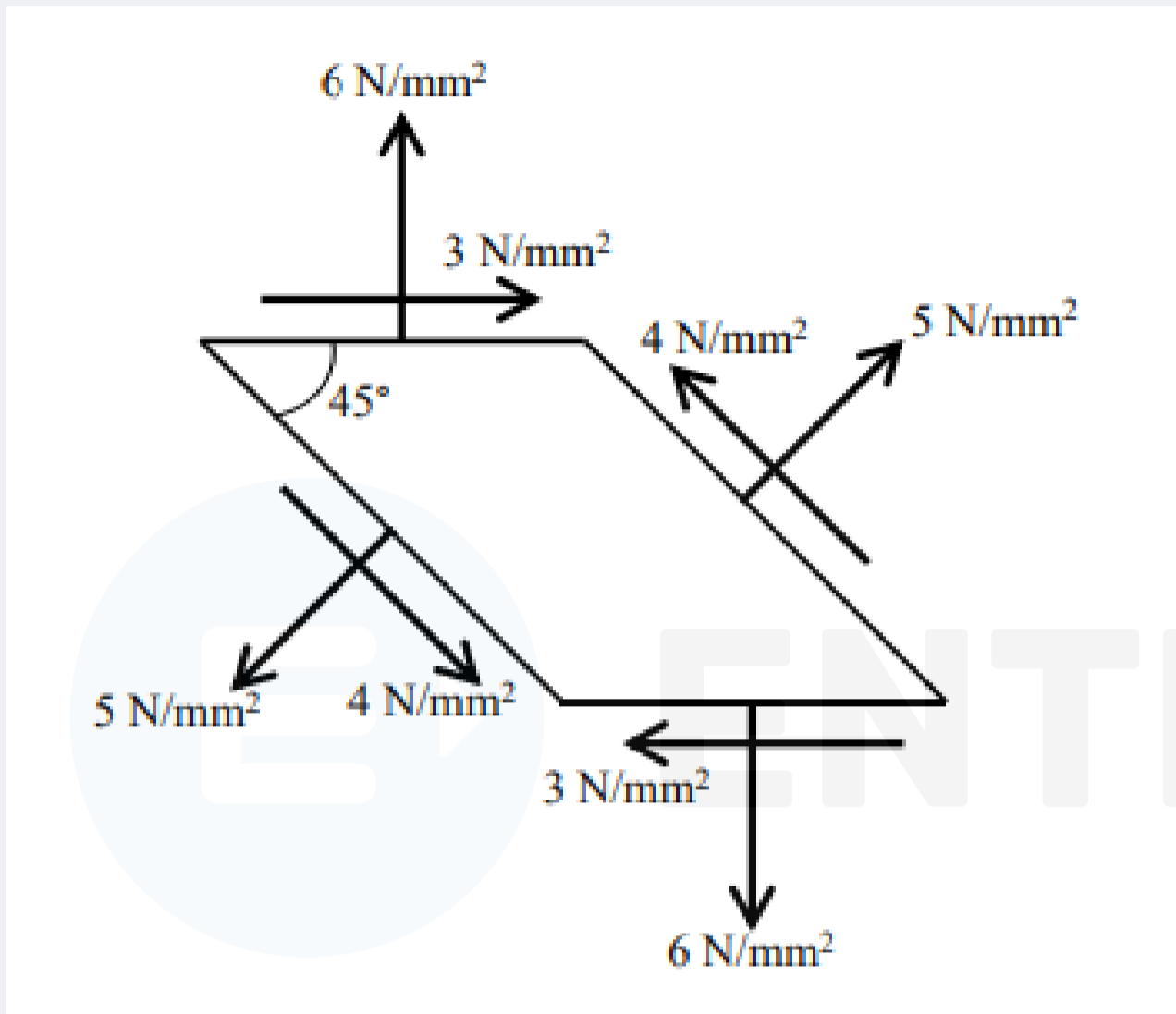
-0.151

Answer range : -0.156--0.146

### Solution

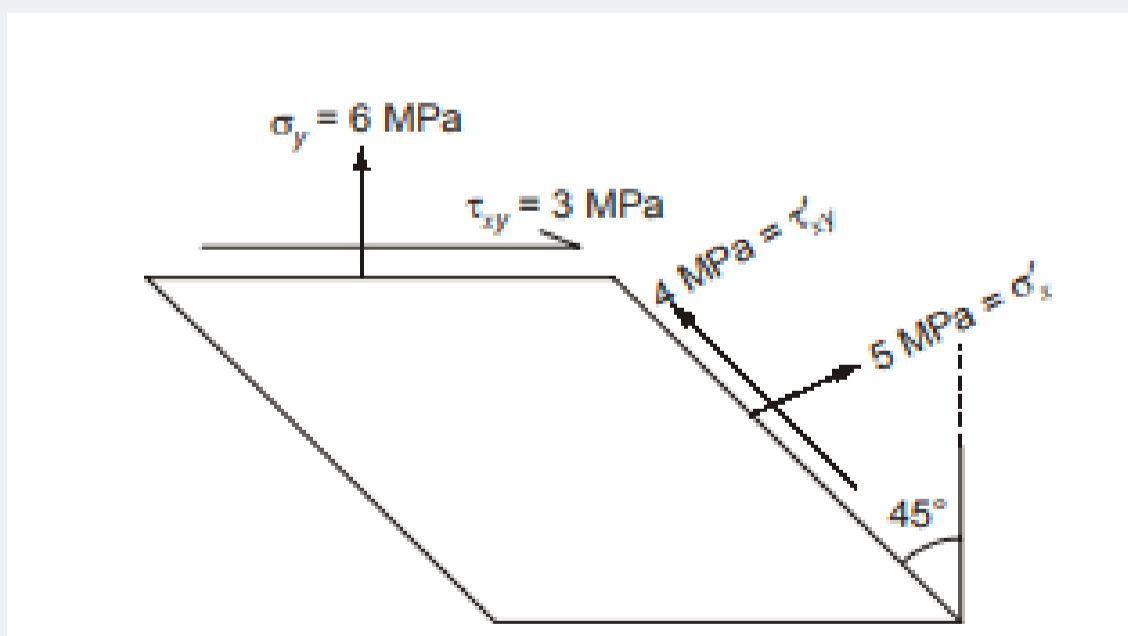
- Given DE:  $\frac{du}{dt} = f(t, u) = 1 - 2tu^2$
- By Euler's Backward method:
- $u_n = u_{n-1} + hf(t_n, u_n)$
- $u_n = u_{n-1} + h(1 - 2t_n u_n^2) \mapsto (1)$
- Given  $h = 0.01$ ,  $u_{n-1} = 1.75$  and  $t_{n-1} = 3.14$
- Substituting these values in eq. (1)  $\Rightarrow$
- $u_n = 1.75 + 0.01 [1 - 2 \times 3.15 \times u_n^2]$
- $u_n = 1.75 + 0.01 - 0.63 u_n^2$
- $0.63 u_n^2 + u_n - 1.76 = 0$
- We get  $u_n = 1.599$  and  $u_{n-1} = 1.75$
- $\therefore u_n - u_{n-1} = -0.151$

53. The infinitesimal element shown in the figure (not to scale) represents the state of stress at a point in a body. What is the magnitude of the maximum principal stress (in  $\text{N/mm}^2$ , in integer) at the point?



7

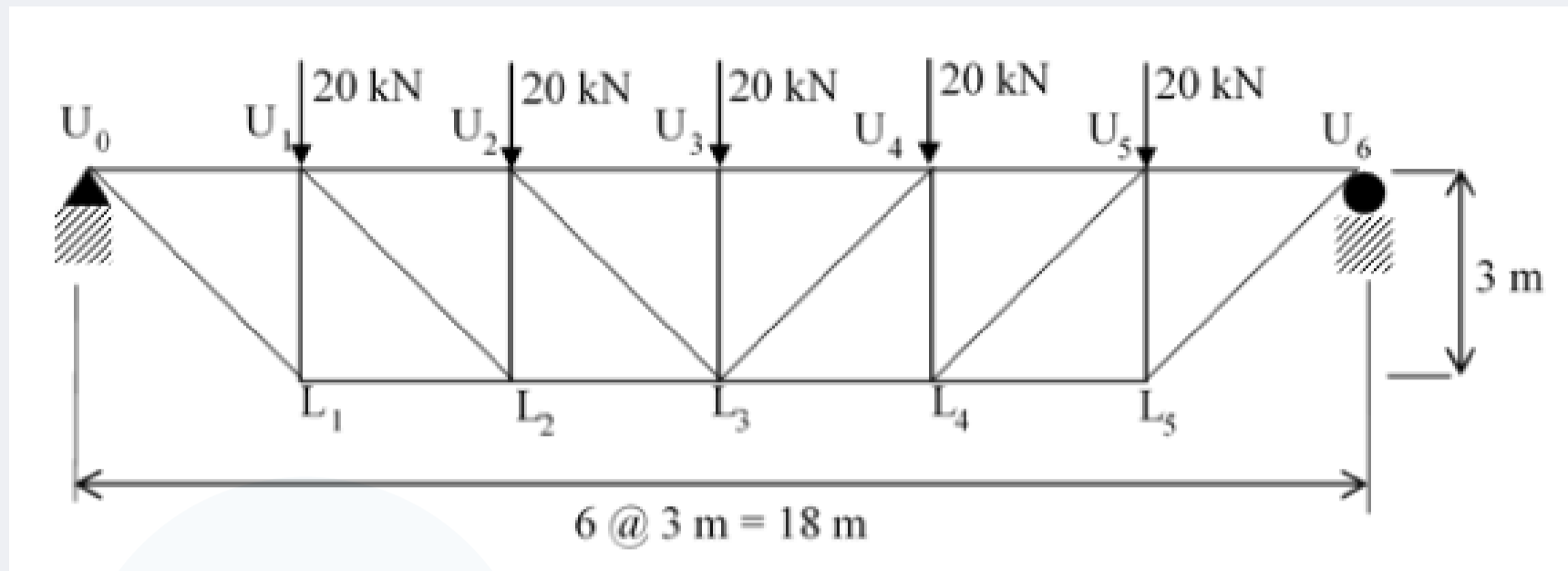
### Solution



- $\sigma'_x = \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta + \tau_{xy} \sin 2\theta \rightarrow (1)$
- Here  $\theta = 45^\circ$
- $\sigma'_x = 5 \text{ MPa}$

- $\sigma_y = 6 \text{ MPa}$
- $\tau_{xy} = 3 \text{ MPa}$
- From (1), we get,
- $\tau_x = -2 \text{ MPa}$
- i.e.  $\sigma_{1/2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}$
- $\sigma_{1/2} = \frac{-2+6}{2} \pm \frac{1}{2} \sqrt{(6+2)^2 + 4 \times 3^2}$
- $\sigma_1 = 7 \text{ MPa}$
- $\sigma_2 = -3 \text{ MPa}$

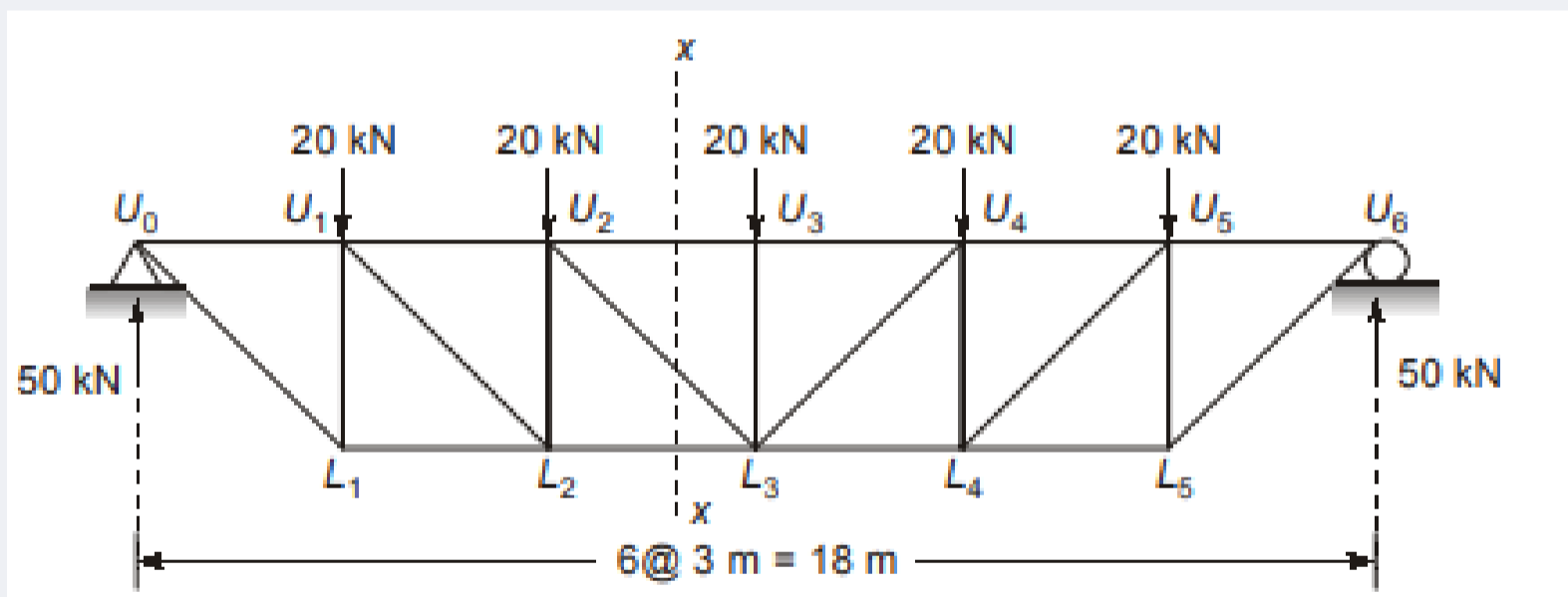
54. An idealised bridge truss is shown in the figure. The force in Member  $U_2L_3$  is: (kN, round off to one decimal place)



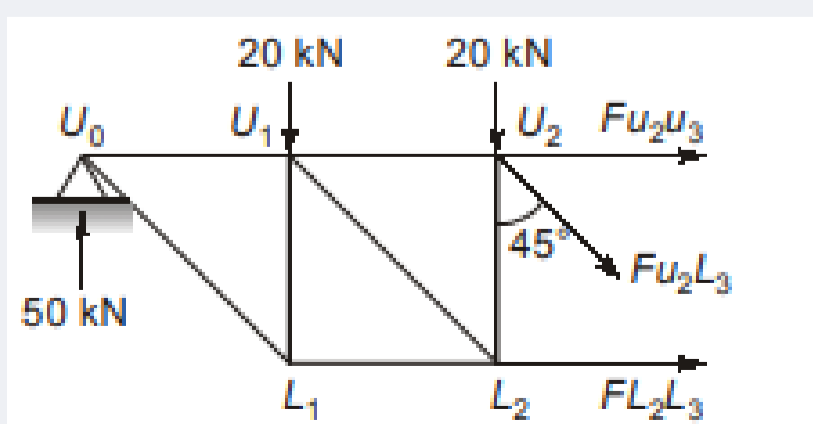
14.1

Answer range : 14.0-14.2

### Solution

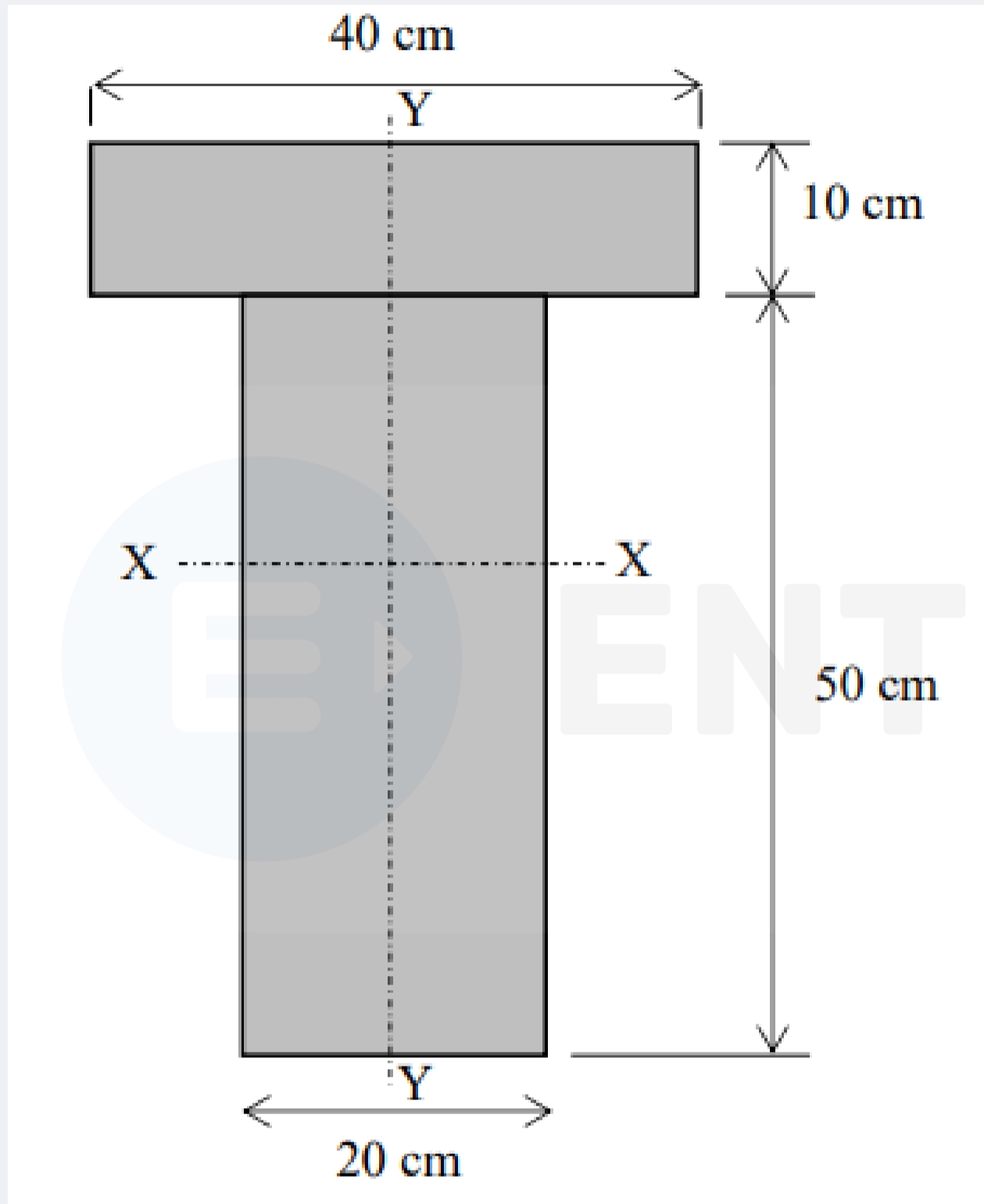


- Because of symmetry vertical reaction at supports =  $\frac{100}{2} = 50$
- Applying method of sections and taking the section as shown in the figure above:



- $\Sigma F_y = 0 \Rightarrow$
- $FU_2L_3\cos 45^\circ + 20 + 20 - 50 = 0$
- $FU_2L_3\cos 45^\circ = 10$
- $FU_2L_3 = 10\sqrt{2} \text{ kN} = 14.12 \text{ kN (Tension)}$

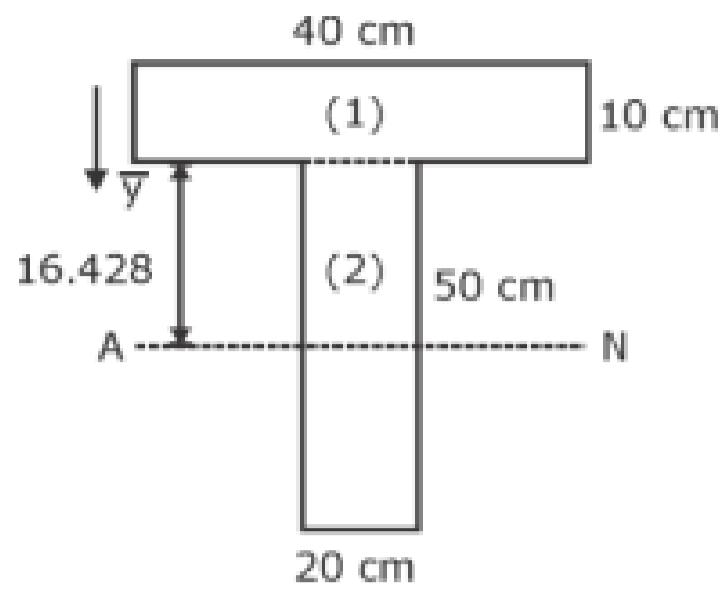
55. The cross-section of a girder is shown in the figure (not to scale). The section is symmetric about a vertical axis (Y-Y). The moment of inertia of the section about the horizontal axis (X-X) passing through the centroid is: (In  $cm^4$ , round off to nearest integer).



468775

Answer range : 468770.0-468810.0

**Solution**



- Taking  $\bar{y}$  from top.
- $$\bar{y} = \frac{A_1 Y_1 + A_2 y_2}{A_1 + A_2}$$
- $$\bar{y} = \frac{40 \times 10 \times 5 + 50 \times 20 \times 35}{40 \times 10 + 50 \times 20}$$
- $$\bar{y} = 26.428 \text{ cm}$$
- $$I_{NA} = \frac{bd^3}{12} + Ah^2$$
- $$I_{NA} = \frac{40 \times 10^3}{12} + 40 \times 10 \times (26.428 - 5)^2 + \frac{20 \times 50^3}{12} + 50 \times 20 \times (26.428 - 10)^2$$
- $$I_{NA} = 468775.2 = 468775 \text{ cm}^4$$



56. A soil having the average properties, bulk unit weight =  $19 \text{ kN/m}^3$ ; angle of internal friction =  $25^\circ$  and cohesion =  $15 \text{ kPa}$ , is being formed on a rock slope existing at an inclination of  $35^\circ$  with the horizontal. The critical height (in m) of the soil formation up to which it would be stable without any failure is: (round off to one decimal place)

[Assume the soil is being formed parallel to the rock bedding plane and there is no ground water effect.]

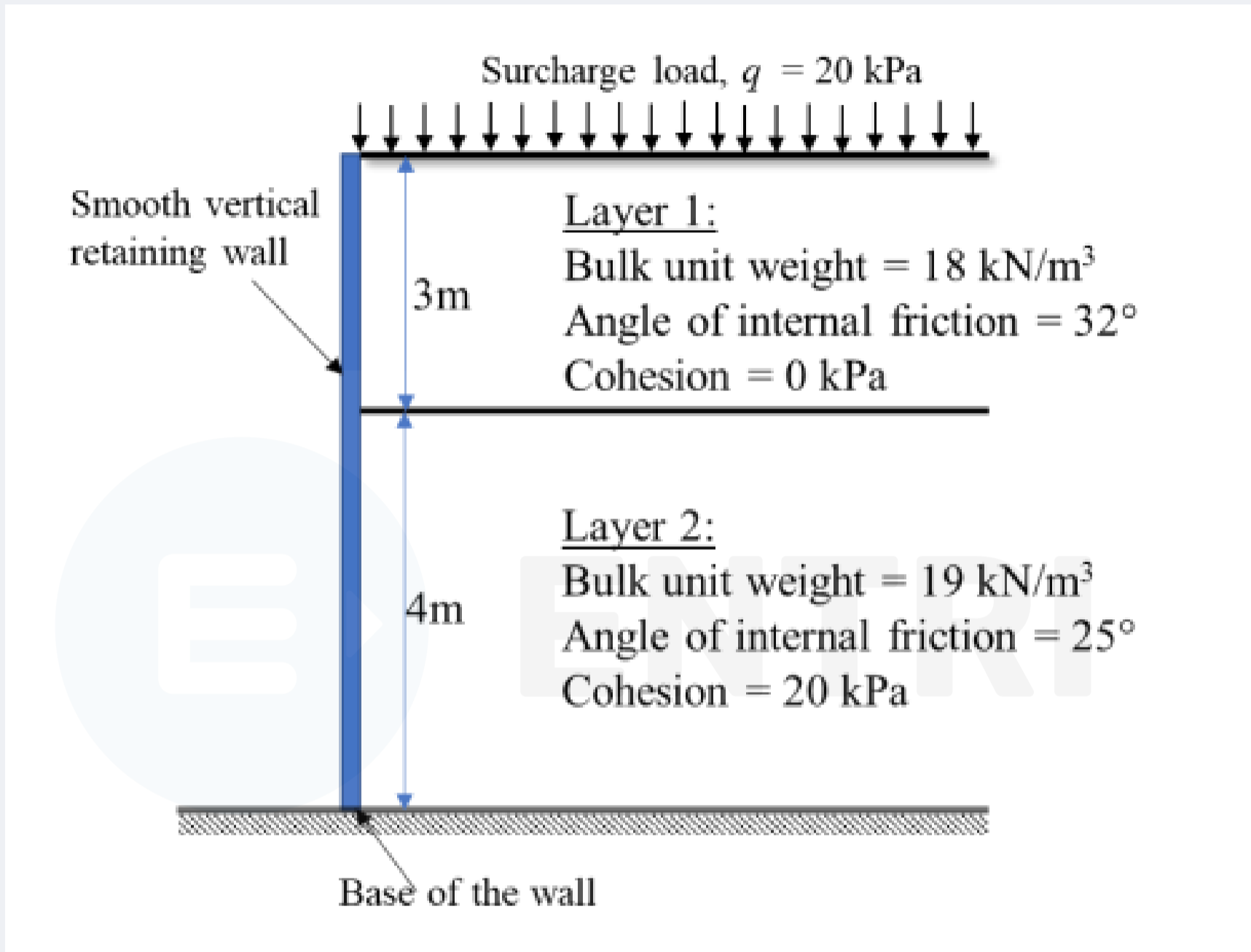
5.03

Answer range : 5.02-5.04

### Solution

- Given  $\gamma_t = 19 \text{ kN/m}^3$
- $\phi = 25^\circ$
- $C = 15 \text{ kPa}$
- $\beta = 35^\circ$
- For critical height of slope:
- $C + \gamma H_c \cos^2 \beta \tan \phi = \gamma H_c \sin \beta \cos \beta \Rightarrow$
- $15 + 19 \times H_c \cos^2 35^\circ \tan 25^\circ = 19 \times H_c \sin 35^\circ \cos 35^\circ \Rightarrow$
- $H_c = 5.03 \text{ m}$

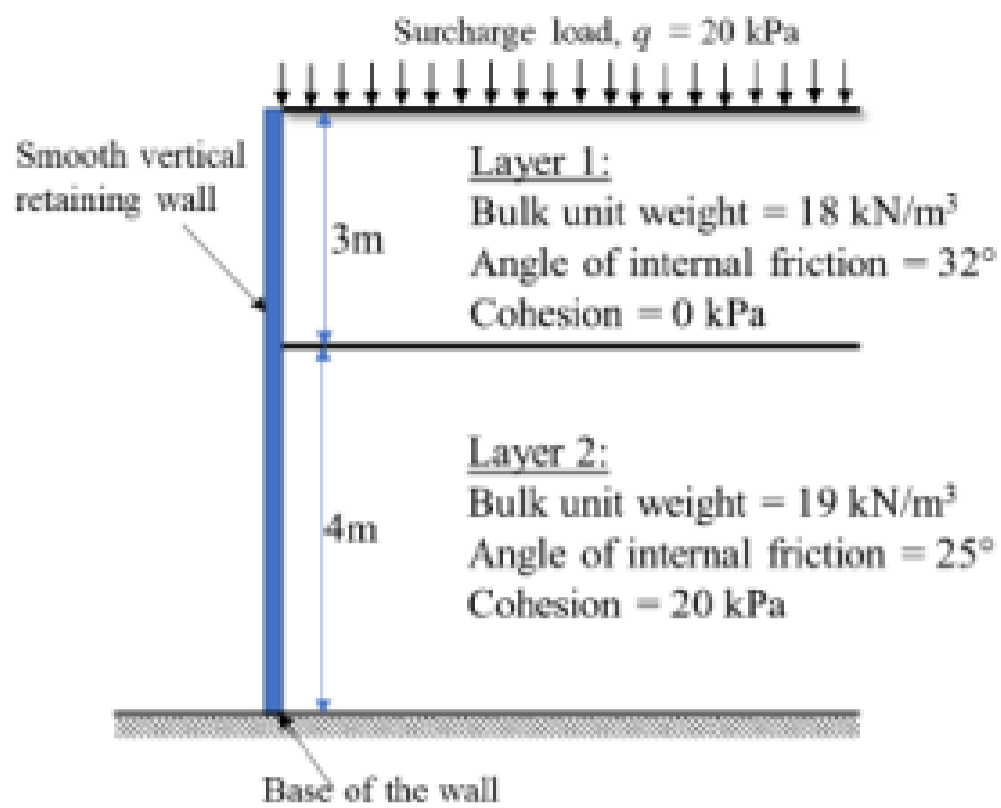
57. A smooth vertical retaining wall supporting layered soils is shown in figure. According to Rankine's earth pressure theory, the lateral active earth pressure acting at the base of the wall is: (in kPa - round off to one decimal place).



35.4

Answer range : 35.2-35.8

**Solution**



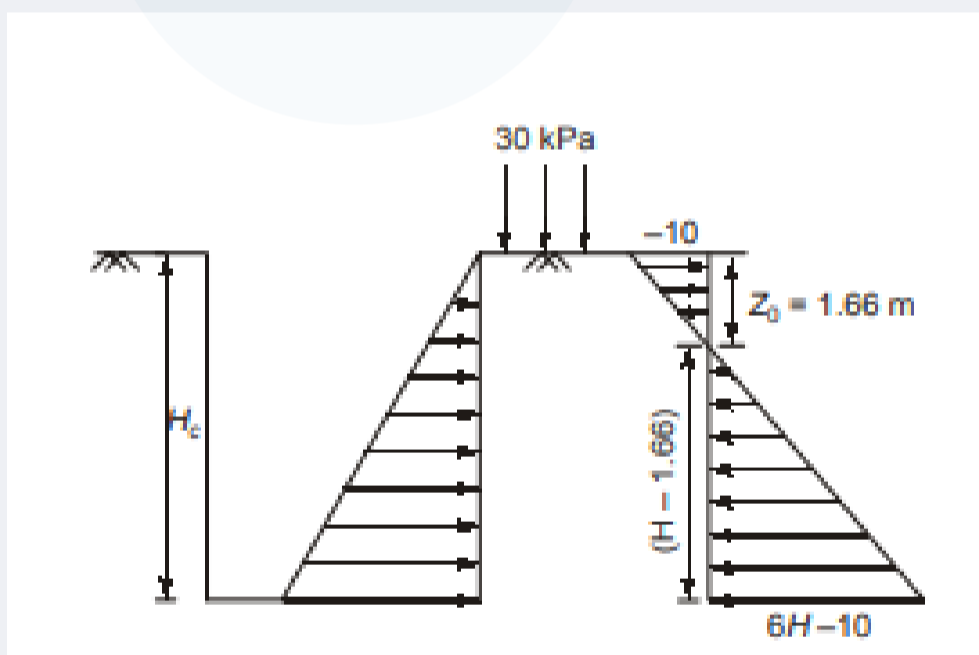
- Active earth pressure at base,
- $\sigma = k_a (\gamma z + q) - 2C \sqrt{k_a}$
- Where,  $k_a = \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{1 - \sin 25^\circ}{1 + \sin 25^\circ} = 0.406$
- $q = 20 + 18 \times 3 = 74 \text{ kN/m}^2$
- $\gamma z = 19 \times 4 = 76 \text{ kN/m}^2$
- $C = 20 \text{ kN/m}^2$
- $\sigma = 0.406 \times (76 + 74) - 2 \times 20 \sqrt{0.406} = 35.413 = 35.4 \text{ kN/m}^2$

58. A vertical trench is excavated in a clayey soil deposit having a surcharge load of 30 kPa. A fluid of unit weight  $12 \text{ kN/m}^3$  is poured in the trench to prevent collapse as the excavation proceeds. Assume that the fluid is not seeping through the soil deposit. If the undrained cohesion of the clay deposit is 20 kPa and saturated unit weight is  $18 \text{ kN/m}^3$ , what is the maximum depth of unsupported excavation (in m, rounded off to two decimal places)?

3.33

Answer range : 3.3-3.36

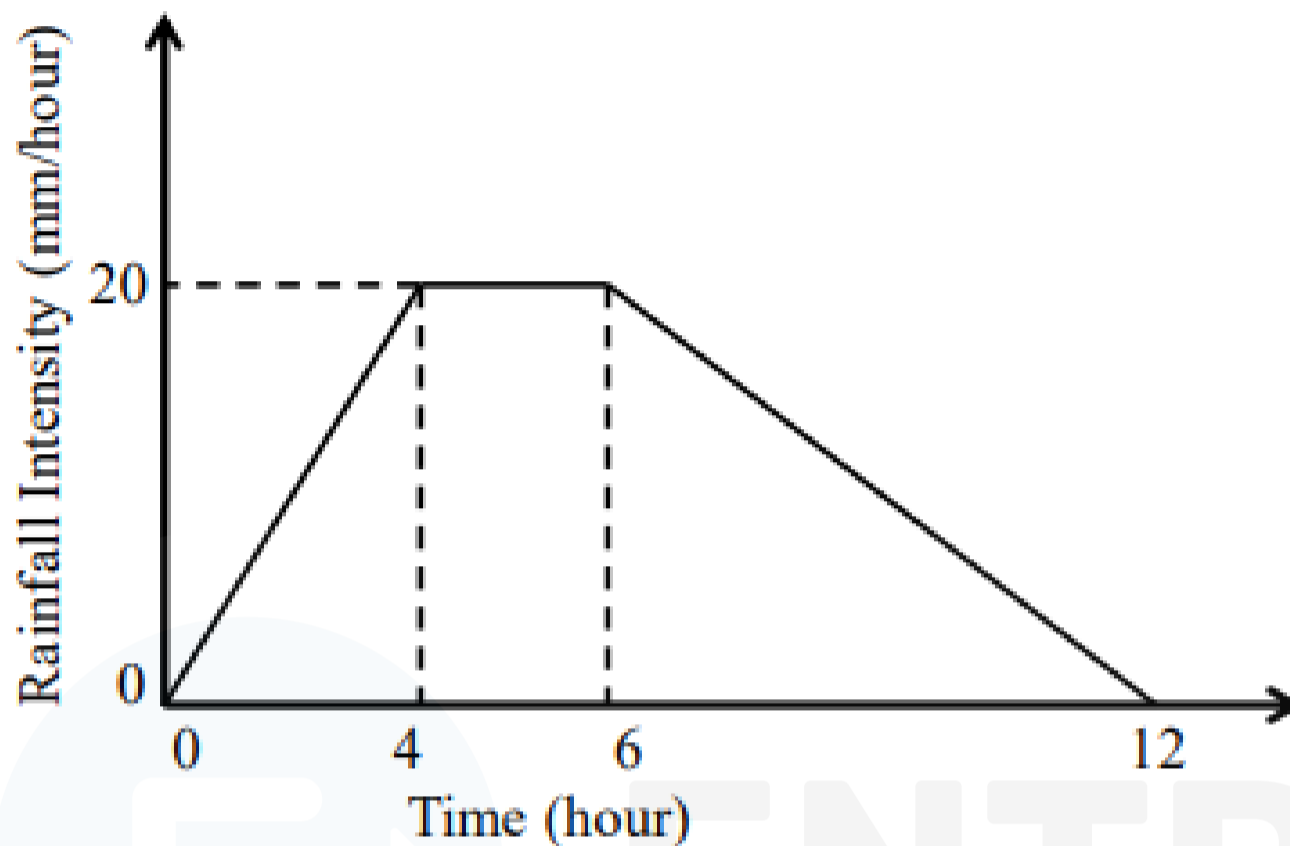
### Solution



- Now, active pressure at depth  $z$ ,
- $\sigma_z = k_a \gamma Z - 2C \sqrt{k_a} - 12z + kq$
- At  $z = 0$
- $k = 1$  for  $\phi = 0^\circ$
- $q = 30 \text{ kN/m}^3$
- $C = 20 \text{ kN/m}^3$
- $\sigma_z = -2 \times C + q = -2 \times 20 + 30 = -40 + 30 = -10 \text{ kN/m}^3$
- $Z_0$
- $\sigma_z = 0$
- $18 Z_0 - 40 - 12 Z_0 + 30 = 0$

- $6 Z_0 - 10 = 0$
- $Z_0 = 1.66 \text{ m}$
- At depth H,
- $\sigma_H = k_a \gamma H - 2C \sqrt{k_a} - 12 H + 30$
- $\sigma_H = 18 H - 40 - 12 H + 30 = 6H - 10$
- For unsupported depth of excavation, total active thrust must be zero.
- So,  $\frac{1}{2} \times 10 \times 1.66 = \frac{1}{2} \times (H - 1.66) \times (6H - 10) \Rightarrow$
- $H = \frac{10}{3} = 3.33 \text{ m}$

59. A 12-hour storm occurs over a catchment and results in a direct runoff depth of 100 mm. The time-distribution of the rainfall intensity is shown in the figure (not to scale). The  $\phi$ -index of the storm is (in mm, rounded off to two decimal places):

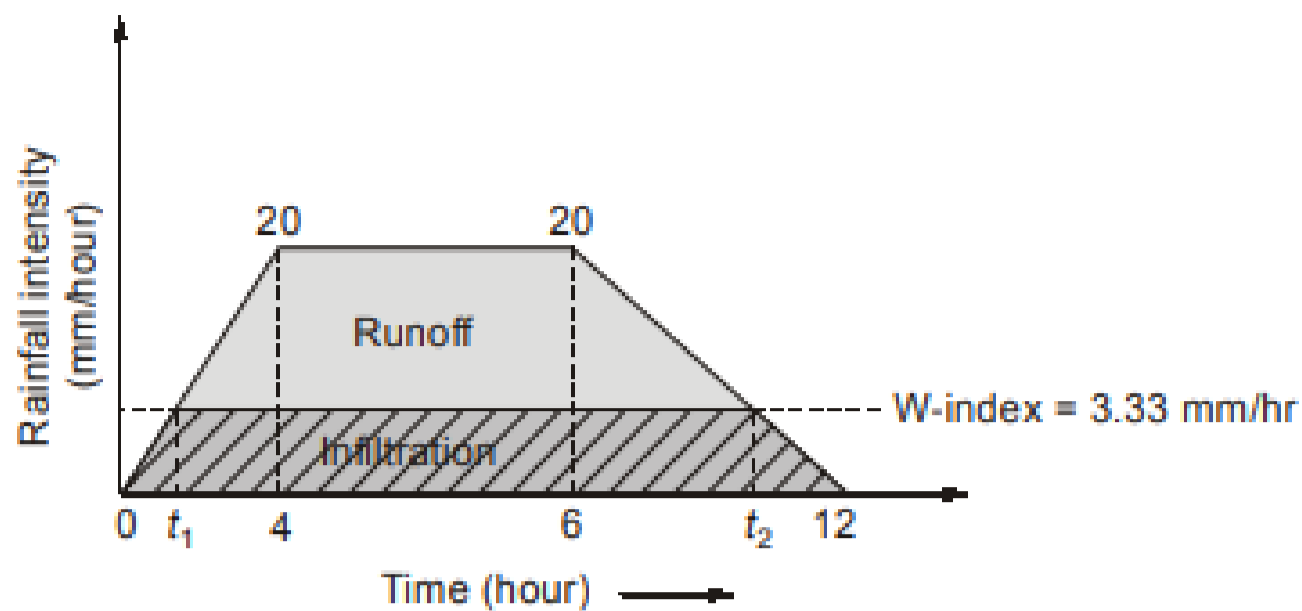


3.6

Answer range : 3.5-3.7

### Solution

- Given: Time = 12 hr
- Runoff,  $R = 100$  mm
- Total depth of precipitation,
- $P = \frac{1}{2} [20 \times 4] + 2 \times 20 + \frac{1}{2} 20 \times 6 = 140$  mm
- Total infiltration,  $I = P - R$
- $I = 140 - 100 = 40$  mm
- $W\text{-Index} = \frac{I}{t} = \frac{140-100}{12} = 3.33$  mm/hr



- $\frac{20}{4} = \frac{3.33}{t_1}$
- $t_1 = 0.67$  hr
- Similarly,  $\frac{20}{6} = \frac{3.33}{t_2}$
- $t_2 = 0.99$  hr
- $\phi$ -Index =  $\frac{\text{Total infiltration in which rainfall excess occur}}{\text{Time period in rainfall which excess occur}}$
- $\phi$ -Index =  $\frac{I - \text{infiltration in which no runoff}}{\text{Time excess}}$
- $\phi$ -Index =  $\frac{140 - [\frac{1}{2} \times 0.33 \times 0.67 + \frac{1}{2} \times 3.33 \times 0.99]}{12 - 0.67 - 0.99}$
- $\phi$ -Index = 3.6 mm/hr
- **CHECK**
- Runoff = Area of hatched portion
- $\frac{1}{2} \times (20 - 3.6) \times (4 - 0.67) + (20 - 3.6) \times 2 + \frac{1}{2} \times (20 - 3.6) \times (6 - 0.99)$   
(HENCE OK)
- $\therefore \phi$ -Index = 3.6 mm/hr

60. A hydraulic jump occurs in a 1.0 m wide horizontal, frictionless, rectangular channel, with a pre-jump depth of 0.2 m and a post-jump depth of 1.0 m. The value of  $g$  may be taken as  $10 \text{ m/s}^2$ . The values of the specific force at the pre-jump and post-jump sections are same and are equal to (in  $\text{m}^3$ , rounded off to two decimal places).

0.62

Answer range : 0.6-0.64

### Solution

- Given,
- $B = 1.0 \text{ m}$
- $y_1 = 0.2 \text{ m}$
- $y_2 = 1.0 \text{ m}$
- $g = 10 \text{ m/s}^2$
- The value of specific force =  $\frac{P+M}{\gamma_w} = \text{a constant}$

$$\Rightarrow = \frac{P_1+M_1}{\gamma_w} - \frac{P_2+M_2}{\gamma_w} = A\bar{y}_1 + \frac{Q^2}{Ag} \mapsto (1)$$

- For horizontal frictionless rectangular channel,
- $\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$
- $Q = \sqrt{\frac{1 \times 0.2 \times (1.2) \times 10}{2}} = 1.095 \text{ m}^3/\text{s/m}$
- Now, specific force =  $0.2 \times 1 \times \frac{0.2}{2} + \frac{1.095^2}{1 \times 0.2 \times 10}$
- Specific force =  $0.62 \text{ m}^3$



61. In Horton's equation fitted to the infiltration data for a soil, the initial infiltration capacity is 10 mm/h; final infiltration capacity is 5 mm/h; and the exponential decay constant is 0.5 /h. Assuming that the infiltration takes place at capacity rates, the total infiltration depth (in mm) from a uniform storm of duration 12 h is: (round off to one decimal place).

70

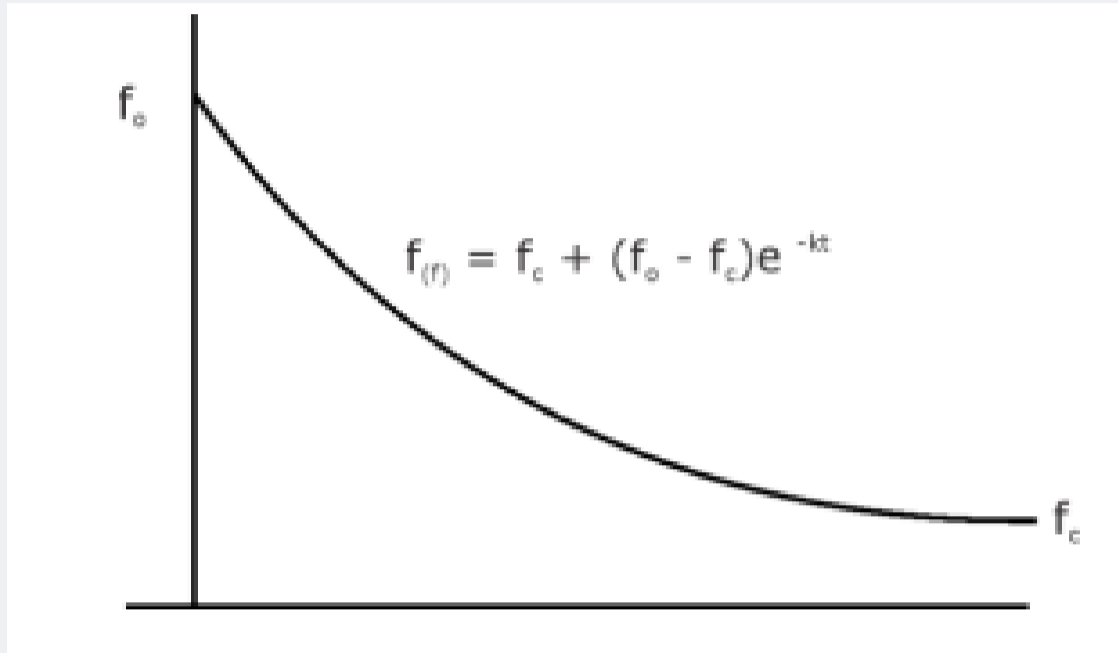
Answer range : 69.5-70.5



ENTRI

## Solution

- Given,
- $f_0 = 10$  mm/hr
- $f_c = 5$  mm/hr
- $k = 0.5$  /hr



- $\therefore$  Total infiltration depth  $= \int_0^{12} 5 + (10 - 5)e^{-0.5t} dt$
- Total infiltration depth  $= \left[ 5t + \frac{5e^{-0.5t}}{-0.5} \right]_0^{12}$
- Total infiltration depth  $= (5 \times 12 - 10 e^{-0.5 \times 12}) - (5 \times 0 - 10 e^{-0.5 \times 0})$
- Total infiltration depth  $= 59.97 + 10 = 69.97 = 70$  mm

62. The composition and energy content of a representative solid waste sample are given in the table. If the moisture content of the waste is 26%, the energy content of the solid waste on dry-weight basis is: (in MJ/kg, round off to one decimal place).

Component	Percent by mass	Energy content as-discarded basis (MJ/kg)
Food waste	20	4.5
Paper	45	16.0
Cardboard	5	14.0
Plastics	10	32.0
Others	20	8.0

18.38

Answer range : 18.34-18.42

### Solution

- Energy content (as discarded basis) =  $(0.2 \times 4.5) + (0.45 \times 16) + (0.05 \times 14) + (0.1 \times 32) + (0.2 \times 8) = 13.6 \text{ MJ/kg}$
- Energy content (on dry basis) for moisture content of 26% =  $\frac{13.6 \text{ MJ/kg} \times 100}{(100 - 26)} = 18.38 \text{ MJ/kg}$

63. A flocculator tank has a volume of  $2800 \text{ m}^3$ . The temperature of water in the tank is  $15^\circ \text{C}$ , and the average velocity gradient maintained in the tank is  $100/\text{s}$ . The temperature of water is reduced to  $5^\circ \text{C}$ , but all other operating conditions including the power input are maintained as the same. The decrease in the average velocity gradient (in %) due to the reduction in water temperature is: (round off to nearest integer)

(Consider dynamic viscosity of water at  $15^\circ \text{C}$  and  $5^\circ \text{C}$  as  $1.139 \times 10^{-3} \text{ N-s/m}^2$  and  $1.518 \times 10^{-3} \text{ N-s/m}^2$ , respectively)

13

### Solution

- Given, Volume of flocculation tank ( $V$ ) =  $2800 \text{ m}^3$
- At  $T = 15^\circ \text{C}$ ,
- $G_{15^\circ \text{C}} = 100/\text{s}$
- $\eta_{15^\circ \text{C}} = 1.139 \times 10^{-3} \text{ N-s/m}^2$
- At  $T = 5^\circ \text{C}$
- $G_{5^\circ \text{C}} = ?$
- $\mu_{5^\circ \text{C}} = 1.518 \times 10^{-3} \text{ N-s/m}^2$
- Temporal mean velocity gradient,  $G = \sqrt{\frac{P}{\mu V}}$
- $P$  and  $V$  are constant.
- So,  $G^2 \mu = \text{constant}$
- $100^2 \times 1.139 \times 10^{-3} = G_{5^\circ \text{C}}^2 \times 1.518 \times 10^{-3}$
- $G_{5^\circ \text{C}} = 86.62 \text{ sec}^{-1}$
- The decrease in velocity gradient =  $\frac{G_{15^\circ \text{C}} - G_{5^\circ \text{C}}}{G_{15^\circ \text{C}}} \times 100 = 13.38 = 13$   
%

64. The wastewater inflow to an activated sludge plant is  $0.5 \text{ m}^3 / \text{s}$ , and the plant is to be operated with a food to microorganism ratio of  $0.2 \text{ mg/mg-d}$ . The concentration of influent biodegradable organic matter of the wastewater to the plant (after primary settling) is  $150 \text{ mg/L}$ , and the mixed liquor volatile suspended solids concentration to be maintained in the plant is  $2000 \text{ mg/L}$ . Assuming that complete removal of biodegradable organic matter in the tank, the volume of aeration tank (in  $\text{m}^3$ , in integer) required for the plant is:

16200

Answer range : 16199.0-16201.0

### Solution

- Given:
- $Q = 0.5 \text{ m}^3 / \text{s}$
- $\frac{F}{M} = 0.2 \text{ mg/mg-day}$
- $S_0 = 150 \text{ mg/L}$
- $X = 2000 \text{ mg/L}$
- $\frac{F}{M} = \frac{Q_0 S_0}{V \times X}$
- $0.2 \text{ d}^{-1} = \frac{0.5 \left( \frac{\text{m}^3}{\text{sec}} \right) \times 86400 \left( \frac{\text{sec}}{\text{d}} \right) \times 150 \left( \frac{\text{mg}}{\text{L}} \right)}{V(\text{m}^3) \times 2000 \frac{\text{mg}}{\text{L}}}$
- $V = 16200 \text{ m}^3$

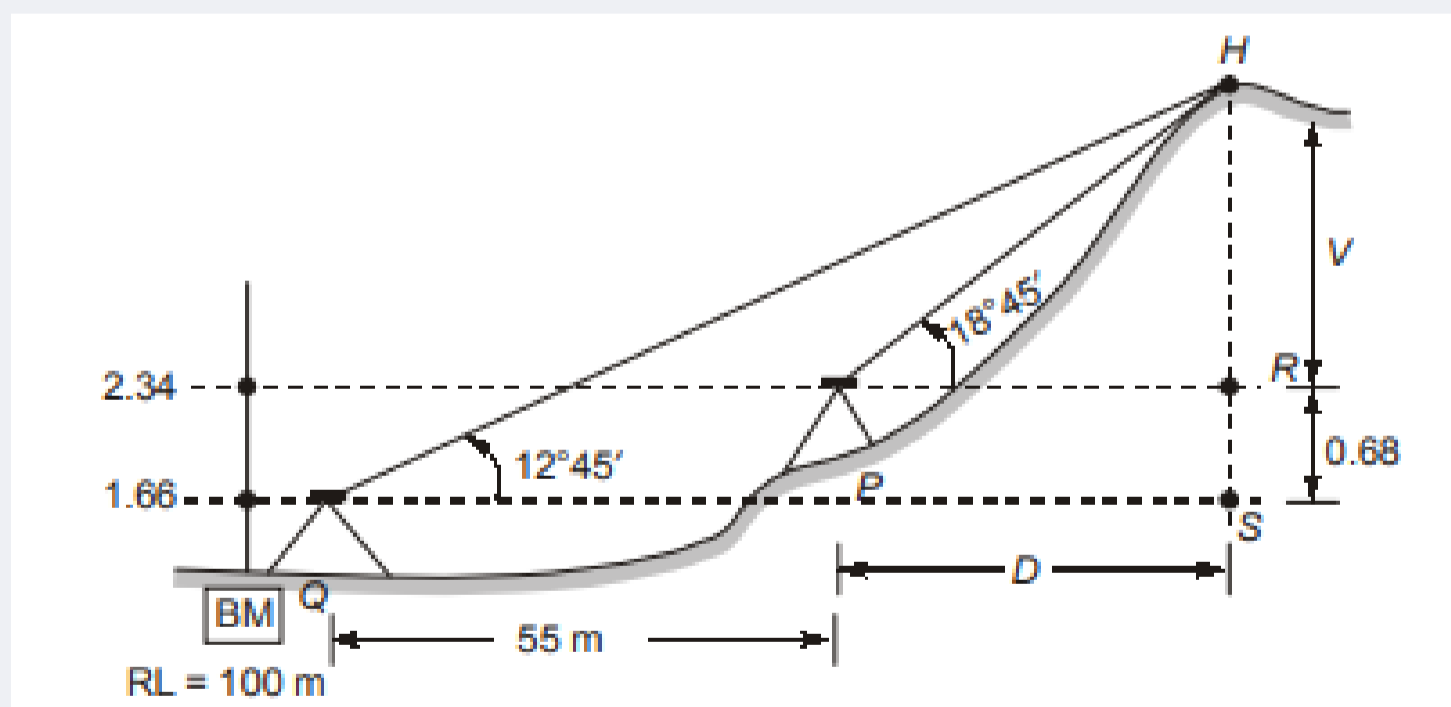
65. Trigonometric levelling was carried out from two stations P and Q to find the reduced level (R. L.) of the top of hillock, as shown in the table. The distance between Stations P and Q is 55 m. Assume Stations P and Q, and the hillock are in the same vertical plane. The R. L. of the top of the hillock (in m) is: (round off to three decimal places)

Station	Vertical angle of the top of hillock	Staff reading on benchmark	R. L. of benchmark
P	$18^\circ 45'$	2.340 m	100.000 m
Q	$12^\circ 45'$	1.660 m	

137.627

Answer range : 137.0-138.0

### Solution



- $\tan 12^\circ 45' = \frac{v+0.68}{D+55} \mapsto (i)$
- $\tan 18^\circ 45' = \frac{v}{D} \mapsto (ii)$
- On solving (i) and (ii), we get
- $v = 35.287 \text{ m}$

- $D = 103.954 \text{ m}$
- $\text{R.L. of hill top} = \text{R.L. of BM} + 2.34 + v$
- $\text{R.L. of hill top} = 100 + 2.34 + 35.287 = 137.627 \text{ m}$



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