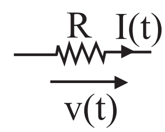
NETWORK THEOREMS

The fundamental laws that govern electric circuits are the Ohm's Law and the Kirchoff's Laws.

Ohm's Law

* Ohm's Law states that the voltage v(t) across a resistor R is directly proportional to the current I(t) flowing through it.

$v(t) \propto I(t)$ or $v(t) = R.I(t)$



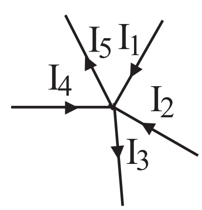
KIRCHOFF'S CURRENT LAW (KCL)

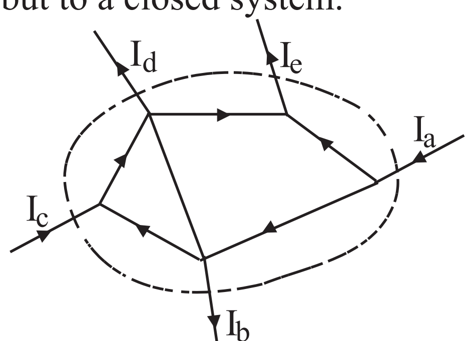
* Kirchoff's current law states that in a node, sum of entering current is equal sum of leaving current. i.e., $\sum I$ at junction point $= 0$
* The theorem is applicable not only to a node, but to a closed system.

$$I_1 + I_2 - I_3 + I_4 - I_5 = 0$$
$$I_1 + I_2 + I_4 = I_3 + I_5$$

Also for the closed boundary,

$$I_a - I_b + I_c - I_d - I_e = 0$$



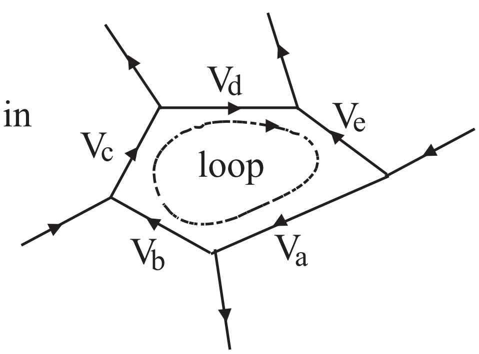


* Kirchoff's voltage law states that the algebraic sum of the voltages around any closed path is zero.

i.e., Around a closed path $\sum V = 0$

Where V are the voltages across the branches in the loop.

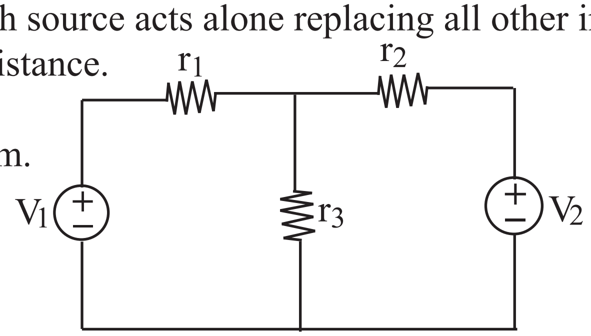
$$v_a + v_b + v_c + v_d - v_e = 0$$



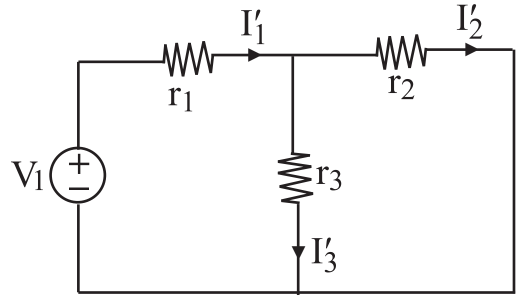
Superposition Theorem

* If a numbers of voltage or current sources are acting simultaneously in a linear network, the resultant current in any branch is the algebraic sum of the currents that would be produced in it, when each source acts alone replacing all other independent sources by their internal resistance.

* In figure to apply superposition theorem.



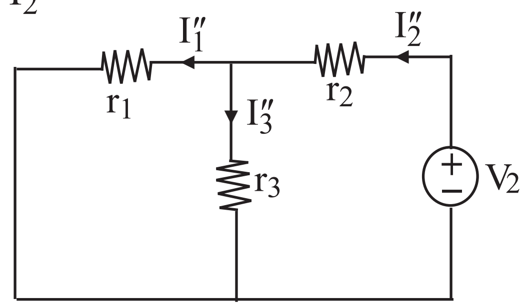
* Let us first take the source V_1 alone at first replacing V_2 by short circuit.



Here, $I'_1 = \dfrac{V_1}{\dfrac{r_2 r_3}{r_2 + r_3} + r_1}$

$I'_2 = I'_1 \dfrac{r_3}{r_2 + r_3}$ and $I'_3 = I'_1 - I'_2$

* Now, removing V_1 by short circuit, let the circuit be energized by V_2 only.



Here, $I''_2 = \dfrac{V_2}{\dfrac{r_1 r_3}{r_1 + r_3} + r_2}$

and $I''_1 = I''_2 \dfrac{r_3}{r_1 + r_3}$ $I''_3 = I_2'' - I_1$

As per superposition theorem, $I_3 = I'_3 + I''_3$ $I_2 = I'_2 - I''_2$
$I_1 = I'_1 - I''_1$

THEVENIN'S THEOREM (or Helmholtz's Theorem)

* Any two terminal bilateral linear d.c. circuit can be replaced by an equivalen circuit consisting of a voltage source and a series resistor.

* Let us consider a simple d.c. circuit as shown in figure (a). We find I_1 by Thevenin's Theorem.

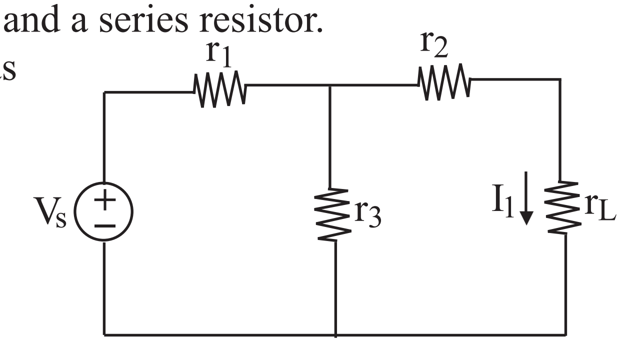


Figure (a)

* Find the equivalent voltage source then r_1 is removed figure (b) and $V_{O.C}$ is calculated

$V_{O.C} = Ir_3 = \dfrac{V_s}{r_1 + r_3}.r_3$ .... (i)

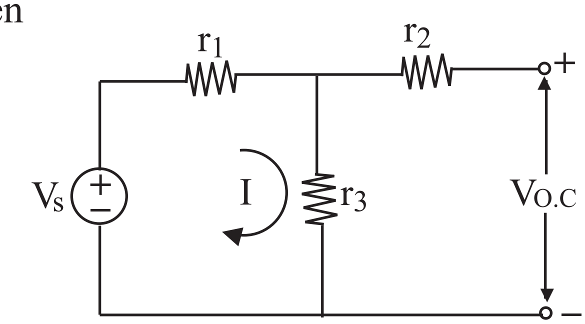


Figure (b)

- Now to find the internal resistance of the network . as shown in figure (c)

$$R_{th} = r_2 + \frac{r_1 r_3}{r_1 + r_3} \quad .....(ii)$$

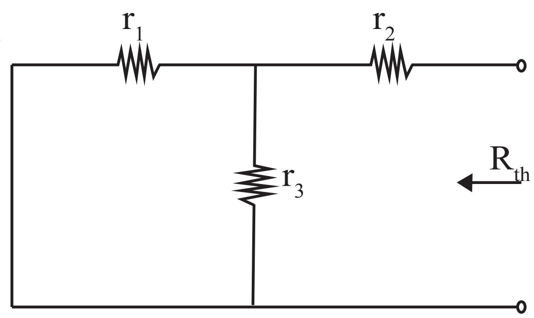


Figure (c)

- As per Thevenin's Theorem, the equivalent circuit being figure (d).

  From equation (i) and (ii)

$$I_1 = \frac{V_{OC}}{R_{Th + rL}} \text{ Amp.}$$

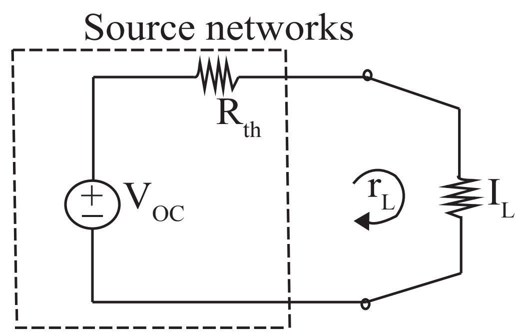


Figure (d)

Norton's Theorem

A linear active network consisting of independent and or dependent voltage and current source and linear bilateral network elements can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance, the current source being the short circuited current across the load terminal and the resistance being the internal resistance of the source network looking through the open circuited load terminals.

A simple d.c. network

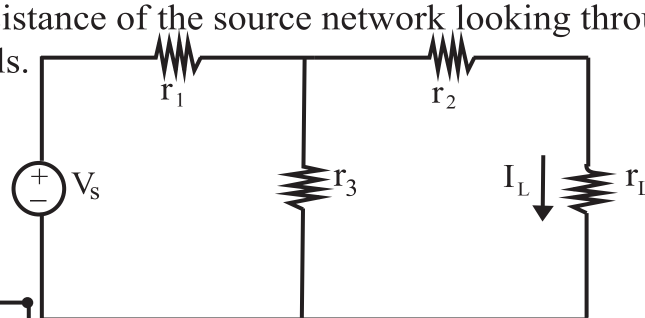


Figure (a)

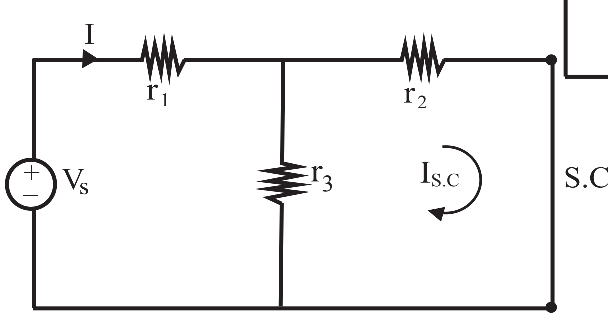
- 

Figure (b) Finding of $I_{S.C}$

$$I = \frac{V_S}{r_1 + \frac{r_2 r_3}{r_2 + r_3}}$$

and $$I_{S.C} = I \frac{r_3}{r_3 + r_2}$$

* R_{int} (Internal Resistance) is

$$R_{int} = r_2 + \frac{r_1 r_3}{r_1 + r_3}$$

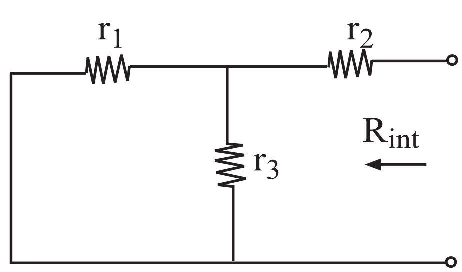


Figure (c) Finding of R_{th} (or R_{int})

* To find load current I_L is

$$I_L = I_{S.C} \frac{R_{int}}{R_{int} + r_L}$$

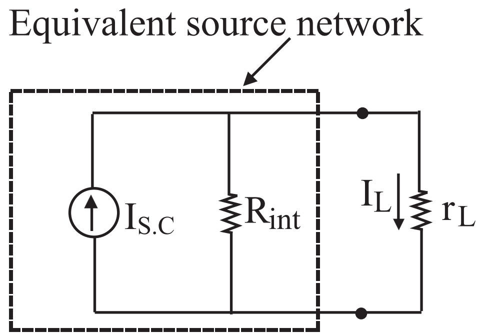


Figure (d) Norton's equivalent circuit

Maximum Power Transfer Theorem

* According to maximum power transfer theorem a resistance load, being connected to a d.c. network, receives maximum power when the load resistance is equal to the internal resistance of the source network as seen from the load terminals.

with reference to figure

$$I = \frac{V_0}{R_{th} + R_L} \quad \text{.... (i)}$$

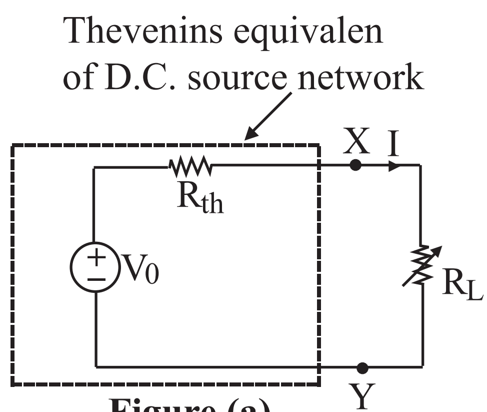


Figure (a)

While the power delivered to the resistive load is

$$P_L = I^2 R_L = \left(\frac{V_0}{R_{th} + R_L}\right)^2 \times R_L \quad \text{... (ii)}$$

P_L can be maximised by varying R_L and hence, maximum power can be delivered when $\frac{dP_L}{dR_L} = 0$

$$\frac{dP_L}{dR_L} = \frac{1}{[(R_{th} + R_L)^2]^2} \left[(R_{th} + R_L)^2 \frac{d}{dR_L}(V_0^2 R_L) - V_0^2 R_L \frac{d}{dR_L}(R_{th} + R_L)^2\right]$$

$$\frac{V_0^2 (R_{th} - R_L)}{(R_{th} + R_L)^3} = 0 \qquad \text{then} \quad (R_{th} - R_L) = 0$$

or, $R_{th} = R_L$

From equation (ii) $$P_{max} = \frac{V_0^2 R_{th}}{(R_{th} + R_{th})^2} = \frac{V_0^2}{4R_{th}}$$

Obviously, the power transfer by the source would be also $\frac{V_0^2}{4R_{th}}$.

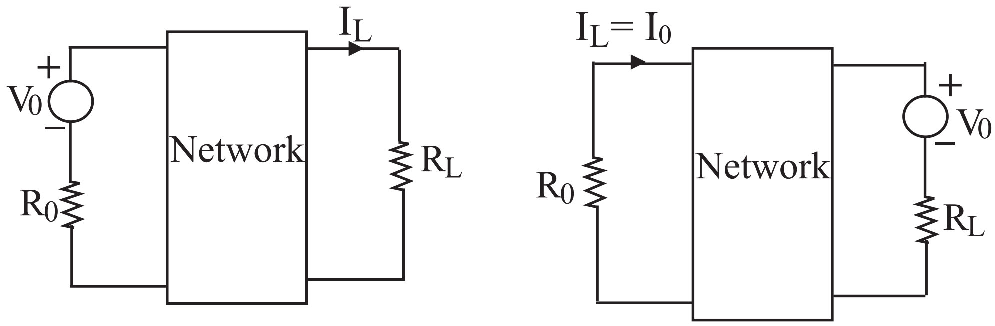
The load power and source power being the same.
The total power supplied is thus $$P = 2\frac{V_0^2}{4R_{th}} = \frac{V_0^2}{2R_{th}}$$

During maximum power transfer the efficiency η becomes

$$\eta = \frac{P_{max}}{P} \times 100 = 50\%$$

Reciprocity Theorem

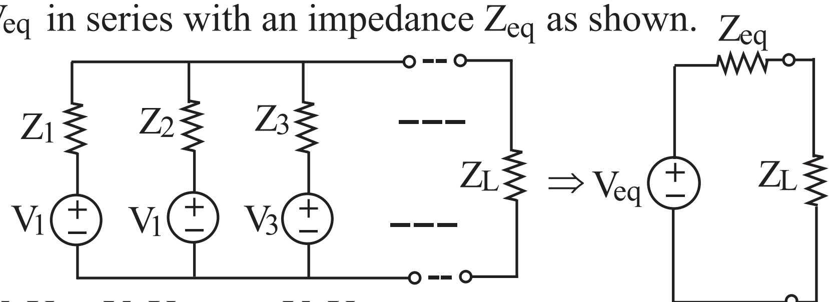
* This theorem states that ay source of emf E, located at one point in a network composed of linear bilateral circuit element produces a current I at the second Point in the network, the same source of emf, E acting at the second point will produce the same current I at the first point.



V_0 of a voltage source in one part of the network driving a current I_L in another part remains the same if the source V_0 and I_L are interchanged.

MILLMAN'S THEOREM

* According to this theorem if number of voltage sources V_1, V_2, V_3 ..... V_n having internal impedance Z_1, Z_2, Z_3 ...... Z_n are connected in parallel supplying a common load Z_1, this arrangement can be replaced by a single voltage source V_{eq} in series with an impedance Z_{eq} as shown.



$$V_{eq} = \frac{V_1 Y_1 + V_2 Y_2 + V_3 Y_3 + \ldots\ldots V_n Y_n}{Y_1 + Y_2 + Y_3 + \ldots\ldots Y_n}$$

$$V_{eq} = \frac{\sum_{K=1}^{n} Y_K V_K}{\sum_{K=1}^{n} Y_K} \quad \text{and} \quad Z_{eq} = \sum_{K=1}^{n} \frac{1}{Y_K}$$

TELLEGEN'S THEOREM

* This theorem states that the algebraic sum of power delivered to each branch of any electric network is zero.

$$\sum_{K=1}^{n} V_K I_K = 0$$

Where,

n = No. of branches of network

V_K = Voltage across K_{th} branch

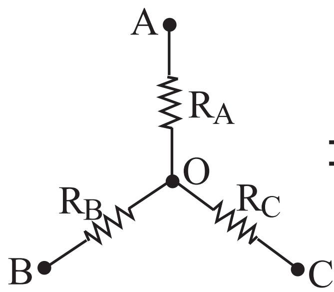
I_K = Current in K_{th} branch

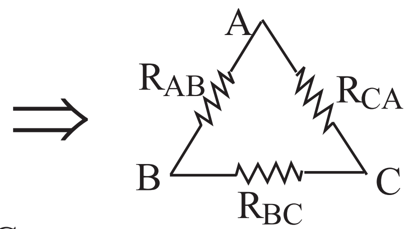
Wye (Star) to Delta Tranformation

$$R_{AB} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

or

$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$





Similarly

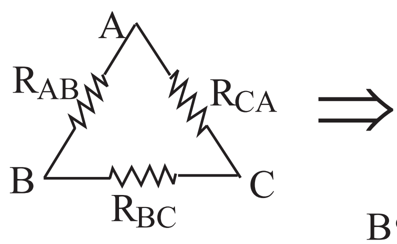
$$R_{BC} = \frac{R_B R_C + R_C R_A + R_A R_B}{R_A} \quad \text{or} \quad R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A}$$

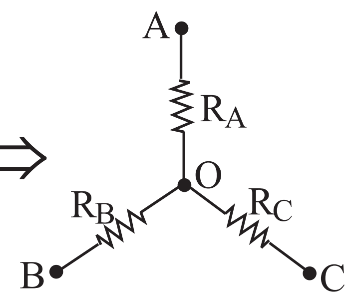
and

$$R_{CA} = \frac{R_C R_A + R_A R_B + R_B R_C}{R_B} \quad \text{or} \quad R_{CA} = R_A + R_C + \frac{R_C R_A}{R_B}$$

Delta to Wye (Star) Transformation

$$R_A = \frac{R_{AB} \times R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$





Similarly

$$R_B = \frac{R_{AB} \times R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_C = \frac{R_{CA} \times R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$

NETWORK ELEMENTS AND GRAPHS

Interconnection of two or more simple circuit elements is called an electric network.

Classification of Network Elements

1. Lumped and Distributed Circuite Elements

- **Lumped circuit element :** Physically separate elements.
  Example : Resistor, Capacitor, Inductor.
- **Distributed element :** A distributed element is one which is not separable for electrical purpose.
  Example : Transmission line has distributed resistance, capacitance and inductance.

2. Active and Passive Elements

- **Active element :** The source of energy is called active element.
  Example: Voltage source, current source.
- **Passive element:** The element which stores or dissipates energy is called passive element.
  Example: Resistor, Inductor, Capacitor

3. Linear and Non linear Elements

- **Linear element:** If the element obeys superposition principle, then it is said to be linear elements.
  Example: Resistor
- **Nonlinear element:** If the given network is not obeying superposition principle, then it is said to be composed of non-linear elements.
  Example: Transistor, Diode.

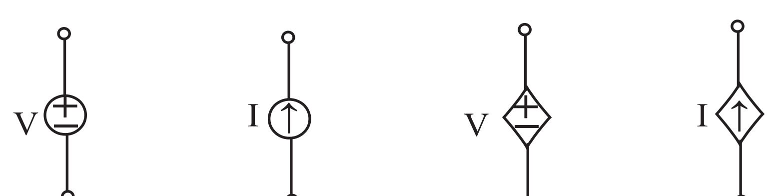
4. Bilateral and Unilateral Element

- **Bilateral element:** In bilateral element, voltage current relation is same for both the directions.
  Example: Resistor
- **Unilateral element:** In unilateral element, voltage current relation is not same for both the directions.
  Example: Diode, Transistors.

Sources

Two types of sources are:

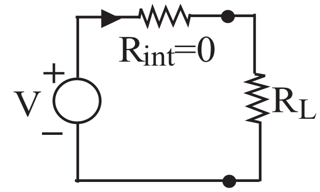
- Independent sources
- Dependent sources



Independent voltage source | **Independent current source** | **Dependent voltage source** | **Dependent current source**

* **Ideal Voltage source:** The sources are independent of current direction and magnitude. It's internal resistance is zero.

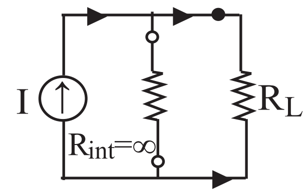
$$\boxed{R_{int} = 0}$$



Ideal voltage source

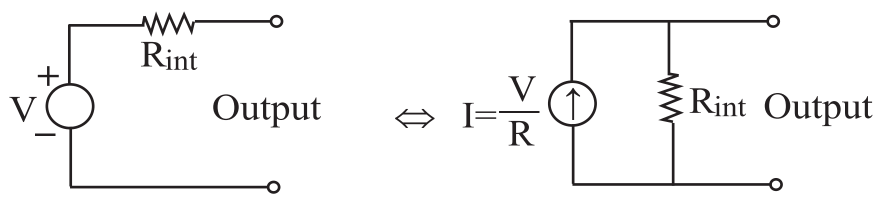
* **Ideal current source:** These source are independent of voltage direction (polarity) and magnitude. It's internal resistance is infinite.

$$\boxed{R_{int} = \infty}$$



Ideal current source

* **Source Conversion:** A voltage source can be replaced by a shunt source as follows.



Resistance (R)

Resistance of a wire depends on its material and it's size. It is given by

$$\boxed{R = \frac{\rho \ell}{A}}$$, where, ρ = Reistivity of a material in Ω cm.
ℓ = Length of the wire, and
A = Area of cross-section of wire.

* The power absorbed by the resistor is given by $P = VI = (IR)I = I^2R$ watt
* When n resistors are connected in series the equivalent resistance is

$$R_{eq} = R_1 + R_2 + R_3 + \text{........} + R_n$$

* When n resistors are connected in parallel the equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \text{......} + \frac{1}{R_n}$$

Inductance (L)

Inductance is the characteristic of an electrical conductor that opposes change in current. The symbol for inductance is L and the basic unit of inductance is the Henry (H).

$$\text{Inductance (L)} = \frac{N\phi}{L} \text{ Henry}$$

where,

N = No. of turns in the coil, ϕ = Flux set by current I.

Also

$$L = \frac{\mu_0 \mu_r N^2 A}{I} \text{Henry}$$

* Current through inductor is given by

$$I(t) = \frac{1}{L}\int_{-\infty}^{t} v\,dt \quad \text{or} \quad I(t) = \frac{1}{L}\int_{-\infty}^{t} v\,dt + I_0 \text{ Amp(A)}$$

where, I_0 being the initial current

* Voltage across inductor is given by

$$V = L\frac{dI}{dt} = N\frac{d\phi}{dt} \text{ volt.}$$

* The power absorbed by the inductor is given by

$$P = VI = LI.\frac{dI}{dt} \text{ watt.}$$

* When n inductors are connected in series the equivalent inductance is

$$L_{eq} = L_1 + L_2 + L_3 + \ldots\ldots + L_n$$

* When n inductors are connected in parallel the equivalent inductance is

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots\ldots. + \frac{1}{L_n}$$

Capacitance (C)

Capacitance is the property of material by virtue of which it opposes the variation in potential between the two sides.

$$C = \frac{q}{v}$$

where, q = Charge, v = potential

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

where,
C = Capacitance, is proportional to the dielectric and area of the plates, and is inversely proportional to the distance between the plates.
$\varepsilon_0 = 8.86 \times 10^{-12}$ F/m; ε_r = Relative permittivity; A = Area of the plate;
d = Spacing between two plates.

* Current through the capacitor is

$$I = C\frac{dv}{dt} \text{ Amp(A)}$$

* Voltage across the capacitor is

$$V = \frac{1}{C}\int_{-\infty}^{t} I(t)dt \quad \text{or} \quad V = \frac{1}{C}\int_{-\infty}^{t} I(t)dt + v_0$$

where, v_0 being the initial voltage

* when n capacitors are connected in series the equivalent capacitance is

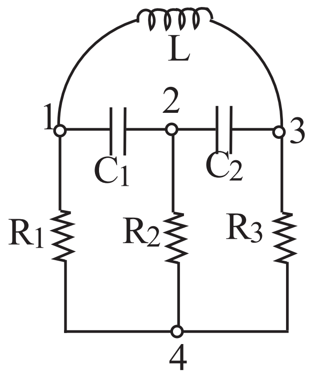
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots.. + \frac{1}{C_n}$$

* When n capacitors are connected in parallel the equivalent capacitance is

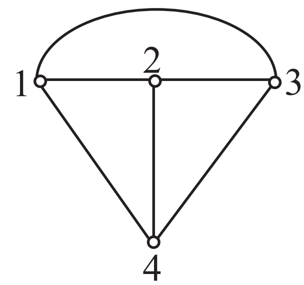
$$C_{eq} = C_1 + C_2 + C_3 + \;......\; + C_n$$

Graph Theory

A graph of any network can be drawn by placing all the nodes which are points of intersection of more than two branches. Consider a network given below:



Given network



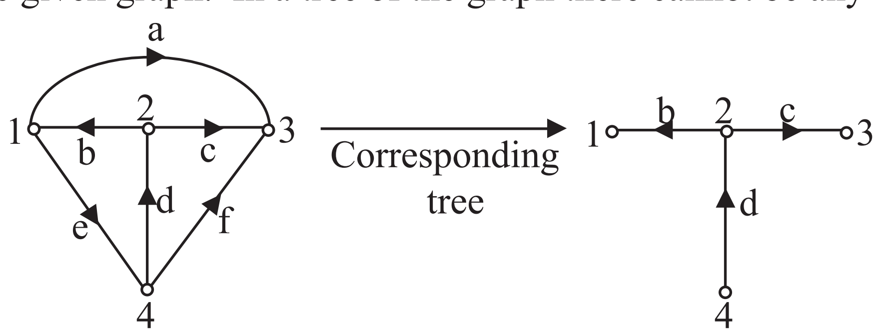
Graph of the given network

* Terminology used in network graph.

* **Branch:** A branch is a line segment representing one network element.

* **Node:** A node point is defined as an end point of a line segment and exists at the junction between two branches.

* **Tree:** It is an interconnected open set of branches which include all the nodes of the given graph. In a tree of the graph there cannot be any closed loop.



Given graph **Tree**

Simple graph and tree

* In the above figure branches b, c, d are called twig while the branches a, e, f, are called links.

* **Twig:** Any branch of a tree is called twig.

$\boxed{\text{Twig} = n - 1}$ where, n = no. of nodes

* **Link or chord**: Branch of graph which is not in the tree.

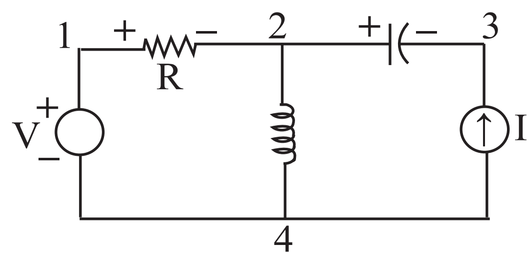
* The number of links L is given by $\boxed{L = b - n + 1}$

where,

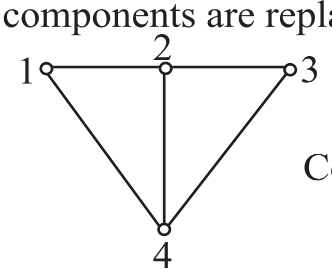
L = number of links; b = number of branches; n = number of nodes.

* The rank of tree is (n - 1). This is also the rank of a graph to which the tree belongs i.e. both and graph has same rank. Where n is the number of nodes in a graph.

Consider the following circuit for example:

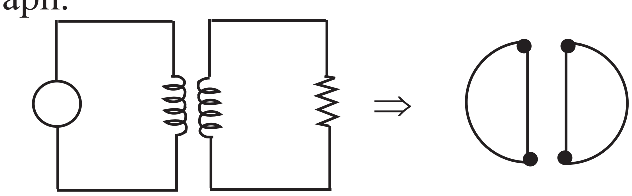


* All the components are replaced by a line segment so a graph is obtained.

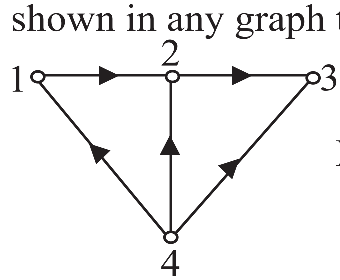


* This type of graph is called connected graph.

* If components are separate then this is called not connected graph or unconnected graph.



* If directions are shown in any graph then this is called directed graph.



* Now we see the direction or load set of a reference.

Loop

If we start from any node and come back to the same node then this path is called loop.

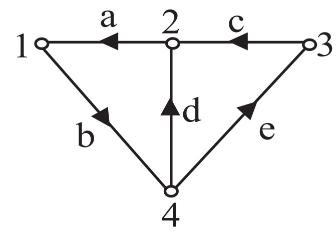
* In loop we doesn't see the direction of graph.

* In loop, the branches may be present inside the loop.

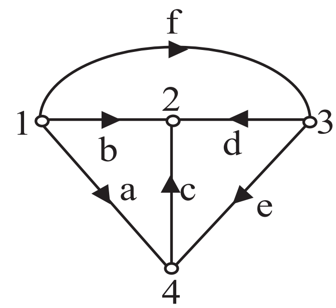
Mesh

Is same as loop but no branch is present inside the mesh.

abec is loop but not mesh.
abd is both loop and mesh.



Complete Incidence Matrix



- * This is a matric between nodes and branches.
- * At any node, if branch is not connected to this node then we place 0 in it.
- * If the branch is connected to any node then we see the direction.
- * For incoming at any node ⇒ -1.
- * For outgoing at any node ⇒ +1.

So, **Nodes** **Branches →**

Nodes ↓	a	b	c	d	e	f
1	1	1	0	0	0	1
2	0	-1	-1	-1	0	0
3	0	0	0	1	1	-1
4	-1	0	1	0	-1	0
Sum	0	0	0	0	0	0

$(n \times b)$

- * The sum of each column must be zero.
- * Complete incidence matrix is a matrix with order $(n \times b)$.
- * If any of the row is absent in the matrix then we can find it because we know sum of all column should be zero. But we should know that which row was absent.

Reduced Incident Matrix

If we remove any row from complete incidence matrix then this matrix is called reduced incidence matrix.

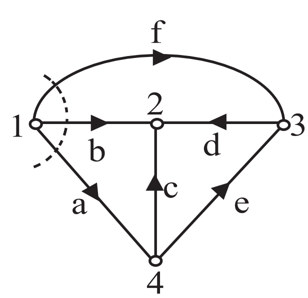
- * Obviously this is of the order of $(n - 1) \times b$.

Nodes **Branches →**

Nodes ↓	a	b	c	d	e	f
1	1	1	0	0	0	1
2	0	-1	-1	-1	0	0
3	0	0	0	1	1	-1

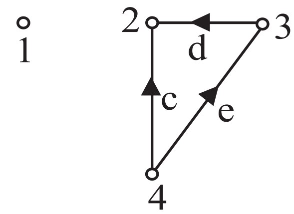
$(n - 1) \times b$

Fundamental Cut Set of a Graph

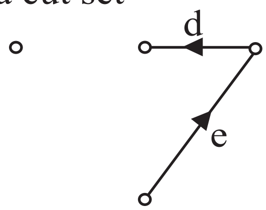


Cut-set of a graph which exactly cut the graph into two parts, with the restriction that by substituting any branch back, then it should be connected.

* cut set (a, b, f)



* (a, b, f, c) this is not a cut set



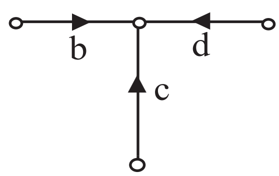
because if we place the branch c back

Minimum number of cut set = Number of nodes

Fundamental Cut-Set (f cut-set)

This is a cut set which contains exactly one twig. (Not more than one or not less than 1).

Let the tree is



$f_1 = (a, b, f)$
$f_2 = (d, e, f)$
$f_3 = (a, c, e)$

So number of fundamental cut set should be (n - 1)

* For the matrix with fundamental cut set we take the reference direction as the direction of twig (+1).

Nodes ↓ **Branches** →

	a	b	c	d	e	f
f_1	1	1	0	0	0	1
f_2	0	0	0	1	-1	-1
f_3	-1	0	1	0	1	0
Sum	0	1	1	1	0	0

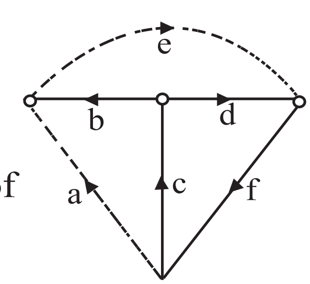
(n - 1) b

* In the matrix of fundamental cut set column with only (+1) entry will be twig and other will be link.

* **Loop:** This is the dual of cut-set.

loop $\overset{\text{duality}}{\Longleftrightarrow}$ cut-set

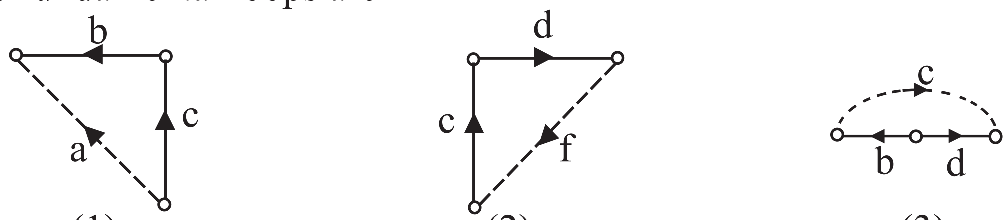
Fundamental Loop (Tie-Set) Matrix



Loop in which only one link will be present.

* For fundamental-loop we take the direction of link as a direction for reference.

So fundamental loops are



(1) (2) (3)

Number of fundamental loop = Number of links = b - n + 1

Loops Branches →

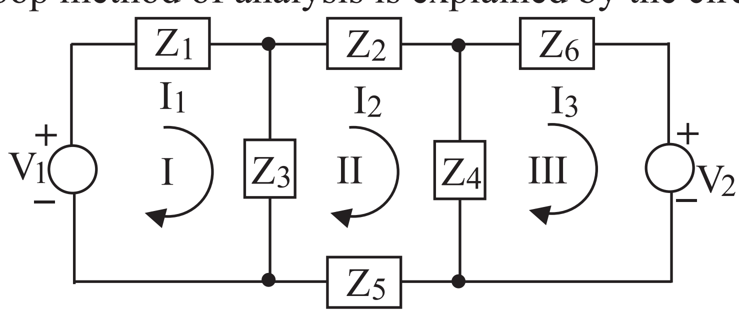
Loops ↓	a	b	c	d	e	f
l_1	1	-1	-1	0	0	0
l_2	0	0	1	1	0	1
l_3	0	1	0	-1	1	0

$[\;]_{l\ b}$

- * Column with one entry will be link and others are twig.
- * Number of equations with variables N - 1 = twigs
- * For example let twig are the voltage variable
- * Links are the shunt variable.

Mesh Analysis

The mesh or loop method of analysis is explained by the circuit below.



Mesh Analysis

In Mesh-I

$I_1Z_1 + (I_1 - I_2)Z_3 - V_1 = 0$ $\therefore$ $(Z_1 + Z_3)I_1 + (-Z_3)I_2 + 0I_3 = V_1$ ...(i)

In Mesh - II

$(I_2 - I_1)Z_3 + I_2Z_2 + (I_2 - I_3)Z_4 + I_2Z_5 = 0$

$(-Z_3)I_1 + (Z_2 + Z_3 + Z_4 + Z_5)I_2 + (-Z_4)I_3 = 0$ ... (ii)

In Mesh - II

$(I_3 - I_2)Z_4 + I_3Z_6 + (V_2) = 0$

$(0)I_1 + (-Z_4)I_2 + (Z_4 + Z_6)I_3 = -V_2$ .....(iii)

Writing equation (i), (ii) and (iii) in the matrix form

$$\begin{bmatrix} Z_1+Z_3 & -Z_3 & 0 \\ -Z_3 & Z_2+Z_3+Z_4+Z_5 & -Z_4 \\ 0 & -Z_4 & Z_4+Z_6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \\ -V_2 \end{bmatrix}$$

Now by cramer's rule Mesh currents I_1, I_2, I_3 will be

$$I_1 = \frac{1}{\Delta} \begin{vmatrix} V_1 & -Z_3 & 0 \\ 0 & Z_2+Z_3+Z_4+Z_5 & -Z_4 \\ -V_2 & -Z_4 & Z_4+Z_6 \end{vmatrix}$$

$$I_2 = \frac{1}{\Delta} \begin{vmatrix} Z_1+Z_3 & V_1 & 0 \\ -Z_3 & 0 & -Z_4 \\ 0 & -Z_2 & Z_4+Z_6 \end{vmatrix}$$

$$I_3 = \frac{1}{\Delta} \begin{vmatrix} Z_1+Z_3 & -Z_3 & V_1 \\ -Z_3 & Z_2+Z_3+Z_4+Z_5 & 0 \\ 0 & -Z_4 & -V_2 \end{vmatrix}$$

where

$$\Delta = \begin{vmatrix} Z_1+Z_3 & -Z_3 & 0 \\ -Z_3 & Z_2+Z_3+Z_4+Z_5 & -Z_4 \\ 0 & -Z_4 & Z_4+Z_6 \end{vmatrix}$$

Generalised mesh equations can be written as

$[Z]\,[1] = [V]$ $\qquad$ $[Z] \rightarrow$ Impedance matrix

having Z_{ij} as elements $i = 1, 2, \ldots\ldots m$ and $j = 1, 2, \ldots\ldots m$

and V is the column matrix of input voltages V_i, $i = 1, 2, \ldots\ldots m$.

The elements Z_{ij} of the impedance matrix [Z] are

(i) Z_{ii}, self impedance of the i^{th} mesh

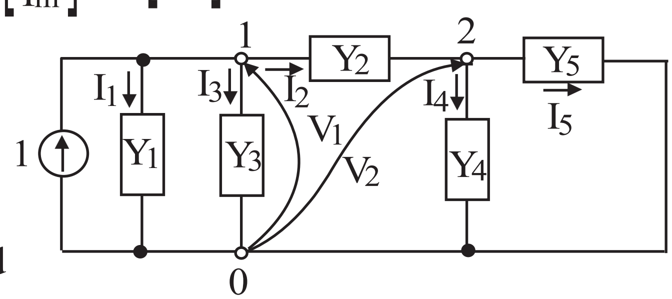
(ii) Z_{ij}, the mutual impedance between i^{th} and j^{th} meshes.

The order of Z matrix will be according to the number of meshes.

Writing the elements of the matrix.

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1m} \\ Z_{21} & Z_{22} & \cdots & Z_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mm} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_m \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_m \end{bmatrix}$$

NODE ANALYSIS



At node 1, writing KCL,

$I - I_1 - I_2 - I_3 = 0$

and in terms of potential and admittances will be

$I - Y_1 V_1 - Y_3 V_1 - Y_2 (V_1 - V_2) = 0$

$$\Rightarrow (Y_1 + Y_2 + Y_3) V_1 + (-Y_2) V_2 = I \qquad .... (i)$$

At node 2, $I_2 - I_4 - I_5 = 0$

$$Y_2 (V_1 - V_2) - Y_4 V_2 - Y_5 V_2 = 0$$

$$\Rightarrow (-Y_2) V_1 + (Y_2 + Y_4 + Y_5) V_2 = 0 \qquad .... (ii)$$

Writing equation (i), (ii) in matrix form as

$$\begin{bmatrix} Y_1 + Y_2 + Y_3 & -Y_2 \\ Y_2 & Y_2 + Y_4 + Y_5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

By cramer's rule

$$\therefore \quad V_1 = \frac{1}{\Delta}\begin{vmatrix} 1 & -Y_2 \\ 0 & Y_2 + Y_4 + Y_5 \end{vmatrix} \qquad V_2 = \frac{1}{\Delta}\begin{vmatrix} Y_1 + Y_2 + Y_3 & 1 \\ -Y_2 & 0 \end{vmatrix}$$

where $\Delta = \begin{vmatrix} Y_1 + Y_2 + Y_3 & -Y_2 \\ -Y_2 & Y_2 + Y_4 + Y_5 \end{vmatrix}$

Generalised node equations can be written as $[Y][V] = [I]$

Square matrix Y is called the admittance matrix,

V is the column matrix of the node voltages with respect to reference node and I is the column matrix of input currents.

The elements Y_{ij}, i = 1, 2, ... m, j = 1, 2, ... m of the admittance matrix Y are

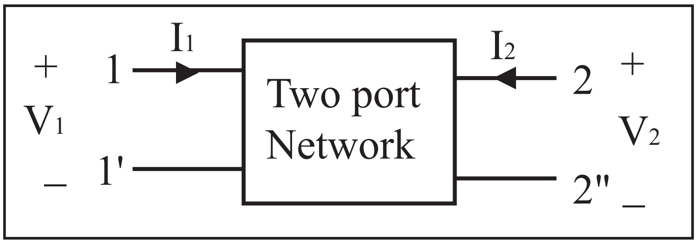
(i) Y_{ij}, the self admittance of the i^{th} node.

(ii) Y_{ij}, the mutual admittance between the i^{th} and j^{th} node of negative sign.

Consider a generalised network with (n + 1) nodes including the reference node, we can write the node equations in matrix form of order $(n \times n)$ using KCL as

$$\begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}$$

One Port and Two Port Network Function



Driving Point Impedance and Admittance Function

(a) Driving impedance function at port 1 :

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)}$$

(b) Driving impedance function port 2 :

$$Z_{22}(s) = \frac{V_2(s)}{I_2(s)}$$

(c) Driving admittance function port 1 : $Y_{11}(s) = \dfrac{I_1(s)}{V_1(s)}$

(d) Driving admittance function at port 2 : $Y_{22}(s) = \dfrac{I_2(s)}{V_2(s)}$

Note : $Z_{11} = \dfrac{1}{Y_{11}}$; $Z_{22} = \dfrac{1}{Y_{22}}$

Impedance and Admittance Transfer Function

(a) $Z_{21}(s) = \dfrac{V_2(s)}{I_1(s)}$; (b) $Z_{12}(s) = \dfrac{V_1(s)}{I_2(s)}$

(c) $Y_{21}(s) = \dfrac{I_2(s)}{V_1(s)}$; (d) $Z_{12}(s) = \dfrac{I_1(s)}{V_2(s)}$

Note : Numerator carries response and denominator carrier excitation.

Voltage and Current Transfer function

(a) Voltage transfer function

1. $G_{21}(s) = \dfrac{V_2(s)}{V_1(s)}$; 2. $G_{12}(s) = \dfrac{V_1(s)}{V_2(s)}$

(b) Current transfer function

1. $\alpha_{21} = \dfrac{I_2(s)}{I_1(s)}$; 2. $\alpha_{12} = \dfrac{I_1(s)}{I_2(s)}$

Consider a Network Function

$$H(s) = \frac{a_0 s^n + a_1 s^{n-1} + \ldots + a_n}{b_0 s^m + b_1 s^{m-1} + \ldots + b_m}$$

$$= \frac{(a_0 - b_0)\,[(s - Z_1)\,(s - Z_2)\,(s - Z_3)\,.\,.\,.\,(s - Z_n)]}{[(s - P_1)\,(s - P_2)\,.\,.\,.\,(s - P_m)]}$$

$$= \frac{P(s)}{Q(s)}$$

Matrix Method for determination of Network Functions

1. Determination of impedance function from nodal analysis

$$I = YV$$

$$Z_{jk} = \frac{\Delta_{Kj}}{\Delta}$$

where $\Delta = |\,Y\,|$ and Δ_{Kj} = co - factor

$$= (-1)^{K+j}\; M_{Kj}$$

M_{Kj} is minor a_{Kj} element pf $|\,Y\,|$

2. Determination of admittance function from mesh analysis

$$V = ZI$$

$$Y_{jK} = \frac{\Delta_{Kj}}{\Delta}$$

where $\Delta = |\,Z\,|$ and $\Delta_{Kj} = (-1)^{K+j}\, M_{Kj}$

3. Determination of current transfer function mesh analysis

$$V = ZI$$

$$\alpha_{12} = \frac{\Delta_{12}}{\Delta_{11}}$$

$$\Delta_{Kj} = (-1)^{K+1}\, M_{Kj}$$

M_{Kj} minor of element a_{Kj} of $|\,Z\,|$

4. Determination of voltage transfer function Nodal analysis

$$I = YV$$

$$G_{12} = \frac{\Delta_{12}}{\Delta_{11}}$$

$$\Delta_{Kj} = (-1)^{K+j}\;\; M_{Kj}$$

M_{Kj} Minor of element of a_{Kj} of $|\,Y\,|$

Properties of driving point impedance and admittance functions or necessary conditions for driving point impedance and admittance function

1. All coefficient of P(s) and Q(s) must be positive and real.
2. Complex and imaginary poles and zeros must be conjugate.

i.e. $F(s) = \dfrac{s + 1}{(s + 3)\,(s + 1 - j)}$

is not a driving point impedance function.

3. (a) Real part of all poles and zeros must not be positive.
   (b) If poles and zeros are imaginary they must be simple (non-repeated).
4. There should not be any missing power of ‘s’ between highest and lowest power in numerator and denominator both except when either all even or odd power are missing.
5. The degree of P(s) and Q(s) can at the most differ by 1.
6. Lowest power of s in numerator and denominator can differ at most ‘1’.

Network Transform Functions

- A transfer function is the ratio of Laplace transform of the output Y(s) to Laplace transform of the input X(s).
- By setting the denominator of the transfer function to zero, and obtaining the roots by solving the equation, we can know the system’s stability by considering by setting s = 0, i.e G(0) give DC gain where G(s) is transfer function.

Procedure for Deriving Transfer Functions

Following assumption is made in deriving the transfer function :

- It is assumed that no loading of one system on other. If the system has more than one non loading element, the transfer function of each element can be determined independently and the overall transfer function can be obtain by multiplying the individual transfer function.
- If the system consisting of element, which load each other, the overall transfer function should be derived by the basic analysis.

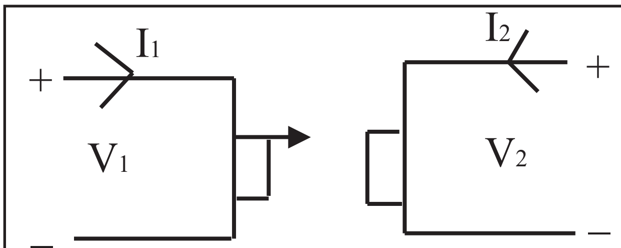
Necessary Conditions for Transfer Function

1. The coefficients in the polynomials P(s) and Q(s) of H(s) = P(s)/Q(s) must be real and those for Q(s) must be positive.
2. Poles and zeros which are complex and imaginary must be conjugate.
3. The real part of the poles must be either negative or zero, if poles are imaginary, then they must be simple (non-repeated). But zeros can have positive real part also.
4. There should not be any missing power of s between lowest and highest power of s in denominator except all odd all even power are missing. Numerator can have i.e. (P(s)) can have missing terms of s and some of the coefficients of may be negative.
5. The degree of P(s) may be as small as zero independent of the degree of Q(s).

6. (a) For G_{12} and α_{12} , the maximum degree of P(s) is the degree of Q(s).
   (b) For Z_{12} and Y_{12} , the maximum degree of P(s) is the degree of Q(s) pulse one.

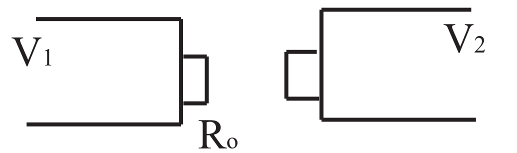
Impedance Matrix of Gyrator

- It is a device that gyrates the current of one port into a voltage at the other and vice versa.



$V_1 = -\alpha I_2$ $\quad V_2 = \alpha I_1$

As impedance transforming element



$V_1 + R_o I_2 = 0$

$V_2 = R_o I_1$

- Matrix form $\begin{vmatrix} 0 & -\alpha \\ \alpha & 0 \end{vmatrix} \begin{vmatrix} i_1 \\ i_2 \end{vmatrix} = \begin{vmatrix} V_1 \\ V_2 \end{vmatrix}$
- If the gyrator is terminated in an impedance Z_L , the driving point impedance

$$Z_d = \frac{Z_{11} Z_L + \Delta z}{Z_{22} + Z_L} \ ; \ Z_{in} = (\alpha^2/Z_L)$$

- If Z_L is capacitor, then $Z_L = (1/sC)$
  $Z_{in} = sC\alpha^2$ give inductor of value $= \alpha^2 C$
- If Z_L is an inductor $Z_L = sL$
  $Z_{in} = (\alpha^2/sL)$ gives capacitor of value $= (L/\alpha^2)$
- Used to simulate inductor from capacitor.

Z or Impedance Parameters

In this, the dependent variables are V_1 and V_2 and independent variable I_1 and I_2 .

Defining equations are :

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

In Impedance Matrix Form

$$\begin{vmatrix} V_1 \\ V_2 \end{vmatrix} = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix}$$

- To obtain the Z-parameters, we open circuit the output and input port alternately.
- With output open circuit $I_2 = 0$

$$V_1 = Z_{11} I_1$$

$$\text{or } Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0} = \text{input impedance}$$

$$V_2 = Z_{21} I_1$$

$$\text{or } Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0} = \text{forward transfer impedance}$$

- With input open circuit $I_1 = 0$

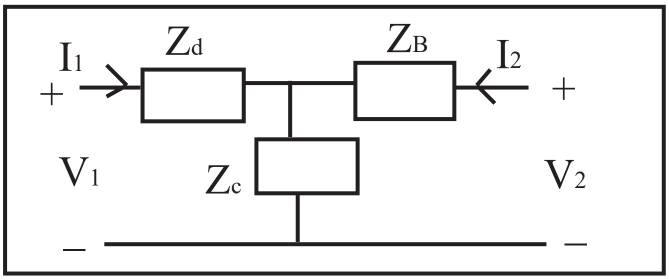
$$V_1 = Z_{12} I_2$$

$$\text{or } Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0} = \text{reverse transfer impedance}$$

$$V_2 = Z_{22} I_2$$

$$\text{or } Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0} = \text{output impedance}$$

Z-parameter for a T-network



$$Z = \begin{vmatrix} Z_A + Z_C & Z_C \\ Z_C & Z_B + Z_C \end{vmatrix}$$

Property of Z-parameter

- Z_{11} , Z_{22} are called **driving point input and output impedance function**.
- Z_{12} and Z_{21} are called **transfer function**.
- Network for which $\mathbf{Z_{11} = Z_{22}}$ called as **symmetrical**.
- Network for which $\mathbf{Z_{12} = Z_{21}}$ are known as **reciprocal network**.
- Reciprocal network need not be symmetrical.

Y or Admittance Parameter

- Also known as admittance parameter.
- Independent variables are voltage V_1 , V_2 and dependent variable current I_1 , I_2 .
- $I_1 = Y_{11} V_1 + Y_{12} V_2$
  $I_2 = Y_{21} V_1 + Y_{22} V_2$
- Admittance in matrix form

$$Y = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}$$

- For reciprocal $Y_{12} = Y_{21}$

Symmetric

A two port is symmetric if port can be interchanged, without any effect on the performance.

$$Z_{11} = Z_{22}$$
$$Y_{11} = Y_{22}$$

Hybrid (h) Parameters

Current (I_1) and voltage (V_2) are the independent variable and current (I_2) and voltage (V_1) are dependent variable.

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

Inverse Hybrid Parameter 9g-parameter)

$$I_1 = g_{11} V_1 + g_{12} I_2$$
$$V_2 = g_{21} V_1 + g_{22} I_2$$

Transmission Parameters (ABCD parameters)

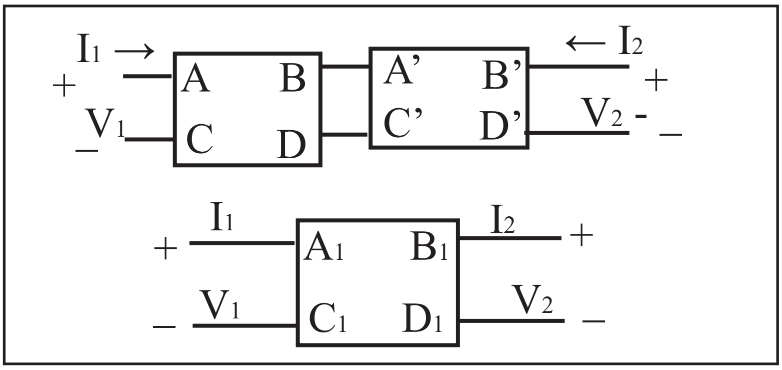
- It relates the voltage and current at input port to volatage and current at output port.
- $V_1 = AV_2 - BI_2$
  $I_1 = CV_2 - DI_2$
- For symmetrical Network $A = D$
- For reciprocal $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = 1$

  = i.e. $AD - BC = 1$

Cascading

Two Port Network Connected in Cascade

Overall ABCD parameter of two cascaded two port network is multiplication of their ABCD parameters.



$$\begin{vmatrix} A_1 & B_1 \\ C_1 & D_1 \end{vmatrix} = \begin{vmatrix} A & B \\ C & D \end{vmatrix} \times \begin{vmatrix} A' & B' \\ C' & D' \end{vmatrix}$$

Two Port Network in Parallel

Overall matrix for two port network connected in parallel is the sum of the individual network y-parameters $(y_A) = (y_B)$

where

(y) = overall y-parameter of combine network

(y_A) = y-parameter of network A

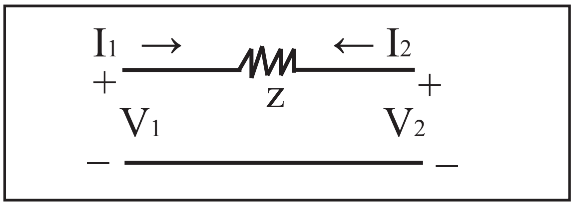
(y_B) = y-parameter of network B.

Two Port Network Connected in Series

The overall matrix for two=port network connected in series is equal to the sum of the individual network Z-parameter matrix

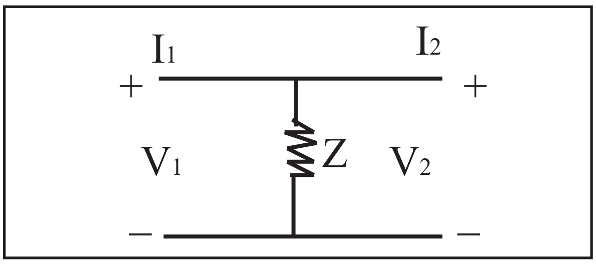
$$(z) = (z_A) + (z_B)$$

Some Special Results



Z-parameter of this network does not exist.

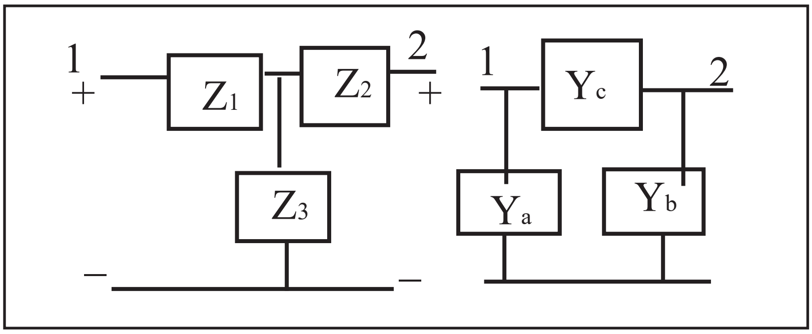
$$y = \begin{pmatrix} 1/z & -1/z \\ -1/z & 1/z \end{pmatrix}, \quad ABCD = \begin{pmatrix} 1 & Z \\ 0 & 1 \end{pmatrix}$$



This network cannot have y-parameter

$$Z = \begin{pmatrix} Z & Z \\ Z & Z \end{pmatrix};\ ABCD = \begin{pmatrix} 1 & Z \\ 0 & 1 \end{pmatrix}$$

Obtain T-equivalent of the π-network



$$Z_{11} = \frac{Z_a (Z_b + Z_c)}{Z_a + Z_b + Z_c}\quad ,\ Z_{12} = Z_{21} = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$

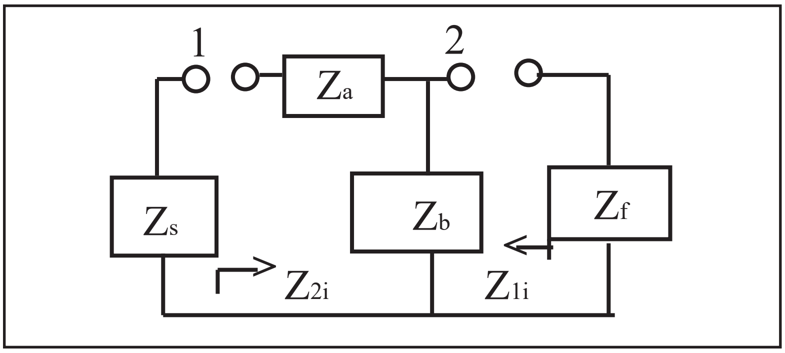
$$Z_{22} = \frac{Z_a (Z_b + Z_c)}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$

$$Z_{21} = \frac{Z_a Z_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c}$$

Impedance Matching



- L-section Z_a and Z_b is inserted between impedances Z_s (source) and Z_{li} (load) so that the source sees an impedfance Z_{li} and seen as impedance Z_{2i} . Such an arrangement is called impedance matching.
- Impedance Z_{li} and Z_{2i} is called image impedance.
- $Z^2_a = Z_{li} (Z_{11} - Z_{2i})$
  $Z_a . Z_b = Z_{1i} - Z_{2i}$
- If $Z_{li} < Z_{2i}$; Z_a and Z_b are reactive for purely resistive image impedances. Also one of the Z, Z_b is inductive and the other capacitive.