Important Algebra Formulas for CAT

Here are some essential algebra formulas that every CAT aspirant should be familiar with:

- **Linear Equations:**
    - Slope of a line: `m = (y2 - y1) / (x2 - x1)`
    - Equation of a line: `y = mx + c` (where m is slope and c is y-intercept)
- **Quadratic Equations:**
    - Standard form: `ax^2 + bx + c = 0`
    - Quadratic formula: `x = (-b ± √(b^2 - 4ac)) / 2a`
    - Discriminant (determines nature of roots): `b^2 - 4ac`
- **Inequalities:**
    - Basic properties: `a > b => a + c > b + c`, `a > b and b > c => a > c`
    - Working with absolute values: `|x| >= 0`
- **Logarithms:**
    - Logarithm laws: `log(a * b) = log(a) + log(b)`, `log(a^n) = n * log(a)`
    - Change of base formula: `log_a(b) = (log_c(b)) / (log_c(a))` (where c is any base)
- **Permutations and Combinations:**
    - Permutation (order matters): `nPr = n! / (n - r)!`
    - Combination (order doesn't matter): `nCr = n! / (r! * (n - r)!)`

CAT Algebra Previous Year Questions

Previous years' questions provide valuable insights into the types of algebra problems that appear in the CAT exam. Let's review some of these questions.

Question 1:

If the sum of two numbers is 30 and their difference is 6, find the product of the numbers.

Solution: Let the numbers be x and y.

Given, x + y = 30 and x - y = 6.

Adding both equations, we get 2x = 36, so x = 18.

Substituting x in the first equation, we get y = 12.

Therefore, the product of the numbers is 18 * 12 = 216.

Question 2:

If the roots of the equation x^2 - 5x + k = 0 are in the ratio 2:3, find the value of k.

Solution*:* Let the roots be 2a and 3a.

Sum of roots = 5a = 5, so a = 1.

Product of roots = 6a^2 = k.

Therefore, k = 6.

Question 3:

Find the value of x for which the expression x^2 - 6x + 11 is minimum.

Solution*:* The given expression is a quadratic equation.

The minimum value occurs at the vertex of the parabola represented by the equation.

The x-coordinate of the vertex is given by -b/2a, where a and b are the coefficients of x^2 and x, respectively.

In this case, a = 1 and b = -6. So, x = -(-6) / 2*1 = 3.

Therefore, the value of x for which the expression is minimum is 3.

Question 4:

If log2(x) + log2(y) = 4 and log2(x/y) = 2, find the value of xy.

Solution*:* Using the properties of logarithms, we can write:

log2(xy) = log2(x) + log2(y) = 4.

Therefore, xy = 2^4 = 16.

Question 5:

x is a positive real number such that x^8 + (1/x)^8 = 47. What is the value of x^9 + (1/x)^9?

Solution: We can't solve directly for x.

Notice the symmetrical nature of the equation.

Multiplying both sides by x gives: x^9 + 1 + (1/x) = 47x.

This simplifies to x^9 + (1/x)^9 = 46. (The answer is 46)

Question 6:

A train travels x km at y km/hr. It then travels the same distance at (y + 2) km/hr. The total time taken is 12 hours. What is the value of xy?

Solution: We can set up two equations for time:

$$x/y + x/(y + 2) = 12.$$

Solve for x and substitute back to find xy.

(The answer can be any value satisfying the equation)

Question 7:

Simplify the expression $x^2 - 1 / x - 1$ for $x \neq 1$.

Solution:

The expression can be simplified by factoring the numerator:

$x^2 - 1 / x - 1 = (x - 1(x + 1)) / x - 1$

For $x \neq 1$, the $x - 1$ terms cancel out, leaving:

$x + 1$

So, the simplified expression is $x + 1$.

Question 8:

If $x + y = 10$ and $xy = 21$, find the value of $x^2 + y^2$.

Solution:

Using the identity:

$x^2 + y^2 = (x + y)^2 - 2xy$

Substituting the given values:

$x^2 + y^2 = 10^2 - 2 \times 21 = 100 - 42 = 58$

So, the answer is 58.

Question 9:

The sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is 7, and the product of the roots is 12. Find the equation.

Solution:

The sum of the roots $\alpha + \beta = -b/a = 7$, and the product $\alpha\beta = c/a = 12$. Assuming $a = 1$:

$x^2 - 7x + 12 = 0$

This is the required equation.

Question 10:

Solve $x^2 - 4x + 4 = 0$.

Solution:

This is a perfect square trinomial:

$(x - 2)^2 = 0$

So, the solution is $x = 2$.

Question 11:

If $\log_{10}x = 2$, find the value of x.

Solution:

From the definition of logarithms:

$\log_{10}x = 2$ implies $x = 10^2 = 100$

So, the value of x is 100.