Amplitude And Frequency Modulation and Demodulation

Modulation is a fundamental requisite of communication system. It is defined as the process by which some characteristics of signal called carrier is varied in accordance with the instantaneous value of another signal called modulating signal. Signals containing information or intelligence to be transmitted are referred to as modulating signals or base signal frequency is greater than the modulating frequency. The signal resulting from the process of modulation is called modulated signal.

Analog (Continuous Wave) Modulation

When the carrier wave is continuous in nature the modulation process is known as continuous wave (CW) modulation or analog modulation. The carrier wave is usually a sinusoidal signal.

Modulating signal → m(t) or f(t) → Audio signal 15 Hz - 15 kHz

Carrier signal → sin $\omega_c t$ or cos $\omega_c t$

ω_o or ω → Carrier frequency

f(t) or m(t) → Band limited signal limited to frequency ω_m

→ Frequency spectrum of this signal extends upto ω_m.

Advantage of Modulation

1. Using modulation, length of receiving antenna is relatively small.
2. Using modulation S/N increases.
3. At low frequency (Audio frequency) attenuation is large, i.e. propagation range is small, this range is increased using modulation at relatively higher frequency.
4. Multiplexing
5. Narrow banding
6. Audio signal 20 Hz to 20 kHz, when transmitted as usual mix-up inseparably. Broadcasting station alone would blanket the air, very less number of channel can be used.
7. Common processing (Tx and Rx).

Amplitude Modulation

Here the amplitude of the carrier is varied in accordance with the instantaneous value of the amplitude of the modulating signal. Keeping frequency and phase of the carrier to be fixed.

$f(t) = v_m(t) = V_m \sin \omega_m(t)$ → mod.signal

$v_c(t) = V_c \sin \omega_m(t)$ → carrier

Modulated Signal (AM SIgnal)

$V(t) = [V_c + K_a V_m (t)] \sin \omega_c t$

$= V_c \sin \omega_c t + K_a V_m , 2 \sin \omega_c t \sin \omega_m t$

$= V_c \sin \omega_c t + K_a \frac{V_m}{2} \left[\cos (\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t \right]$

Modulation Index (m_a = 0.4 to 0.6)

The extent of amplitude variation in AM about an unmodulated carrier amplitude is measured in terms of a factor called modulation index. It is define as :

$$m_a = \frac{V_m (max)}{V_c} = \frac{K_a V_m}{V_c} = \frac{V_m}{V_c} \text{ (if } K_a = 1)$$

$$V_m = \frac{m_a V_c}{K_a} \text{ or } K_a V_m = m_a V_c$$

This factor m_a is also known as : (i) depth of modulation or (ii) degree of modulation or (iii) modulation factor

Power Contained in the AM Signal

$V(t) = V_c \sin \omega_c t$

$= \frac{M_a v_c}{2} [\cos (\omega_c - \omega_m) t - \cos(\omega_c + \omega_m) t]$

Total Tx power = Power in the carrier + Power USB + Power LSB

= Power (Carrier) + 2 Power (SB)

Power in carrier $\propto V_c^2 = P_c$

$$\text{Power (USB)} \propto \left(\frac{m_a V_c}{4}\right)^2 = \frac{m_a^2}{4} P_c$$

$$\text{Power (LSB)} \propto \left(\frac{m_a V_c}{2}\right)^2 = \frac{m_a^2}{4} P_c$$

Power in AM signal, i.e. AM - DSB/ FC

$$P_t = P = P_c + \frac{m_a^2}{4} P_c + \frac{m_a^2}{4} P_c$$

$\text{Power} = P_c\left(1 + \frac{m_a^2}{2}\right)$ → Only for the sinusoidal type of modulating signal

Example : Power saving in different AM system.

Let m_a = 100% i.e. $m_a = 1$

Case 1 : AM - DSB/FC

Power $P_t = P_c\left(1 + \frac{m_a^2}{2}\right) = \frac{3P_c}{2}$

Power required to transmit AM - DSB/FC

Case 2 : AM - DSB/SC

Power saving = P_c

Power transmitted = Power in 2 SB

$$= P_c\left(\frac{m_a^2}{2}\right) = \left(\frac{P_c}{2}\right)$$

% Power saving w.r.t AM-DSB/FC

$$= \frac{P_c}{(3P_c/2)} = 100 = \frac{200}{3} = 76\%$$

Case 3 : AM - SSB/FC

Power saving = $P_c\frac{m_a^2}{4} = \frac{P_c}{4}$

Power transmitted = $P_c\left(1 + \frac{m_a^2}{2}\right) = \frac{3P_c}{4}$

% Power saving w.r.t. AM-DSB/FC

$$= \frac{(P_c/4)}{(3P_c/2)} \times 100 = \frac{50}{3} = 17\%$$

Case 4 : AM - SSB/SC

Power saving = $P_c + \frac{m_a^2 P_c}{4}$

$$= P_c\left(1 + \frac{m_a^2}{2}\right) = \frac{5P_c}{4}$$

Power transmitted = $P_c\frac{m_a^2}{4} = \frac{P_c}{4}$

% Power saving w.r.t AM-DSB/FC

$$= \frac{(5P_c/4)}{(3P_c/2)} \times 100 = 83\%$$

Efficiency of Transmission AM-DSB/FC

$\eta = 33.3\%$

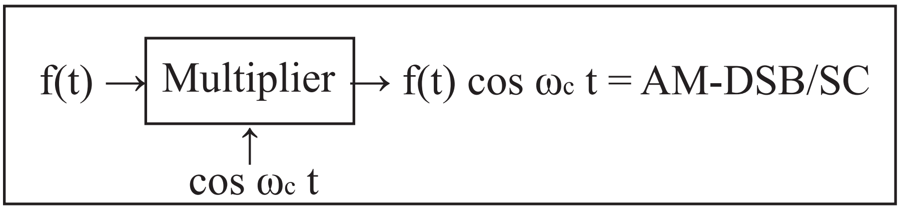
For broadcasting AM, DSB/FC is always preferred since the receiver configuration is the simplest.

AM-DSB/SC

Modulation signal = f(i)

Carrier = $\cos \omega_c t$

Frequency components

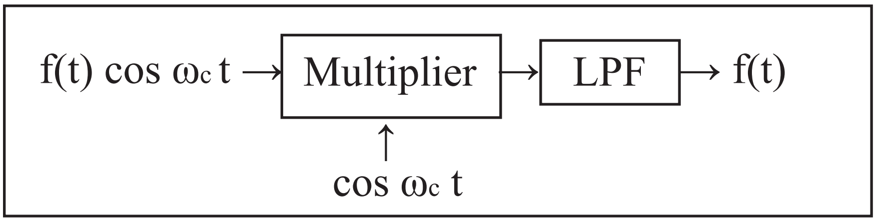


$$f(t)\cos \omega_c t = f(t)\,\frac{(e^{j\omega_c t} + e^{-j\omega_c t})}{2}$$

$$= FT[1/2\, f(t)\, e^{j\omega_c t}] + FT[1/2\, f(t)\, e^{-j\omega_c t}]$$

$$= (1/2)\,[F(\omega + \omega_c)] + F(\omega - \omega_c)$$

Principle of Demodulator



After the multiplier

$= f(t)\cos^2 \omega_c t$

$= (1/2)\, f(t)\,[1 + \cos 2\omega_c t]$

$= (1/2)\, f(t) + (1/2)\, f(t) \cos 2\omega_c t$

After LPF

$(1/2)\, f(t)$ is passed and $(1/2)\, f(t) \cos 2\omega_c t$ is attenuated

$\therefore$ $f(t)$ is recovered

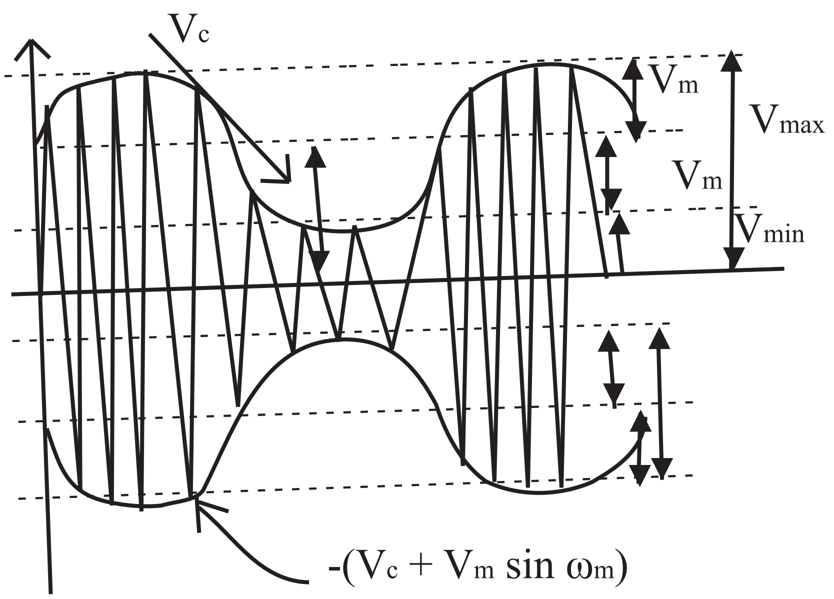
$f(t)\cos^2\omega_c t$

$= (1/2)\, f(t) + (1/2)\, 2 \cos 2\omega_c t$

$= (1/2)\, F(\omega) + (1/4)\,[F(\omega + 2\omega_c) + F(\omega - 2\omega_c)]$

Representation of AM

$$V(t) = V_c \sin \omega_c t + \left(\frac{K_a V_m}{2}\right) \cos(\omega_c - \omega_m)t - \left(\frac{K_a V_m}{2}\right) \cos(\omega_c + \omega_m)t$$



$$V_m = \frac{V_{max} - V_{min}}{2}$$

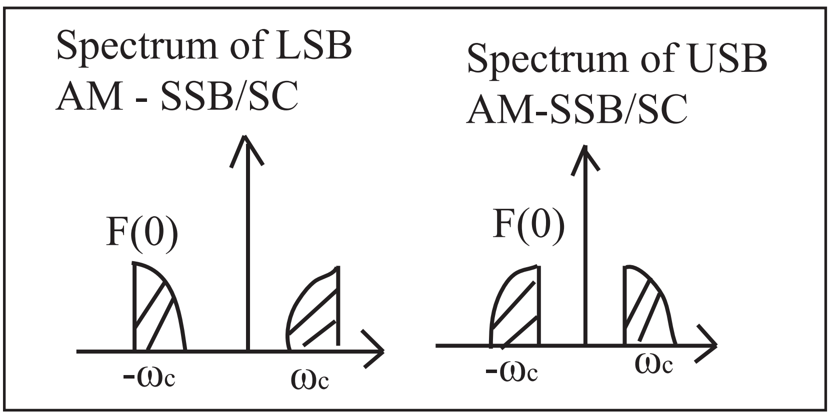
$$V_c = V_{max} - V_m = \frac{V_{max} + V_{min}}{2}$$

Modulation index

$$m_a = [V_m / V_a] = \frac{[(V_{max} - V_{min}) / 2]}{[(V_{max} + V_{min}) / 2]}$$

$$= \frac{[V_{max} - V_{min}]}{[V_{max} + V_{min}]}$$

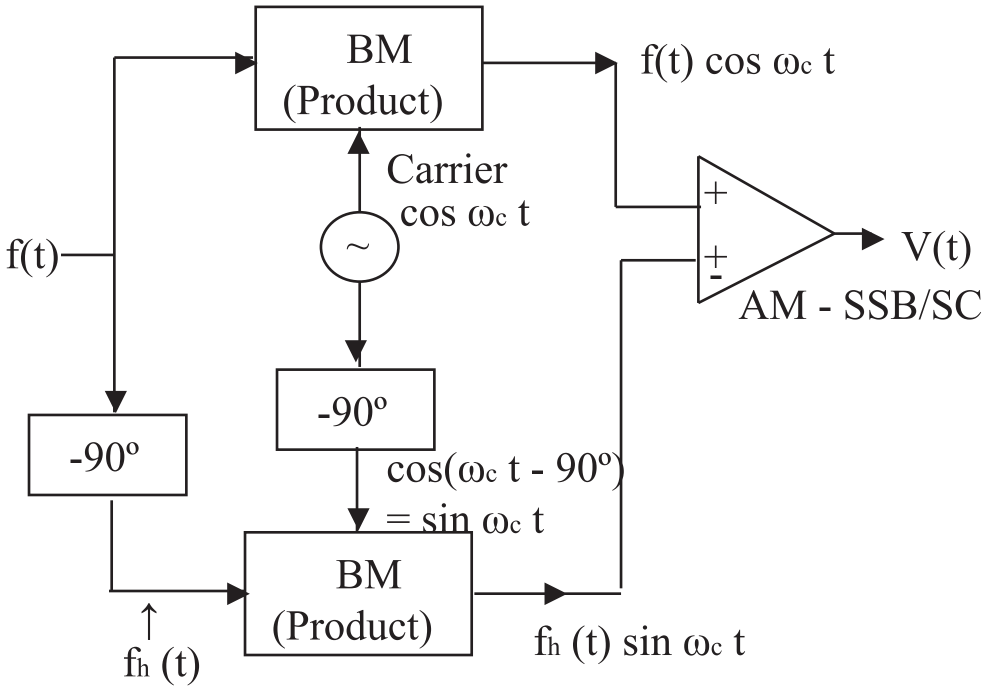
AM - SSB/SC



There are three method to Generate SSB/SC

1. The Filter System
2. The "Third" Method or Weaver Method

Phase Shift Method of Generating AM-SSB/SC Signal



Where f_h (t) is the Hilbert transform of f(t)

Special Case :

Let $f(t) = \cos \omega_m t$

→ It holds good for any general signal

$V(t)|_{SSB/SC} = \cos \omega_m t \, . \, \cos \omega_c t + \sin \omega_m t \, . \, \sin \omega_c t$

$= \cos (\omega_c - \omega_m) t$

= LSB is generated

$V(t)|_{SSB/SC} = f(t) \cos \omega_c t + f_h (t) \sin \omega_c t$

→ LSB is generated

$= f(t) \cos \omega_c t - f_h (t) \sin \omega_c t$

→ USB is generated

In general representation of AM.SSB/SC

$V(t) = f(t) \cos \omega_c t \pm f_h (t) \sin \omega_c t$

Detection of AM - SSB/SC Signal

Basic principle

$V(t) = f(t) \cos \omega_c t \pm f_h \, \omega_i \, t$ - AM - SSB/SC

FOr synchronous detection

$V(t) = V(t) \cos \omega_c t$

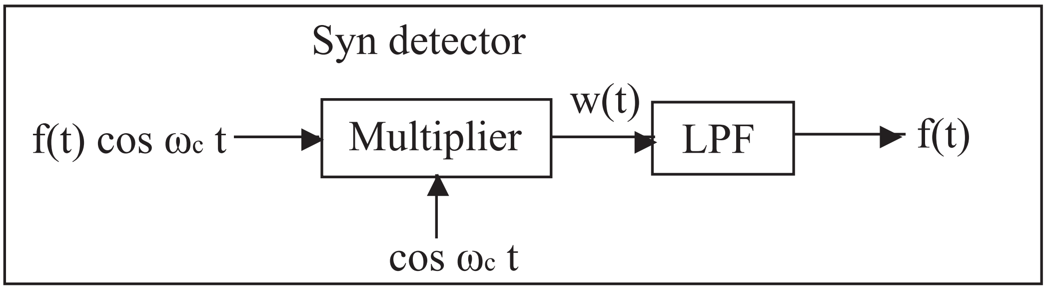
$= f(t) \cos^2 \omega_c t \pm f_h (t) \sin \omega_c t \, . \, \cos \omega_c t$

$$= \frac{1}{2} [f(t) (1 + \cos 2\omega_c t)] \pm \frac{1}{2} [f_h (t) \sin 2\omega_c t]$$

$$= \frac{1}{2} f(t) + \frac{1}{2} [f(t) (1 + \cos 2\omega_c t)] \pm [f_h (t) \sin 2\omega_c t]$$

When this signal is passed through a LPF having cut off frequency of ω_m and centered at $\omega = 0$ origin, the original modulating signal will then be recovered. The other frequency sprectrum of f(t) centered at $\pm 2\omega_c$ will be just filtered out.

Therefore original modulating signal is recovered.



Angle Modulation

In general modulated carrier signal can be represented mathematically as

$$x_c (t) = A(t) \cos[\omega_c t + \phi(t)]$$

where, A(t) and $\phi(t)$ are called the instantaneous amplitude and phase angle of the carrier. When A(t) is linearly related to the message signal f(t) the result is amplitude modulation. If $\phi(t)$ or its derivative is linearly related to f(t), then we have phase or frequency modulation.

FM : Here the frequency of the carrier is varied in accordance with the instantaneous value of the amplitude of the modulating signal.

PM : Here the phase of the carrier is varied in accordance with the instantaneous value of the amplitude of the modulating signal.

Let, modulating signal = f(t)

$$\text{Carrier} = A \cos \omega_c t = A \cos \theta(t)$$

For angle modulation, the modulated carried is represented by

$$x_c (t) = A \cos [\omega_c t + \phi(t)]$$

where, A and ω_c are constant and the phase angle $\phi(t)$ is function of the message signal f(t). If we rewrite above equation

$$x_c (t) = A \cos \theta(t)$$

where $\theta(t) = [\omega_c t + \phi(t)]$

Then we can define the instan taneous radian frequency of x_c (t) define by ω_i as

$$\omega_i = \frac{d\,\theta(t)}{dt} = \omega_c + \frac{d\,\phi(t)}{dt} \quad \omega_c + \omega\,\Delta$$

The function $\phi(t)$ and $[d\,\phi(t) / dt]$ are known as the instantaneous phae deviation and instantaneous frequency deviation of $x_c(t)$. The quantity $\Delta\omega = \delta$ define by

$$\Delta\omega = |\,\omega_i - \omega_c\,|_{max}$$

Is called the maximum (or peak) radian frequency deviation of the angle modulated signal

where, $\theta(t) = [\omega_c t + \phi(t)]\ \omega_c \qquad 1 \rightarrow$ frequency

$$\frac{d\,\theta(t)}{dt} = \omega_c \quad \frac{d\,\phi(t)}{dt} \quad ; \quad \phi(t) \rightarrow \text{Phase}$$

$$\theta(t) = \int \left[\omega_c + \frac{d\,\phi(t)}{dt}\right] dt$$

FM : Frequency is varied with f(t)

$\omega'_c = \omega_c + k_f\, f(t) \rightarrow$ instantaneous frequency

$$\omega'_c = \left[\omega_c + \frac{d\,\phi(t)}{dt}\right]$$

$$K_f\, f(t) = \frac{d\,\phi(t)}{dt} \quad ; \quad \phi(t) = \int K_f\, f(t)\, dt$$

Where K_f = the frequency deviation constant expressed in radian/sec for FM

$$\theta'(t) = \int \omega'_c\, dt = \int [\omega_c + K_f\, f(t)]\, dt$$

$$= \int \left[\omega_c + \frac{d\,\phi(t)}{dt}\right] dt$$

FM signal $f(t) = A \cos\theta'(t)$

$$= A \cos \int [\omega_c + K_f\, f(t)]\, dt$$

$f_{FM}(t) = A \cos \omega_c t + K_f \int f(t)\, dt$

PM : The instantaneous phase deviation of the carrier is proportional to the message signal; that is

$\phi(t) = K_p\, f(t)$

$\theta(t) = [\omega_c t + \phi(t)]$

$\theta'(t) = [\omega_c t + K_p\, f(t)]$

$\omega'_c = \omega_c + K_P\, d\,\frac{f(t)}{dt} \rightarrow$ instantaneous frequency

where, K_p is the phase deviation constant expressed in radian/unit of f(t) for PM.

PM signal $= A \cos \theta'(t)$

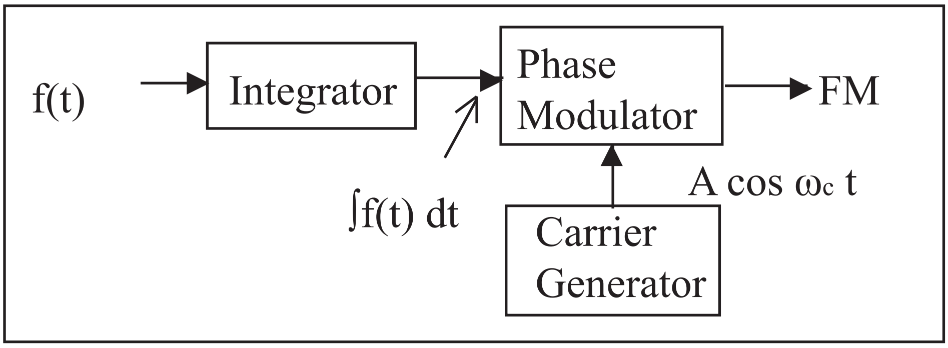
$f_{PM}(t) = A \cos [\omega_c t + K_p\, f(t)]$

$f_{FM}(t) = A \cos [\omega_c t + K_p \int f(t)\, dt]$

Relationship between PM and FM

PM and FM are closely related in the sense that the net effect of both is variation in total angle. In PM the phase angle varies linearly with f(t) where as in FM phase angle varies linearly with the integral of f(t).

1. Indirect Method



2. Direct method of frequency Modulation → Reactance Modulation

Defination of Modulation Index

For FM deviation is proportional to modulating voltage E_m , i.e

$$[\,\Delta\omega = \delta] \propto E_m \quad ; \quad \left(\frac{\delta}{E_m}\right) = K_f$$

$\delta = K_f \,.\, E_m$ Hz or $\delta = K_f \,.\, E_m$ rad

$m_f \,.\, \omega_m = \Delta\omega = \delta = K_f \,.\, E_m$

Definition of Modulation Index for FM

$$m_f = \frac{K_f \,.\, E_m}{\omega_m}$$

For PM

$m_p = (K_p \,.\, E_m)$

In general, modulation index β for angle modulation is defined as

$$\beta = \begin{cases} K_p \;\; E_m & \text{for PM} \\ \dfrac{K_f \;\; E_m}{\omega_c} & \text{for FM} \end{cases}$$

Relation between Δ, f_m , M_cf

δ → Frequency deviation

m_f → Modulation index

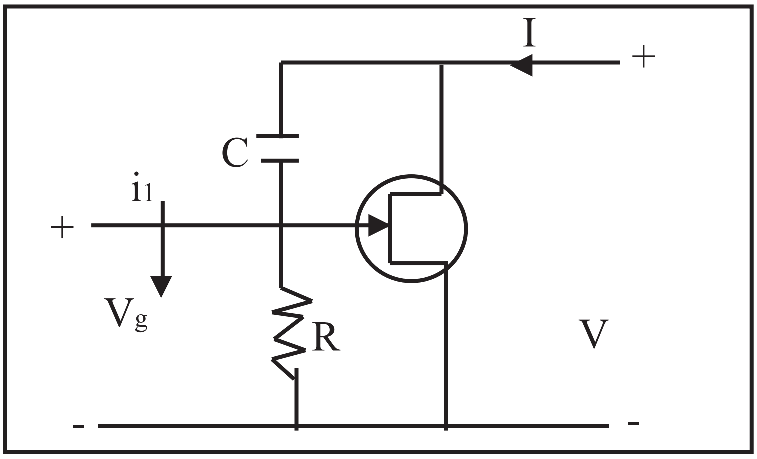
f_m → Modulation index

$\delta = m_f \,.\, f_m$

In general angle modulation index is given by

$$\beta = \frac{\Delta\omega}{\omega_m}$$

Reactance Modulator



$$Z_o = \frac{1}{j\omega C_{eq}}$$

where, $C_{eq} = g_m RC$

Therefore, the output impedance of the given circuit is capacitive

If $X_c >> R$

Let, $\left(\frac{X_c}{R}\right) = n$ and $(n >> 1)$

$$Z_o = \frac{-j X_c}{g_m R} = \frac{1}{j g_m} \frac{X_c}{R}$$

$$Z_o = \left(\frac{n}{jg_m}\right) \quad ; \quad C_{eq} = \left(\frac{g_m}{n\omega}\right)$$

Basic Principle of Reactance Modulator

1. The modulating signal is applied at the gate terminal of the FET.
2. The trans conductance g_m of FET varies w.r.t. to the input voltage V_g or V_m

$$f = \frac{1}{2\pi \sqrt{(LC)}}$$

3. This FET circuit has output impedance which is purely Capacitive in nature, which is equal to $C_{eq} = g_m RC$
4. The C_{eq} is connected effectively in parallel with a fixed frequency carrier oscillator. The frequency of carrier oscillator is given by

$$f = \frac{1}{2\pi \sqrt{(LC)}}$$

where effect of C_{eq} is taken then

$C' = C_{eq} + C_o$

Frequency at the output

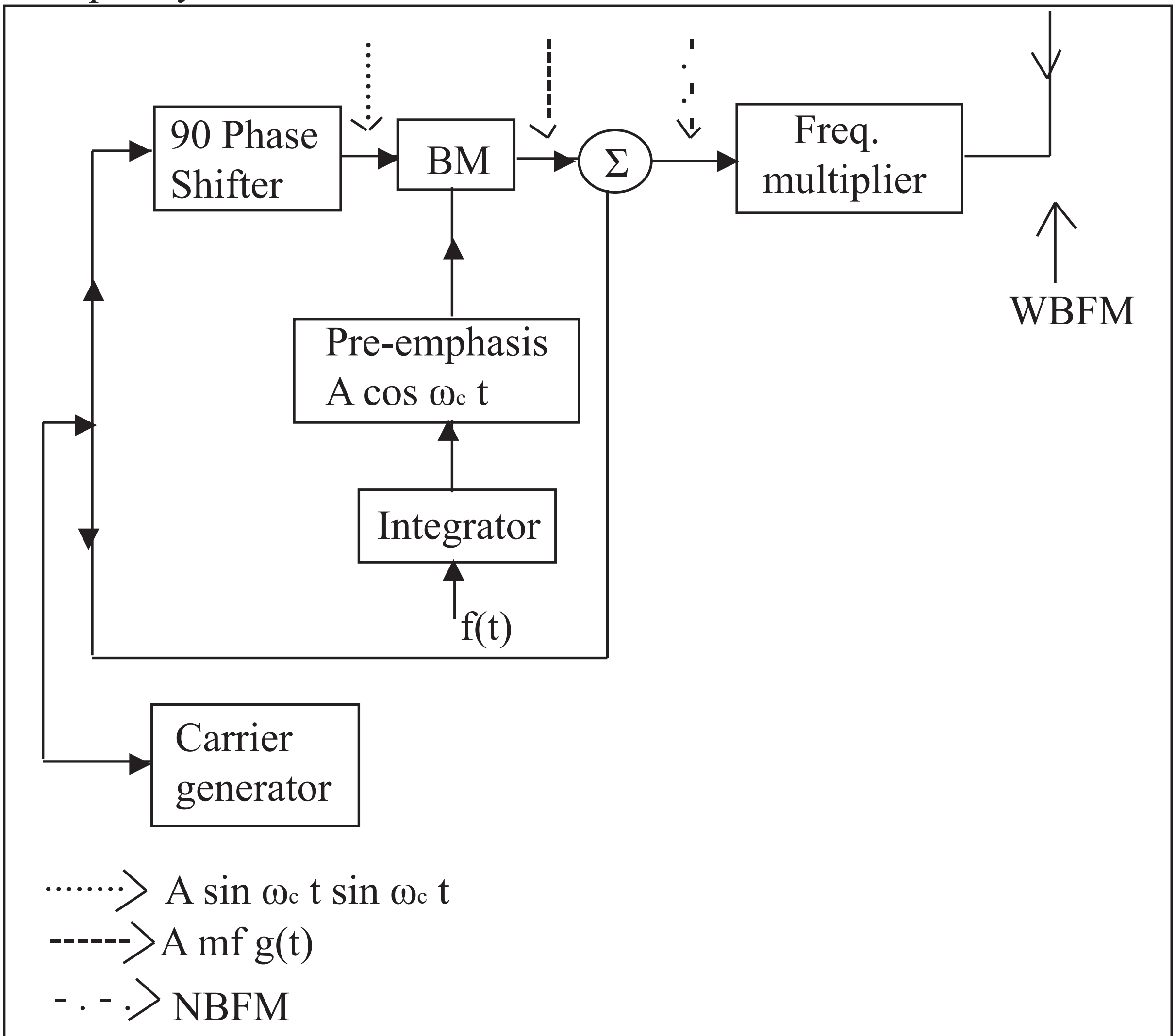
$$= \frac{1}{2\pi \sqrt{[L_o (C_o + C_{eq})]}}$$

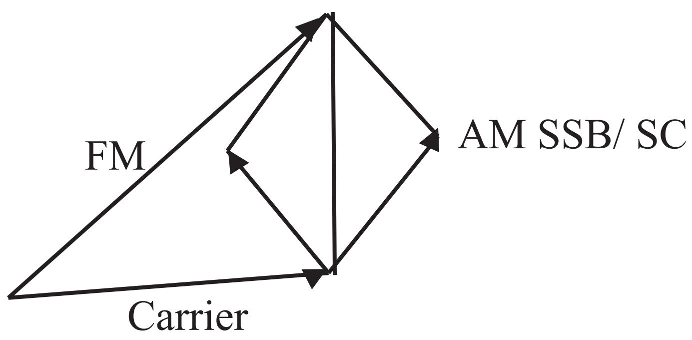
5. Hence the frequency of the Carrier is effectively varied in accordance with the amplitude of input modulating signal $V_m = V_g$
6. The result is a frequency modulated signal.

Armstrong Method

FM Signal Range = 88 MHz to 108 MHz ≅ f_c

Frequency deviation $\delta = \pm 75$ kHz





Generation of narrow Band FM, using Balance Modulator.

Narrow Band FM is multiplied than the result is multiplication of frequency and modulation index which result as a wide Band FM and known as

$$\phi_{FM}(t) = A \cos\omega_c t - A m_f g(t) \sin \omega_c t$$

$$\phi_{PM}(t) = A \cos\omega_c t - A m_f f(t) \sin \omega_c t$$

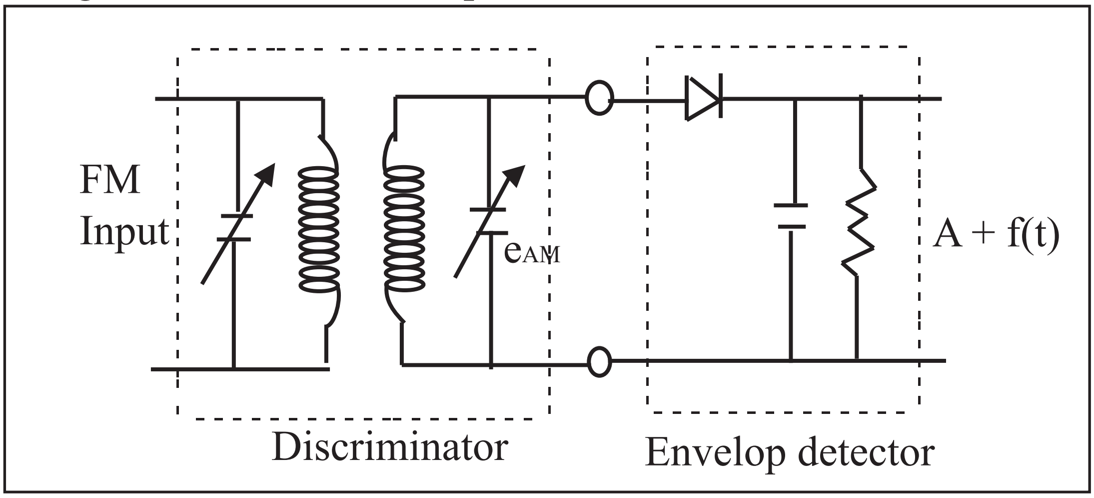
Note : The ecpression for FM and PM are very much similar to the expression of AM signal, with only slight modification. Hence the narrowband FM is same as AM.

AM + NBFM → SSB with carrier

FM Decodulators

The FM detector performs the extraction of modulating signal in two steps :

1. It converts the frequency modulated signal into a corresponding AM signal by using frequency dependent circuit, i.e. circuit whose output voltage depends on input frequency. Such circuit is called as frequency discriminator.
2. The original modulating signal f(t) is recovered from this AM signal by using a linear diode envelop detector.



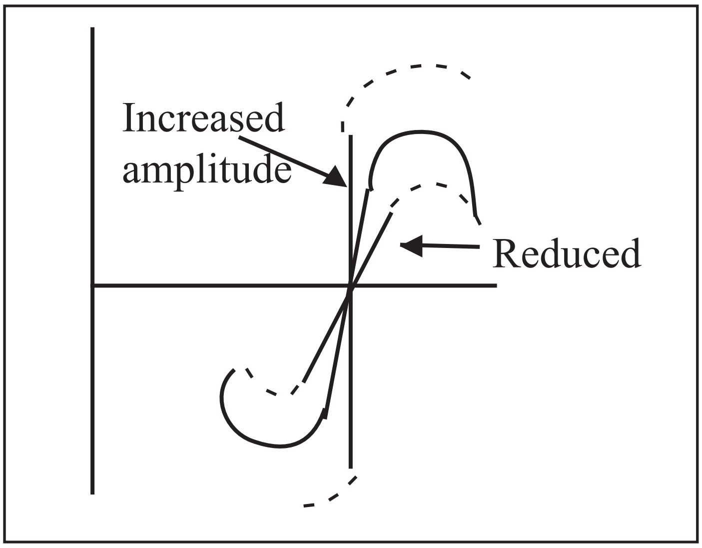
Feature of Foster-Seeley discriminator (phase discriminator)

1. Foster Seeley discriminator is used in FM receiver AFC system. AFC system consist in.
   a. Picking up voltage between the junction of load and the reference terminal.
   b. Apply this DC voltage as the controlling voltage to the reactance modulator which is placed in shunt with the tuned circuit of local oscillator automaticaally in such a way that IF center.
   c. Frequency remanins constant. Separating out the DC component with the help of suitable filter.
2. The discriminator curve is almost linear over a range of ± 200 kHz. The performance is much better than the other type of FM Detector.

ENTRI

Disadvantage

Any variation in the amplitude of the input FM signal due of noise modifies the discriminator characteristic. The distortion is reduced using a limiter circuit which precede the discriminator stage.



Frequency Spectrum of FM Wave

Let, $f(t) = V_m \cos \omega_m t$

carrier $= A \sin \omega_c t$

$f_{FM}(t) = A \cos (\omega_c t + m_f \int f(t)\, dt)$

$f\omega_{FM}(t) = A \sin (\omega_c t + m_f \sin \omega_m t)$

→ It is sin of a sin function

→ The solution for this function involve the use of Bassel function

$f_{FM}(t) = A \sin (\omega_c t + m_f \sin \omega_m t)$

$= A J_o (m_f) \sin \omega_c t + J_1 (m_f)([\sin \omega_c t + \omega_m]t - \sin(\omega_c - \omega_m)t) + J_2 (m_f)$

$([\sin \omega_c t + 2\omega_m]t - \sin(\omega_c - \omega_m)t) \ldots$

It is seen that the output consist of a carrier and apparently infinite sidebands each preceded by J coefficient. These are Bessel function.

Statement of Sampling theorem

1. Frequency domain statement : A band limited signal of finite energy which has no frequency components higher than f_m Hz is completely described by specifying the value of signal at the instant of time separated by

$$T_s = \left(\frac{1}{2f_m}\right) = \left(\frac{1}{2\omega_m}\right) \text{ sec}$$

Sampling Theorem Mathematical Proof

The f(t) is band limited signal which is limited to frequency $\pm\, \omega_m$.

Let, $f(t) \longleftrightarrow F(\omega)$

Also $\delta_{TS}(t)$ = Sampling signal

$$F_s(\omega) = \omega_0 \sum_{-\infty}^{\infty} F(\omega - n\,\omega_o)$$

Ordinary AM

In ordinary AM, the signals can be demodulated by synchronous detector or by envelope detector. The modulated signal has the form.

$$X_c(t) = A_c[1 + m_a x(t)] \cos \omega_c t$$

where ma is the modulation index and $m_a \leq 1$ and $|x(t)| \leq 1$

If the synchronous detector includes an ideal dc suppressor, the receiver output $y_0(t)$ will be

$$y_0(t) = A_c m_a x(t) + n_c(t)$$

$$= x_0(t) + n_0(t)$$

where $x_0(t) = A_c m_a x(t)$ and $n_0(t) = n_c(t)$

So $$\left(\frac{S}{N}\right)_0 = \frac{S_0}{N_0} = \frac{Acm_a^2 S_x}{2\eta B}$$

The input signal power is

$$S_i = \frac{1}{2} EA_c^2[1 + m_a x(t)]^2$$

Since x(t) is assumed to have a zero mean,

$$S_i = \frac{1}{2} A_c^2(1 + m_a^2 S_x)$$

$$S_0 = A_c^2 m_a^2 S_x = \frac{2m_a^2 S_x}{1 + m_a^2 S_x} S_i$$

and $$\left(\frac{S}{N}\right)_0 = \frac{S_0}{N_0} = \frac{m_a^2 S_x}{1 + m_a^2 S_x}\left(\frac{S_i}{\eta B}\right)$$

$$= \frac{m_a^2 S_x}{1 + m_a^2 S_x}\,\gamma$$

Because $m_a^2 S_x \leq 1$

$$\left(\frac{S}{N}\right)_0 \leq \frac{\gamma}{2}$$

Angle Modulation

In angle modulation, the angle of the carrier wave is varied in accordance with the message signal. The amplitude of the carrier wave is maintained constant. An important feature of the angle modulation is that it can provide better discrimination against noise and interference than amplitude modulation. This improvement is achieved at the expense of increased transmission bandwidth.

The modulated carrier can be expressed as

$$s(t) = Ac \cos(2\pi f_c\, t + \phi(t)]$$

where Ac and f_c are constants and $\phi(t)$ is a function of the message signal m(t).

$$s(t) = Ac \cos\theta(t)$$

where $\theta(t) = 2\pi f_c\, t + \phi(t)$

The instantaneous radian frequency of s(t) is

$$\omega_i = \frac{d\theta(t)}{dt} = 2\pi f_c + \frac{d\varphi(t)}{dt}$$

$$\text{or } f_i = \frac{1}{2\pi}\frac{d\varphi(t)}{dt}$$

$$\omega_i - \omega_c = \frac{d\varphi(t)}{dt}$$

The function $\phi(t)$ is called the instantaneous phase deviation and $\frac{d\varphi(t)}{dt}$ is called instantaneous frequency deviation. The maximum frequency deviation is given by

$$|\,\omega_i - \omega_c\,|_{max} = \left|\frac{d\varphi(t)}{dt}\right|_{max}$$

$$\text{Or} \quad |\,\Delta f\,|_{max} = \frac{1}{2\pi}\left|\frac{d\varphi(t)}{dt}\right|_{max}$$

The two basic types of angle modulation are phase modulation and frequency modulation. In P.M, the instantaneous phase deviation of the carrier is proportional to the message signal, i.e. $\phi(t) = K_p\, m(t)$, where K_p is the phase deviation constant expressed in radians per unit of m(t).

In FM, the instantaneous frequency deviation of the carrier is proportional to the message signal, i.e. $\frac{d\varphi(t)}{dt} = K_f\, m(t)$ or

$$\phi(t) = K_f \int_{t_0}^{t} m(\lambda)\, dx + \varphi(t_0)$$

Where K_f is the frequency deviation constant, expressed in radians per second per unit of m(t) and $\phi(t_0)$ is the initial phase angle at $t = t_0$. It is usually assumed that $t_0 = -\infty$ and $\phi(-\infty) = 0$.

Thus the PM signal can be expressed as

$$s(t) = Ac \cos[\omega_c\, t + K_p\, m(t)]$$

The FM signal can be expressed as

$$s(t) = Ac \cos [\omega_c t + K_f \int_{-\infty}^{t} m(\lambda)\, dx$$

For P.M

$$\omega_i = \omega_c + K_p \frac{dm(t)}{dt}$$

For F.M

$$\omega_i = \omega_c + K_f\, m(t)$$

Thus in PM, the instantaneous frequency ω_1 varies linearly with the deviation of the modulating signal and in FM ω_1 varies linearly with the modulating signal.

Narrow Band Angle Modulation

The angle modulated signal is

$$s(t) = Ac \cos [\omega_c t + \phi(t)]$$

$$s(t) = \text{Re } [Ac \underset{e}{J} (\omega_c + \phi(t)]$$

$$= \text{Re } [Ac\, e^{j\omega ct}\, e^{j\phi(t)}]$$

$$= \text{Re } \left\{ A_c e^{j\omega ct} \left[1 + j\phi(t) - \frac{\phi^2(t)}{2!} - \ldots\ldots + j^{n} \frac{\phi^{n}(t)}{nt} + \ldots\ldots \right] \right\}$$

$$= A_c [\cos \omega_c t - \phi(t) \sin \omega_c t - \frac{\phi^2(t)}{2!}$$

$$\text{Cos}\omega_c t + \frac{\phi^3(t)}{2!} \sin \omega_c t + \ldots\ldots]$$

If $\phi(t) << 1$, the higher terms of $\phi(t)$ can be neglected.

Then $s(t) = A_c \text{ Cos } \omega_c t + A_c\, \phi(t) \text{ Sin } \omega_c t$

For P.M, $\phi(t) = K_p\, m(t)$

So the narrow band P.M is

$$s(t) = A_c \cos \omega_c t + A_c K_p\, m(t) \text{ Sin } \omega_c t$$

For F.M, $\phi(t) = K_f \int_{-\infty}^{t} m(\lambda)\, d\lambda$

So the narrow band F.M is

$$s(t) = A_c \cos \omega_c t - A_c \left[\int_{-\infty}^{t} m(\lambda)\, dx \right] \text{Sin } \omega_c t$$

Sinusoidal Modulation

If the message signal m(t) is pure sinusoid, that is

$$m(t)\begin{cases} A_m \operatorname{Sin}\omega_m t \text{ for PM} \\ A_m \operatorname{Cos}\omega_m t \text{ for FM} \end{cases}$$

$$s(t) = A_c \operatorname{Cos}[\omega_c t + K_p A_m \operatorname{Sin}\omega_m t] \text{ for P.M}$$

$$s(t) = A_c \operatorname{Cos}[\omega_c t + \beta \operatorname{Sin}\omega_m t]$$

Where $\beta = K_p A_m$ is the modulation index for P.M

$$s(t) = A_c \operatorname{Cos}\left[\omega_c t + K_f \int_{-\infty}^{t} A_m \operatorname{Cos}\omega_m \lambda \, d\lambda\right] \text{ for F.M}$$

$$= A_c \operatorname{Cos}\left[\omega_c t + \frac{K_f A_m}{\omega_m} \operatorname{Sin}\omega_m t\right]$$

$$s(t) = A_c \operatorname{Cos}[\omega_c t + \beta \operatorname{Sin}\omega_m t]$$

$$\text{where } \beta = \frac{K_f A_m}{\omega_m} \text{ for F.M}$$

The β is defined only for sinusoidal modulation and can be expressed as

$$\beta = \frac{\Delta\omega}{\omega_m} \text{ or } \frac{\Delta f}{f_m}$$

where Δf is the maximum frequency deviation

Wide Band Modulation

In a sinusoidal angle modulation, the modulated signal can be expressed as

$$s(t) = A_c \operatorname{Cos}(\omega_c t + \beta \operatorname{Sin}\omega_m t)$$

It can be written as $s(t) = A_c \operatorname{Re}(e^{j\omega_c t} e^{j\beta \operatorname{Sin}\omega_m t})$

The function $e^{j\beta \operatorname{Sin}\omega_m t}$ is a periodic function with period

$T_m = \frac{2\pi}{\omega_m}$. By using fourier series

$$e^{j\beta \operatorname{Sin}\omega_m t} = \sum_{n=-\infty}^{+\infty} C_n e^{-jn\omega_m t}$$

The fourier coefficient C_n can be written as

$$C_n = \frac{\omega_m}{2\pi} \int_{-\pi/\omega_m}^{+\pi/\omega_m} e^{j\beta \operatorname{Sin}\omega_m t} e^{-jn\omega_m t} \, dt$$

Put $\omega_m t = x$

$$C_n = \frac{1}{2\pi} \int_{-\pi}^{+\pi} e^{j(\beta \operatorname{Sin}x - nx)} \, dx$$

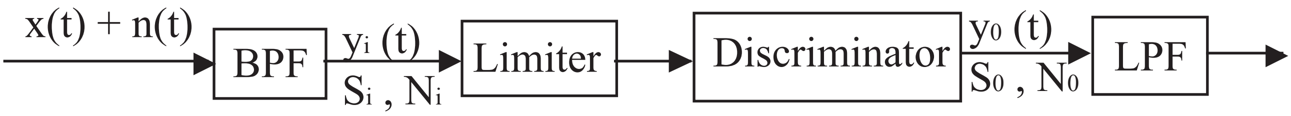
$$= J_n(\beta)$$

Signal To Noise Ratio (SNR)

The transmitted signal X_c (t) can be expressed as

$$X_c(t) = Ac\ \text{Cos}\ [\ \omega_c t + \phi(t)]$$

$$\text{Where } \phi(t) = \begin{cases} k_p X(t) \text{ for PM} \\ k_f \int_{-\infty}^{t} x(\lambda)\, d\lambda \text{ for FM} \end{cases}$$



The figure shows a modal for the angle demodulation system. The predetection filter bandwidth is approximately $B_T = 2(D + 1)$, where D is the deviation ratio and B is the bandwidth of the message signal. The detector input is

$$y_i(t) = X_c(t) + n_i(t)$$

$$= Ac\ \text{Cos}\ [\omega_c(t) + \phi(t)] + n_i(t)$$

The carrier amplitude remains constant, therefore

$$S_i = E[X^2c(t)] = \frac{1}{2} Ac^2 \text{ and } N_i = \eta B_T$$

$$\text{So } \left(\frac{S}{N}\right)_i = \frac{Ac^2}{2\eta B_T}$$

Because n_i (t) is narrow band,

$$n_i(t) = V_n(t)\ \text{Cos}\ [\omega_c t + \phi_n(t)]$$

where V_n (t) is Rayleigh - distributed and ϕ_n (t) is uniformly distributed [0, 2π]

$$y_i(t) = v(t)\ \text{Cos}\ [\omega_c t + \theta(t)]$$

where $v(t) = \{[Ac\text{Cos}\phi + v_n(t)\ \text{Cos}\phi_n(t)]^2 + [Ac\ \text{Sin}\phi + v_n(t)\ \text{Sin}\phi_n(t)]^2\}^{1/2}$

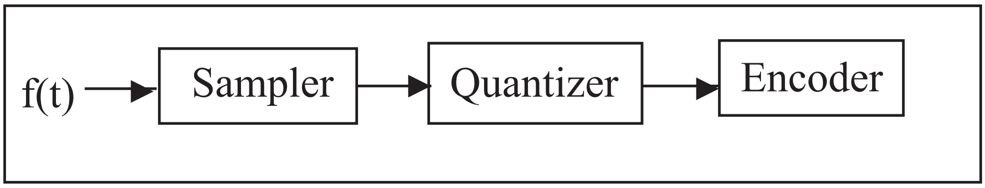
$$\text{And } \theta(t) = \tan^{-1} \frac{Ac\text{Sin}\phi + V_n(t)\ \text{Sin}\phi_n(t)}{Ac\text{Cos}\phi + V_n(t)\ \text{Cos}\phi_n(t)}$$

The limiter suppresses any amplitude variation in v(t). Hence in angle modulation SNR are derived from consideration of θ(t) only. The detector is assumed to be ideal.

PULSE CODE MODULATION

Basic Principle of PCM

The essential processes of PCM are sampling quantizing and encoding as shown in figure.



Sampling is the process in which a continuous time signal is sampled by measuring its amplitude t discrete instants. Representing the sampled values of the amplitude by a finite set of levels is called quantizing. Designating each quantized level by a code is called encoding.

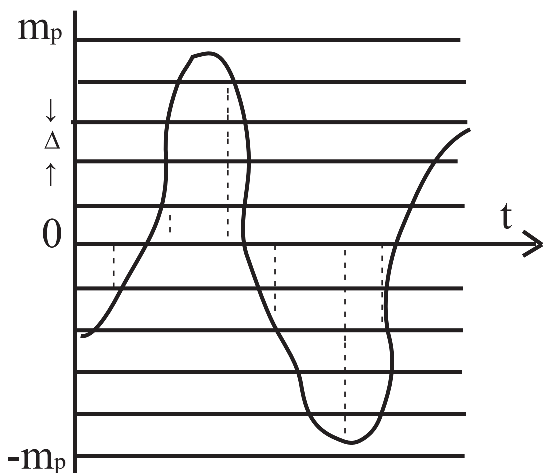
While sampling converts a continuous time signal to a discrete-time signal, quantizing converts a continuous amplitude sampke to a discrete-amplitude sample. Thus sampling and quantizing operations transform an analog signal to a digital signal.

The quantizing and encoding operations are usually performed in the same circuit, which is called analog-to-digital (A/D) converter. The combined use of quantizing and encoding distinguishes PCM from analog pulse modulation techniques.

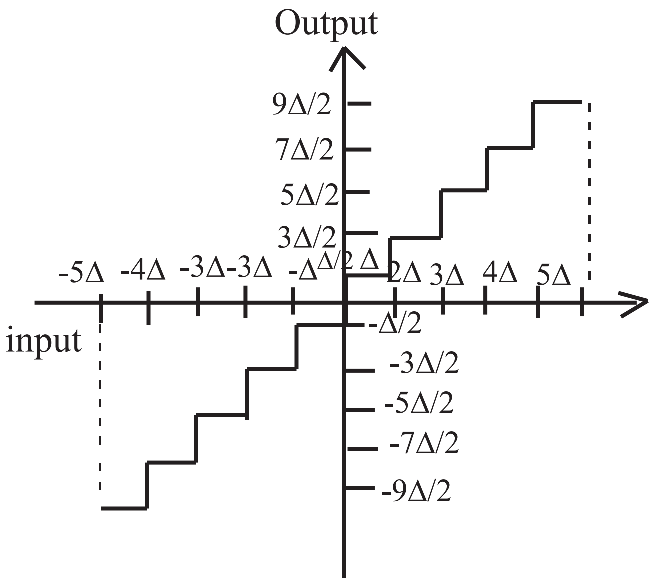
In the following sections, we discuss the operations of sampling, quantizing, and encoding.

A. Uniform Quantizing

An example of the quantizing operation is shown in figure. We assume that the amplitude of m(t) is confined to teh range ($-m_p$, m_p). As illustrated in figure, this range is divided in L.



(a)



Uniform Quantizing

Uniform Quantizing

Zones, each of step size ($\Delta = s$) given by

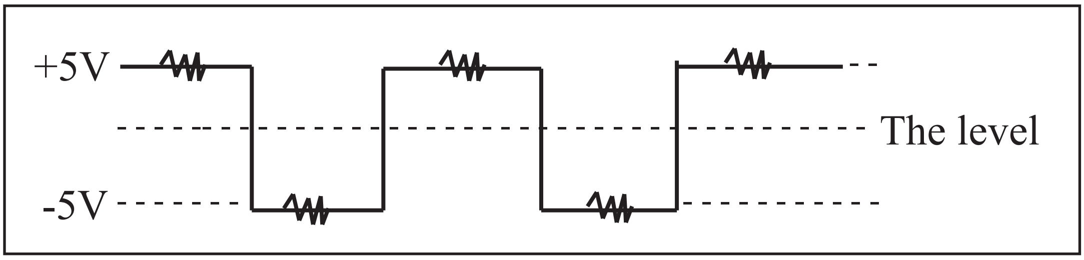
$$\Delta = \frac{2m_p}{L} \qquad (1)$$

where $s = \Delta$ and $L = M$

A sample amplitude value is approximated by the midpoint of the interval in which it lies. The input- output characteristics of a uniform quantizer are shown in above figure.

Reception of PCM Signal

Basic principle



$S/N \cong 2^{2n}$ where n = number of bits sample

If [n = 3] : $(S/N) = 2^{2(3)} = 2^6 = 64$

BW = (1/2) [Bit rate]

$= (1/2)\,[\,nf_s\,]\,;\ f_s = 2\,f_m$

Quantization error = $(\Delta^2/12)$ where, Δ = step size

Number of steps = 2^n = number of levels = M

$$S/N = 2^{2n} = (2^n)^2 \; M^2$$

$$\therefore M = 2^n$$

$$S/N = M^2$$

If total range of input $\pm V$

the step-size $S = \frac{2V}{2^n} = \frac{2V}{M}$

if $n \uparrow \rightarrow S/N \uparrow$

$\rightarrow BW \uparrow$

$\rightarrow$ Design of quantizer becomes complex

$\rightarrow$ Quantization error $\downarrow$

Quantizing Noise

The difference between the input and output signals of the quantizer becomes the quantizing error, or quantizing noise. It is apparent that with a random input signal, the quantizing error q_e varies randomly within the interval $(s = \Delta)$.

$$\frac{-\Delta}{2} \leq q_e \leq \frac{\Delta}{2} \qquad (2)$$

Assuming that the error is equally likely to lie anywhere in the range $(-\Delta/2, \Delta/2)$, the mean-square quantizing error (q_c^2) is given by

$$q_c^2 = 1 \int_{-\Delta/2}^{\Delta/2} q_e^2 \, dq_e = \frac{\Delta^2}{12} \qquad (3)$$

Substituting equation (1) into equation (3) we have

$$(q\epsilon^2) = \frac{m_p^2}{3L^2} \qquad (4)$$

The peak-to-peak excursion of the quantizer input is 2A. Form Equation. The quantizer step size is

$$\Delta = -\left(\frac{2S}{L}\right)$$

Then from equation, the average quantizing noise power is

$$N_q = q_e^2 = \frac{\Delta^2}{12} = \left(\frac{2A}{L}\right)^2 \times 1 = \frac{A^2}{3L^2}$$

Signal power $S_o = \left(\frac{A^2}{2}\right)$

The output signal to Quantizing noise ratio of PCM system for a full-scale test tone is therefore

$$(SNR)_0 = \left(\frac{S}{N_q}\right)_0 = \frac{(A^2/2)}{A^2/(3L^2)} = \frac{3}{2} L^2$$

Expressing this in decibels, we have

$$\left(\frac{S}{N_q}\right)_0 dB = 10 \log \left(\frac{S}{N_q}\right)_0 = 1.76 + 20 \log L$$

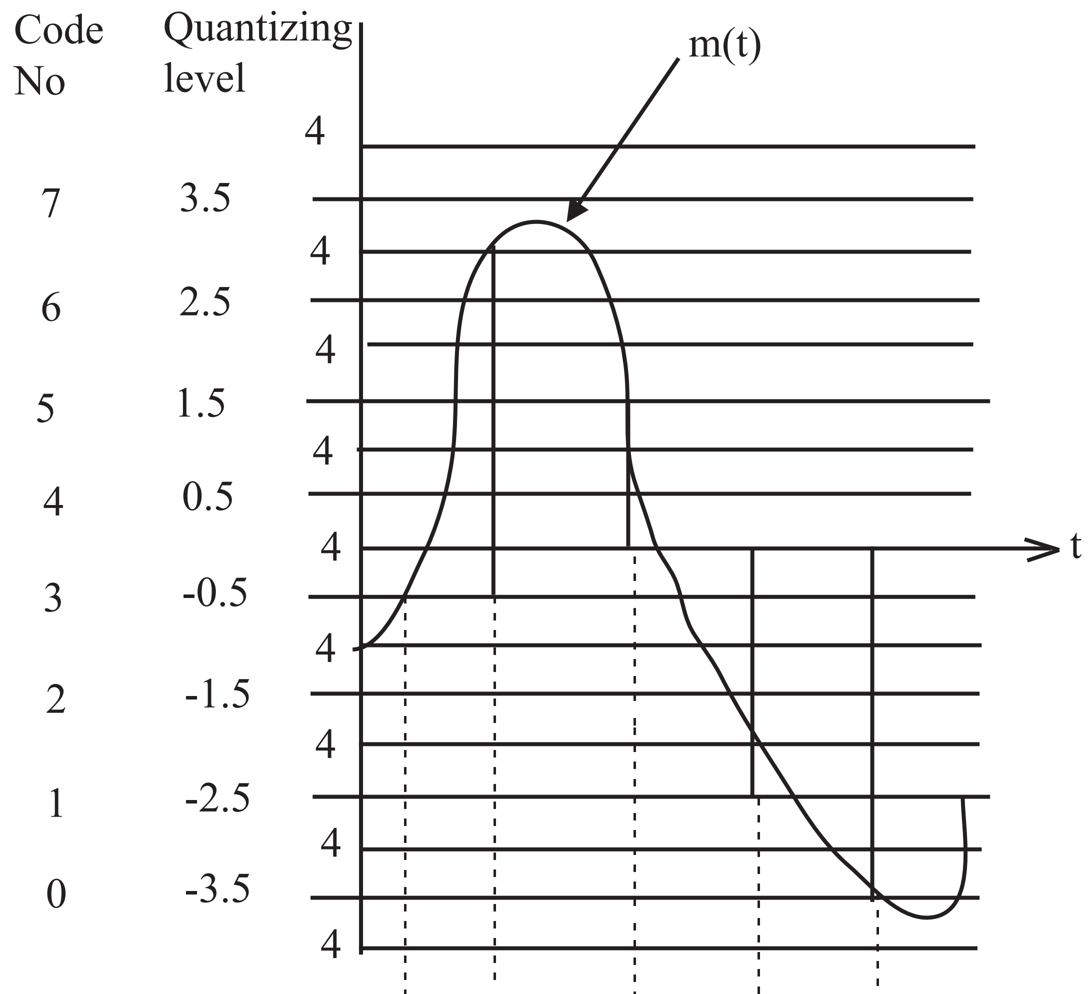
$$= 1.76 + 20 \log 2^n \; [L = 2^n]$$

$$= 1.76 + 6.02 \; xndB$$

Where L = M is the number of Quantizing levels.

Above equation indicates that each bit in the code word of a binary PCM system contributes 6 dB to the output signal-to-Quantizing ratio. This is called the **6 dB rule.**

Encoding



Sample value	-0.25	3.1	1.5	-2.4	-3.7
Quantized value	-0.5	3.5	1.5	-2.5	-3.5
Code No	3	7	5	1	0
Binary code	011	111	101	001	000

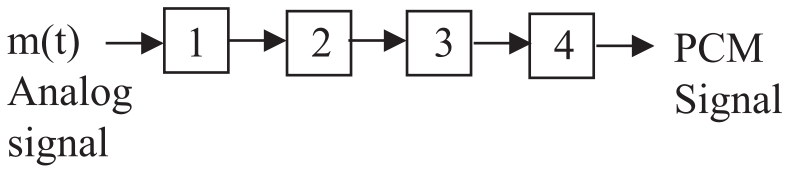
An encoder in PCM translates the quantized sample into a code number. Usually the code number is converted to its representation in binary sequence. The binary sequence is converted to a sequential string of pulses for transmission. In this case the system is referred to as binary PCM. The essential features of binary PCM are shown in Figure.

Bandwidth Requirements of PCM

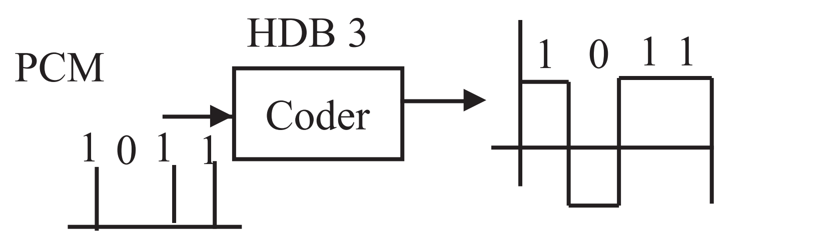
$$2f_{PCM} = nf_s$$

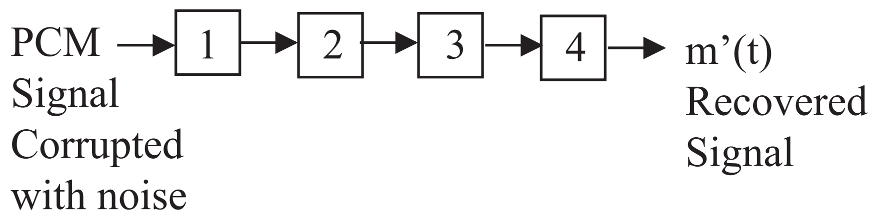
$$\text{or,}\quad f_{PCM} = \frac{n\,f_s}{2} \geq n\,f_m\ \text{Hz}$$

PCM Encoder



(1) Compressor (2) Sampler
(3) Quantizer (4) Binary Encoder





1. Limiter or 2. Level quantizer
2. Decoder 3. Expander 4. LPF

1. A PCM system has very high S/N ratio but at the cost of complex circuit. Specially design of is quantizer most complex.
2. To reduce the complexity of the circuit and still maintain same S/N ratio we use another coded modulation called delta modulation.

Advantages of PCM

1. PCM offers a considerable increase in S/N ratio at the receiver.
2. The decision by receiver is to take pulse is available or not; nothing to be known concerning its amplitude width etc.
3. PCM permits amplification of encoded signal without significant distortion.
4. PCM system promot flexibility and increasing system utilization design for one system can be used for the other too.
5. Using many repeater station PCM communication range can be increased.
6. PCM/ TDM system applicable
7. PCM-AM is common.

Disadvantage

PCM requires large bandwidth compare to analog system.

DIGITAL MODULATION

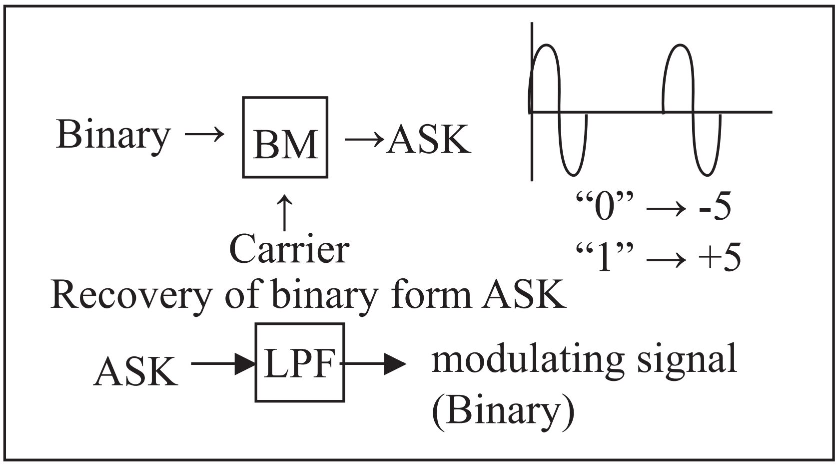
Amplitude-shift Keying (ASK)

In ASK, the modulated signal can be expressed as

$$x_c(t) = \begin{cases} A\cos\omega_o t & \text{symbol } 1 \\ 0 & \text{symbol } 0 \end{cases}$$

Note that the modulated signal is still an on-off signal. Thus ASK is also known as on-off keying.

ASK (Amplitude Shift Keying)



Not in use due to fading & does not give a satisfactory value for probability of error.

Frequency Shift Keying and Pulse Shift Keying For Digital

Frequency-Shift Keying (FSK)

In FSK, the modulated signal can be expressed as

$$x_c(t) = \begin{cases} A\cos\omega_1 t & \text{Symbol 1} \\ A\cos\omega_2 t & \text{Symbol 0} \end{cases}$$

FSK (Frequency shift keying)

$$= A\cos(\omega'_c + d(t)\,\omega')\,t$$

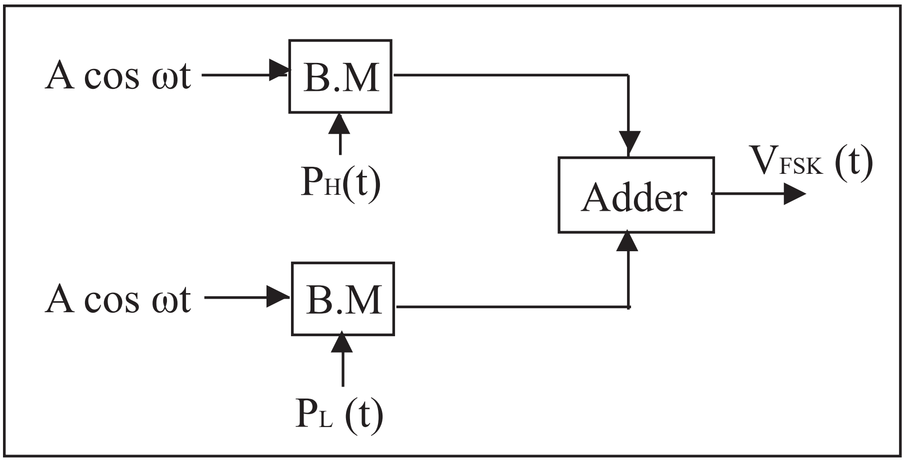
$$V_{PSK} = A\cos(\omega_o \pm \omega')t$$

$$= A\cos(\omega_o + \omega') \rightarrow \text{“1”}$$

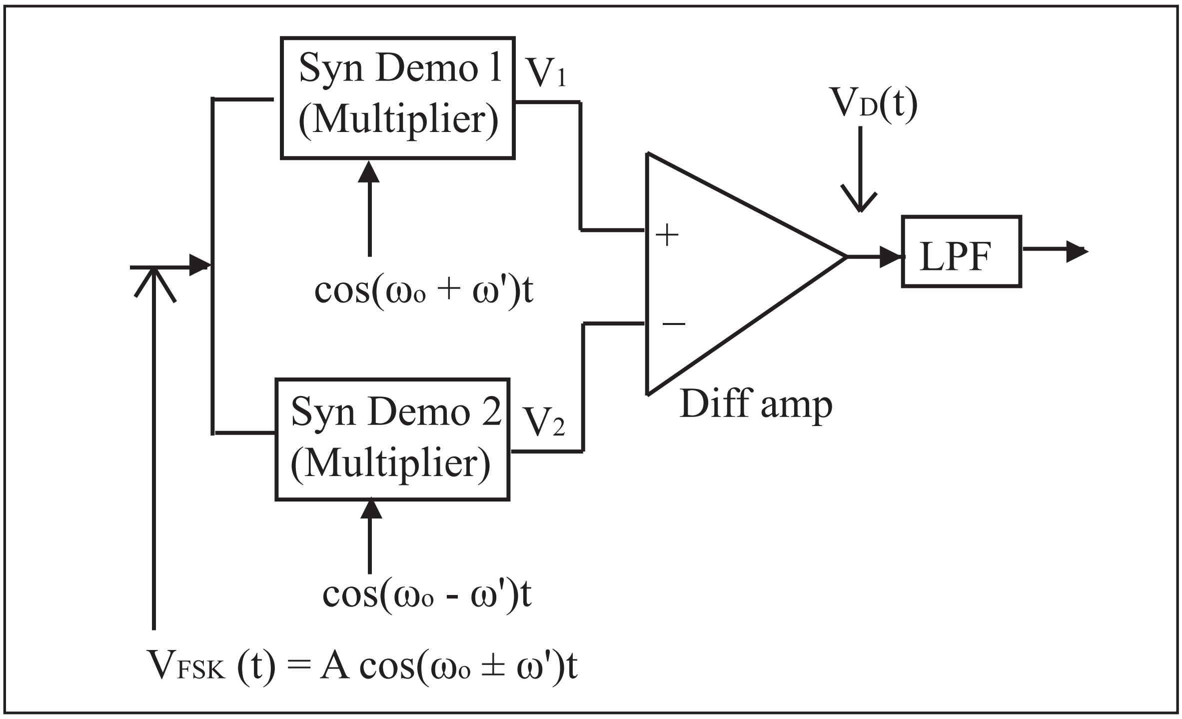
$$= A\cos(\omega_o - \omega')t \rightarrow \text{“0”}$$

$\omega_H = \omega_o + \omega'$; $\omega_L = \omega_o - \omega'$

d(t)	$P_H(t)$	$P_L(t)$
+1	+1 V	0 V
-1	0 V	+1 V



Demodulation



BPSK (Binary Phase Shift Keying)

$V_{PSK}(t) = A\cos(\omega_c t + \phi)$

Let $\phi = 0^o$; $V_{PSK}(t) = A\cos\omega_c t$

$\phi = 180^o$; $V_{PSK}(t) = A\cos\omega_c t$

Power of binary signal $P_S = \left(\frac{1}{2}\right)A^2$

$$A = \sqrt{2P_S}$$

Modulating signal $v(t) = \pm V$

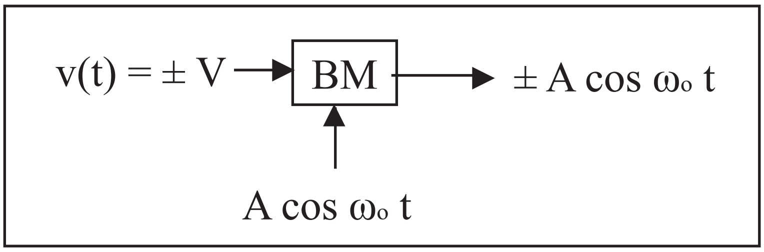
$\phi = 0^o$

$V_{PSK}(t) = A\cos\omega_c t$

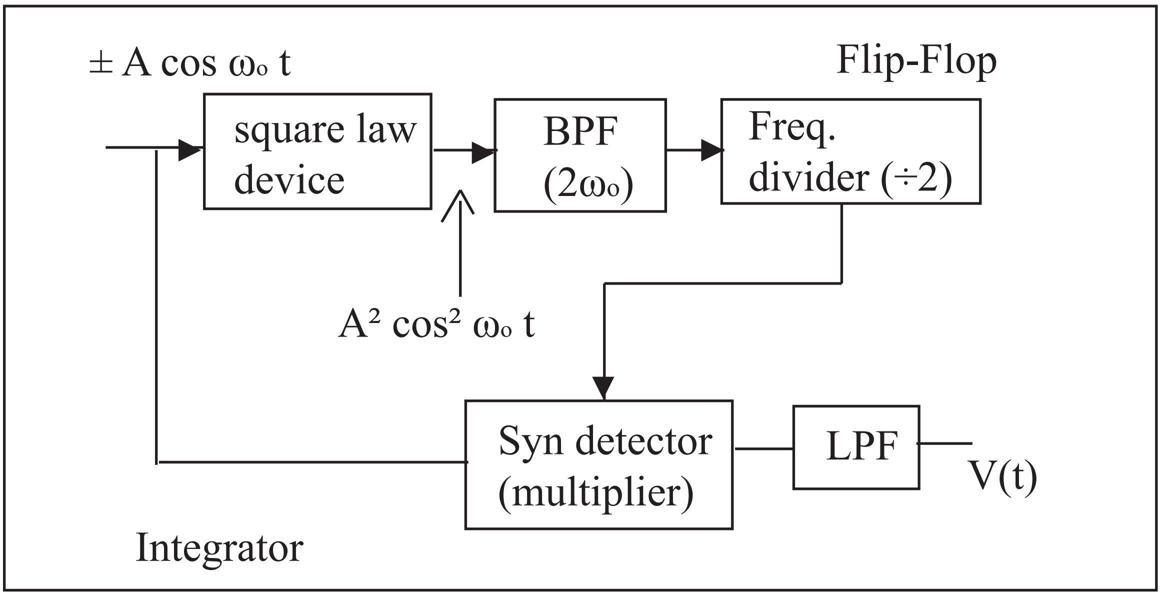
$V_{PSK} = \frac{v(t)}{V} A\cos\omega_c t$

if $v(t) = +V(1)$ $V_{PSK}(t) = A\cos\omega_c t$

$v(t) = -V(0)$ $V_{PSK}(t) = A\cos\omega_c t$



Recovery of PCM signal form BPSK



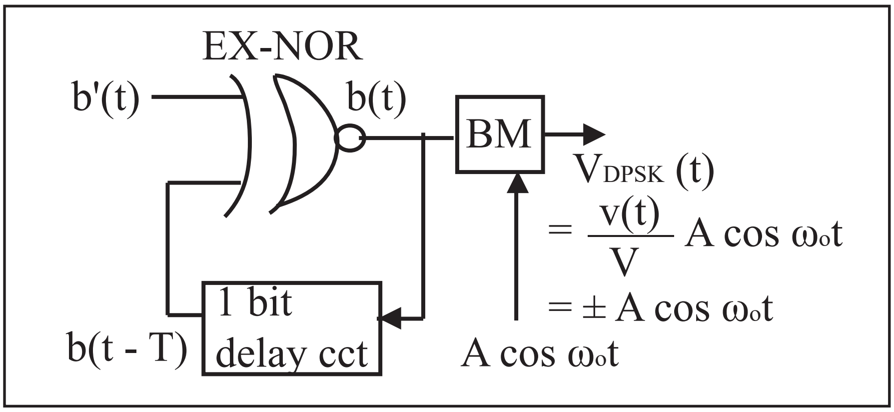
Advantage of PSK

1. Smaller Bandwidth
2. Signaling speed fastest up to 2400 baud.
3. It is better for working under noisy condition.

Disadvantage of PSK

We need a synchronous detector and the associated circuit so that the receiver configuration of PSK system becomes more complex. To avoid this we use differential PSK i.e. (DPSK) system which does not need synchronous detector.

DPSK (Differential Phase Shift Keying)



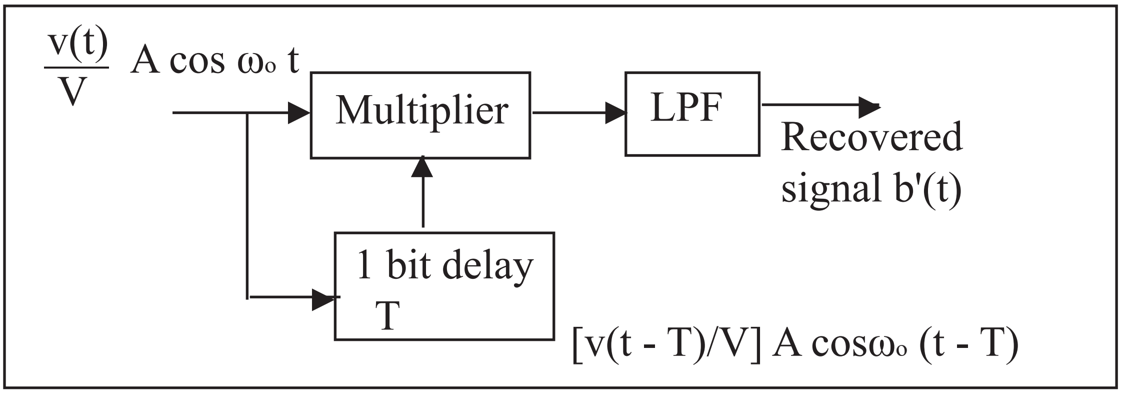
$b'(t)$ = input modulating bit stream

$b(t)$ = arbitrary generated bit stream

ω_o = carrier frequency

T = delay equal to period of 1 bit

Demodulation of DPSK



Characteristic of DPSK

1. The DPSK system avoids the use of synchronous detector. This is possible by generation of arbitrary waveform b(t) at the modulator. The generation is simple.
2. The error rate at DPSK is more so that S/N ratio is less than PSK system. Bit error occur in pair or single.

Quadrature PSK (QPSK)

[+ 45° + 135° - 135° - 45°]

We have seen that when a data stream whose bit duration is T_b is to be transmitted by PSK the channel bandwidth must be normally $2f_b$ where $f_b = (1/T_b)$.

QPSK allows bits to be transmitted using the half bandwidth required by PSK.

$V(t) = b_e(t) A \cos \omega_o t + b_o(t) A \sin \omega_o t$

for PSK $BW = 2 \times (1/T_b)$

for QPSK $BW = 2 \times (1/2T_b)$

PSK BW = (1/2) FSK BW

QPSK BW = (1/2) PSK BW

= (1/4) FSK BW

when $b_o = +1$; $S_o(t) = A \sin \omega_o t$

$b_o = -1$; $S_o(t) = -A \sin \omega_o t$

and $b_e(t) = +1$; $S_e(t) = A \sin \omega_o t$

$b_e(t) = -1$; $S_e(t) = -A \sin \omega_o t$

These four signals are represented as phase