1 Properties of Fluid

Substance in liquid or gaseous phase is called fluid. They are capable of deforming continuously under the action of shear stress.

Note:
In solids stress is proportional to strain but in fluid stress is proportional to strain rate.

Types of fluid

Ideal fluid

- They do not have viscosity, surface tension and are incompressible
- Bulk modulas is infinite.

Real fluid

- Fluids which are not ideal are called real fluids.
- As ideal fluits do not exist practically so all are real fluids

Some specific fluid properties

1. Density $= \dfrac{\text{mass}}{\text{volume}}$ measured in kg/m^3.
2. Liquid density is constant while that of gas is directly proportional to pressure and inversely to temperature
3. Specific gravity/relative density $= \dfrac{\text{Density of substance}}{\text{Density of water at 4°C}}$
4. If R.D < 1 then fluid is lighter than water.
5. Specific weight $= \dfrac{\text{Weight of substance}}{\text{Volume of substance}}$, denoted by $\gamma = \rho g$ in N/m^3
6. $\gamma_{\text{water}} = 9810 \dfrac{N}{m^3} = 9.81 \dfrac{KN}{m^3}$
7. $\gamma_{\text{mercury}} = 13.6\ \gamma_w$
8. Specific volume $= \dfrac{1}{\text{Density}}$

Vapour Pressure and Cavitation

When liquid molecules are healed then the molecules on the surface of the liquid start converting into gaseous form called vapour. These vapours molecules exert partial pressure in the space called **vapour pressure**.

When the **absolute pressure** above a liquid surface becomes less than or equal to the vapour pressure then boiling starts. It leads to formation of cavity inside the liquid surface, and if this occurs in flowing liquid then this cavity is washed away in region of **higher pressure** where water from the cavity surrounding's rushes in, to fill the cavity and this bubble burst's. This phenomenon of formation and collapsing is called **cavitation**.

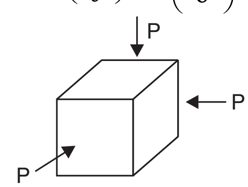
Note:

1. Higher temperature, more chances of cavitation.
2. Mercury has very low vapour pressure, hence used in pressure measuring equipment.

At 100°C, vapour pressure of water = Atmospheric pressure.

Bulk modulus: Bulk modulus of elasticity $(k) = \dfrac{-dp}{\left(\dfrac{dv}{v}\right)} = \dfrac{-dp}{\left(\dfrac{d\delta}{\delta}\right)}$

Compressibility $= \dfrac{1}{K} = \dfrac{d\delta}{\delta dp}$



Note:

If density does not change with pressure *i.e.*, $\dfrac{\partial\delta}{dp} \neq 0$, then fluid is in compressible

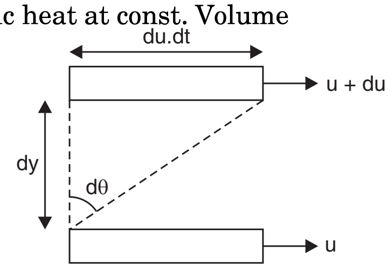
- Isothermal bulk modulus K_T = Pressure
- Adiabatic bulk modulus $K_A = \gamma \times$ pressure,

  γ – Adiabatic. Index

$$\gamma = \frac{C_P}{C_V} = \frac{\text{Specific heat at const. Pressure}}{\text{Specific heat at const. Volume}}$$

Viscosity: It's the measure of resistance of fluid to deformation.

It is due to the internal frictional forces that develop between different layers of fluid when they are forced to **move relative to each other**.



Shear strain = $d\,\theta = \dfrac{du \cdot dt}{dy}$

Rate of change of shear strain $\dfrac{d\theta}{dt} = \dfrac{du}{dy}$ velocity gradient

In Newtonian fluids,

$\tau\ \alpha$ Rate of change of shear strain

$$\tau\ \alpha\ \frac{d\theta}{dt} \propto \frac{du}{dy} \Rightarrow \tau = \mu\ \frac{du}{dy}$$

μ = absolute viscosity, dynamic viscosity or coefficient of viscosity

Unit of $\mu = \dfrac{\text{NS}}{\text{m}^2}$ or $\dfrac{\text{kg}}{\text{m.s}}$ or pascal sec (SI) or poise

(CGS unit) or $\dfrac{\text{Dyne} - \text{sec}}{\text{cm}^2}$

$$1\ \frac{\text{NS}}{\text{m}^2}\ (\text{SI}) = 10 \text{ poise}$$

Note: $\mu_{water} \approx 50\ \mu_{air}$

$$\text{Kinematic viscosity} = \frac{\text{dynamic viscosity}}{\text{density}} = \frac{\mu}{g} \Rightarrow \upsilon = \frac{\mu}{g}$$

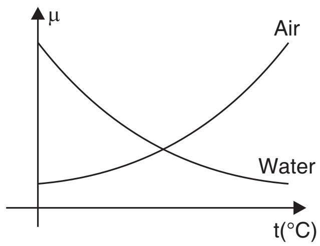
SI unit = $\dfrac{\text{m}^2}{\text{sec}}$ CGS unit = $\dfrac{\text{cm}^2}{\text{sec}}$ or stoke

1 m^2/s (SI) = 10^4 stoke

Note:
$\nu_{air} \approx 15.2\ \nu_{water}$ at 20°C
μ_{water} = 1 centipoise at 1°C

Viscosity of liquids is due to **cohesion** but for gases it is due to **molecular momentum transfer**.

Effect of temperature and pressure on fluid's viscosity



$$\mu_{liq} = \frac{\mu_0}{1 + \alpha t + \beta t^2}$$

$$\mu_{gas} = \mu_0 + \alpha t - \beta t^2$$

α, β are constants t = temp in °C

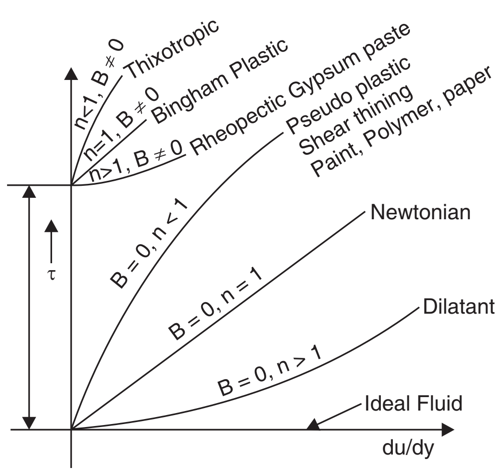
For liquids, μ does not depends on pressure except at high pressure.

For gases, μ_{gas} also doesn't depends on pressure but as δ is proportional to pressure

So, $$\nu_{gas} \propto \frac{1}{\text{Pressure}}$$

Newtonian and Non-Newtonian fluids

- If $\tau = \mu \dfrac{du}{dy}$ then Newtonian fluids otherwise not



- $\tau = A\left(\frac{du}{dy}\right)^n + B$ General shear equation
- Slope of the curve gives **apparent viscosity**.
- Study of Now-Newtonian fluid is called **Rheology.**
- Pseduo plastic are **shear thinning** while Dilatants are **shear thickening** fluids.
- Eg

(*a*) Thixotropic	Ink, ketchup, Enamels
(*b*) **B**ingham plastic	Toothpaste, sewage, sludge, Drilling mud, gel (i.e., Related to **B**ath room).
(*c*) Rheopectic	Gypsum in water and Bentonite slurry.
(*d*) **P**seudo Plastic	**P**aint, **P**olymer, **P**aper, **P**ulp, **B**lood, syrup
(*e*) Dilatant	Quick sand, Butter, sugar in water

Note:

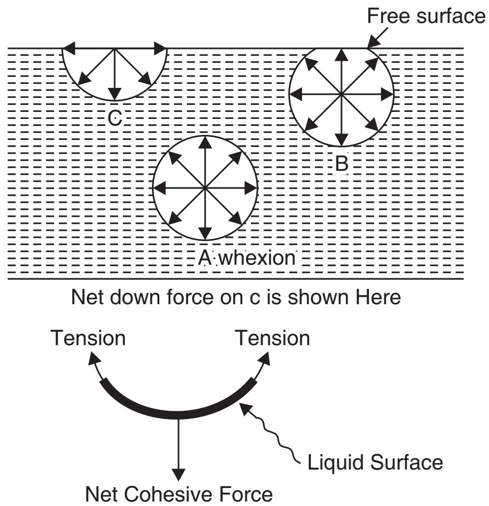
Tujhe	Bachk	Rchna	Padega	Nahitoh	Dil tot jaeyga
↓	↓	↓	↓	↓	↓
Thixotropic	Bingham	Rheopectic	Pseudo	Newtonian	Dilatant

No-slip condition of viscous fluid: When ever a viscous fluid flows over a solid surface, the relative velocity between the solid surface and the adjacent fluid particle is zero. This condition is known as No-slip condition.

Note:

1. Wetting property is due to surface tension.
2. No slip condition is due to fluid viscosity.
3. Ideal fluids → No-viscosity → No "No slip" condition

Surface tension and capillary effect: It occurs at the **liquid-gas interface** or at the interface of **two immiscible liquids** where a thin film is apparently formed due to attraction of liquid in the surface which is similar to tension in stretched membrane known as surface tension measured as $\frac{\text{force}}{\text{length}}$ Unit $\frac{\text{N}}{\text{m}}$



Note:
Surface tension occurs due to cohesion only.

$$\text{Surface tension} = \frac{\text{Work done}}{\text{Change in area to work done}}$$

$\sigma_{\text{water/air}}$ 0.0786 N/m, At critical point it becomes zero

P = Gauge pressure

σ = Surface tension

d = diameter

Then

(*a*) Pressure inside jet $P = \dfrac{2\sigma}{d}$

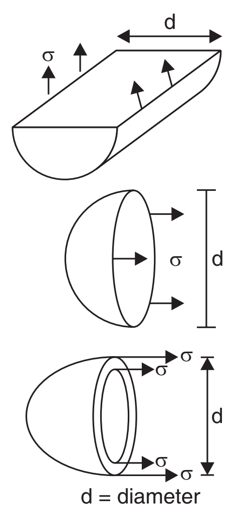
P = Gauge pressure

(*b*) Pressure in side $P = \dfrac{4\sigma}{d}$

σ = Surface tension

(*c*) Pressure inside bubble $P = \dfrac{8\sigma}{d}$

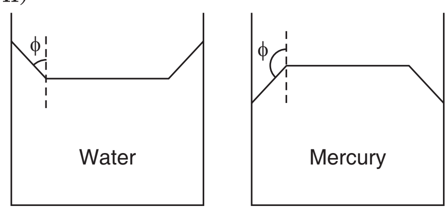
d = diameter.



Rise or fall in the surface of liquid when a **small** diameter tube is inserted into the liquid is called capillary rise or capillary depression respectively.

Note:
Capillary effect is due to **Adhesion** and **surface tension** both

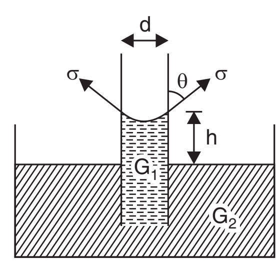
Force of attraction between molecules of different types is called adhesion while in molecules of same type it is called cohesion (for eg clay is a cohesive soil)



Note:

$\phi < 90°$	Cohesion < Adhesion	Wetting of surface	Concave top surface	Rise in capillary tube
$\phi > 90°$	Adhesion < cohesion	Does not wets the surface	Convex top surface	Drop in capillary tube

$$h = \frac{4\sigma\cos\theta}{(G_1 - G_2)\gamma_w d}$$



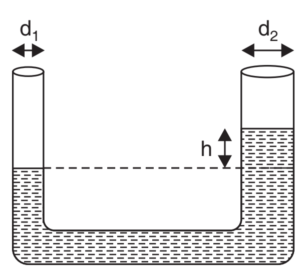
θ for water glass = 0°, mercury glass = 130°, Kerosene glass = 26°

Special case :-

$$h = \frac{4\sigma\cos\theta(d_2 - d_1)}{G\gamma_w d_2^2}$$

- $d_2 > d_1$, capillary rise is positive, that is water liquid rises in capillary tube of larger diameter (if $\phi < 90°$ otherwise vice versa)
- $d_1 = d_2$ No capillary rise as $h = 0$
- If $d_2 >>> d_1$, then neglect d_1 wrt d_2

$$h = \frac{4\sigma\cos\theta}{G\gamma_w d_2}$$



2 Pressure and its Measurement

Normal force exerted by a fluid per unit area is called pressure. It is a **scalar Quantity** *i.e.*, it has magnitude but no direction.

Atmospheric Pressure: Pressure exerted by atmosphere. It is measured by **Barometer** At mean sea level it is equal to 10.3 m head of water or 76 cm head of mercury (specific gravity 13.6)

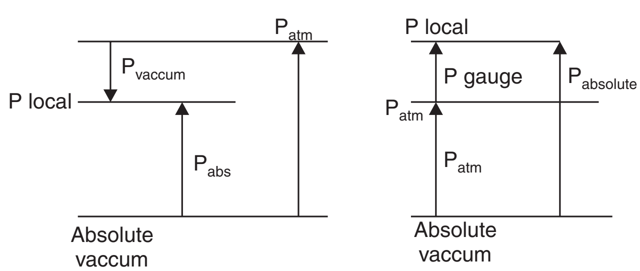
Note:
It head of water is '*h*' *m* then equivalent pressure is $\gamma_\omega h$ and if head of mercury is '*h*' *m* then equivalent pressure will be $\gamma_{Hg}\, h$

Absolute Pressure: Pressure with respect to absolute zero or complete Vaccum is called absolute pressure. It's the actual pressure and measured by **Aneroid Barometer.**

Gauge Pressure: Pressure with respect to atmospheric pressure as datum. It is measured using **Manometer** or **Bourdon gauge**

Steps to draw :-

1. Draw P_{atm} always above $P_{absolute}\ V_{acc}$.
2. Then draw P_{gauge}/P_{vaccum} wrt to P_{atm} depending upon gauge reading



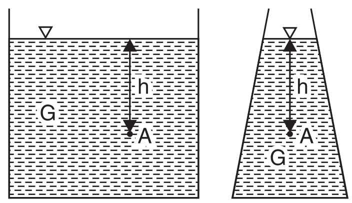
$$P_{absolute} = P_{atm} - P_{vaccum}$$
$$P_{absolute} = P_{atm} + P_{gauge}$$

Note:
'*h*' *m* of water vaccum means pressure of $-h\,\gamma_\omega$

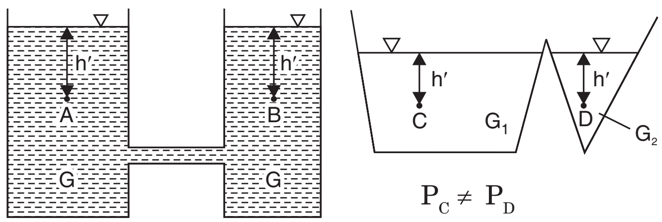
- A moving fluid exert a tangential force on the surface apart from normal forces. However in absence of motion, fluid will exert normal force only.

Pressure Variation in Vertical direction for fluid at rest

$P_{gauge} = G\,\gamma_{\omega}\,h$ (G is Specific gravity of liquid)



Pressure at a point in a fluid at rest is independents of shape and cross-section of container in which it is kept. It varies in vertical direction and remains constant in horizontal direction.



$P_A = P_B$

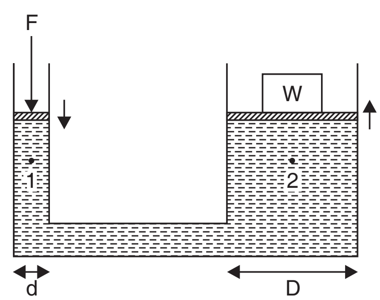
$P_C \neq P_D$
(as $G_1 \neq G_2$)

Note:
As per **Pascal's law,** pressure applied to a **confined** fluid increases the pressure throughout by the same amount.

$$P_1 = P_2$$

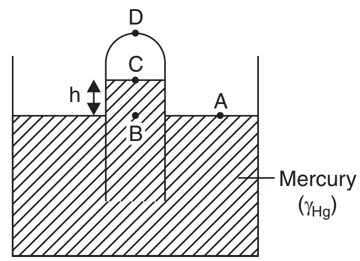
$$\frac{F}{\frac{\neq}{4}d^2} = \frac{W}{\frac{\neq}{4}D^2}$$

Hence $$W = F\left(\frac{D}{d}\right)^2$$



Principal of conservation of energy is still valid as if higher weight will be lifted up by small distance when smaller force comes down by large distance.

Toricceli-Barometer



- Used to measure Atmospheric pressure
- A mercury filled tube is inverted to a mercury container that is open to atmosphere

$$P_A = P_B = P_{atm}$$
$$P_C = \text{Absolute zero}$$
$$P_B = P_C + \gamma_{Hg}\, h$$

Hence $\quad P_{atim} = \gamma_{Hg}\, h$

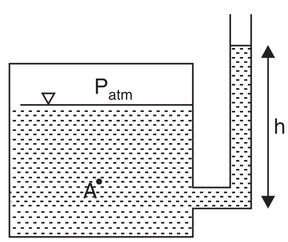
- If a hole is made at D, level of C will come down to B.

Measurement of fluid pressure

- Monometer
  - Based on principle of balancing a column of fluid by the same or other column
  - Simple manometer — Used to measure pressure at a point
    - Piezometer
    - U-Tube manometer
    - Single column manometer
  - Differential manometer — Used to measure the pressure difference
    - Inverted differential manometer
    - Micro manometer
- Mechanical gauges
  - Mechanical pressure measuring instruments with a deflecting needle (used in filling air in tyres)

1. Piezometer

- –ve pressure cannot be measured
- Very long column of piezometer is required if pressure is large.
- Cannot measure the gas pressure.



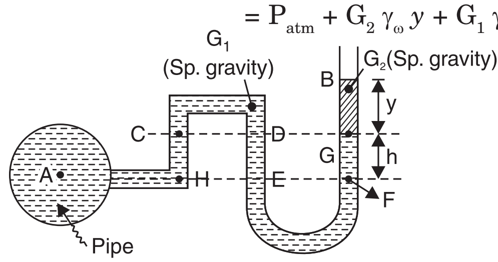
$P_A = \gamma_h$

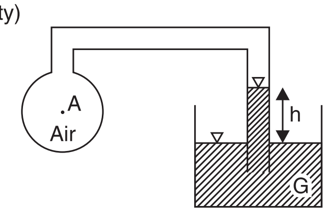
2. U-Tube manometer

- for large pressure measurement
- for measuring gas pressure
- for measurement of –ve pressure

Pressure at A = Pressure at H

$$= P_{atm} + G_2\,\gamma_\omega\, y + G_1\,\gamma_\omega\, h$$





$P_A = G\gamma_\omega h$

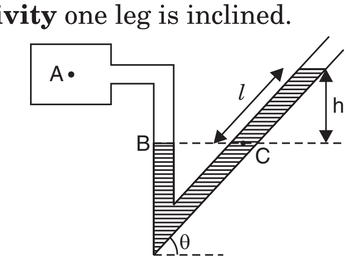
Note:
Liquid in U-tube manometer, should have specific gravity more than the liquid whose pressure is to be measured.

- Liquid should also have small thermal coefficient and small vapour pressure
- Liquid should be completely immiscible with the liquid whose pressure is to be measured.

Special Case: To increase the **sensitivity** one leg is inclined.

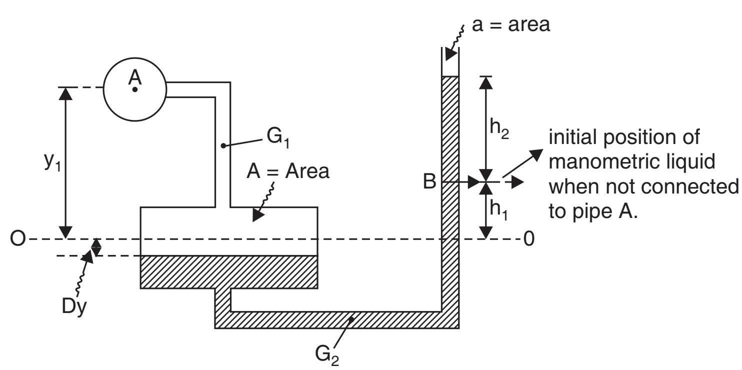
$P_A = P_B = P_C = G\,\gamma_\omega\, h = G\,\gamma_\omega\,(l \sin\theta)$

measured reading of tube = 'l'



3. Single Column Manometer:

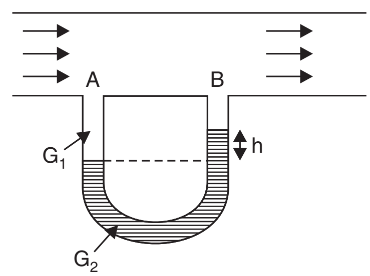
In this case **only one reading** is required as against the two readings on two limbs in case of U-tube manometer.



$$\% \text{ error} = \frac{G_2 \gamma_\omega h_2}{G_2 h_2 \gamma_\omega + (G_2 - G_1)\frac{a}{A} h_2 \gamma_\omega} \times 100 = 1\text{–}2\%$$

$$P_A = G_2 h_2 \gamma_\omega + (G_2 - G_1)\frac{a}{A} h_2 \gamma_\omega$$

Concept of differential manometer

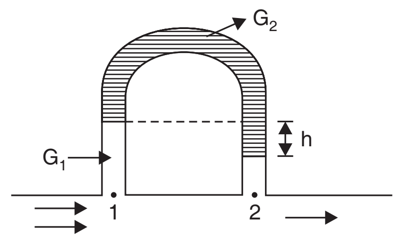


$$P_A - P_B = (G_2 - G_1)\gamma_\omega h$$

$$P_A - P_B = (\text{head loss})\, G_1 \gamma\omega$$

$$\frac{P_A - P_B}{G_1 \gamma_\omega} = \left(\frac{G_2}{G_1} - 1\right)h$$

Inverted differential manometer (for measuring small pressure difference)



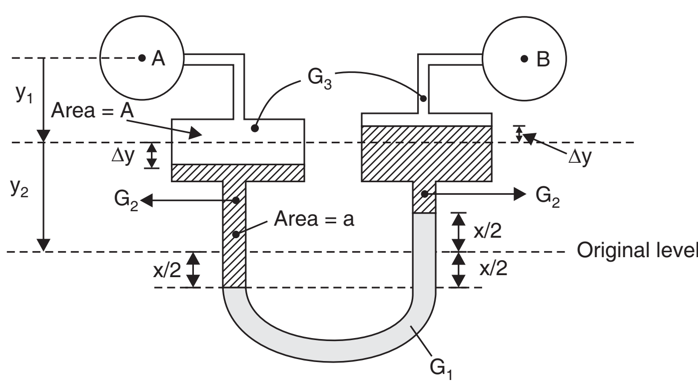
$$P_1 - P_2 = (G_1 - G_2)\gamma_\omega h$$

Here, $G_1 > G_2$ otherwise fluid G_2 will drop in pipe as soon as flow stops.

Here also sensitivity can be increased by inclining the gauge tube.

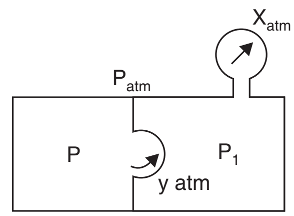
2. **Micromanometer:** measures very small pressure difference or for measuring the pressure difference with high precision.

   $P_A - P_B = (G_1 - G_2)\, x\, \gamma_\omega$



Note:
Multi fluid manometer is used in measuring pressure in a pressurized water tank.

$$P_1 = P_{atm} + x_{atm}$$
$$P = P_1 + y_{atm}$$
$$P = P_{atm} + x_{atm} + y_{atm}$$
$$P_{atm} = 1 \text{ atm}$$

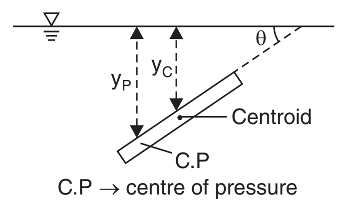


Factual facts about pressure

1. Longer runway's needed at higher altitude due to reduced drag and lift.
2. Aeroplane cruise as higher altitude's because of less drag, which increases fuel efficiency.
3. Nose bleeding starts at higher altitude because of difference in body's blood pressure and atmosphere pressure.
4. Motor capacity reduces at higher altitude.
5. Cooking takes longer time at higher altitudes.

3 Hydrostatic–Forces

Forces at every point on the **plane** surface can be added algebrically to obtain the magnitude of resultant force on the plane surface.



It the surface is **Curved,** then at every point, the direction of force due to stationary fluid is **Normal** to the surface

$$y_P = y_C + \frac{I_G \sin^2\theta}{A y_C}$$

I_G = MOI about the centroidal axis

y_P = Centre of pressure from liquid surface

y_C = Centroid from the liquid surface

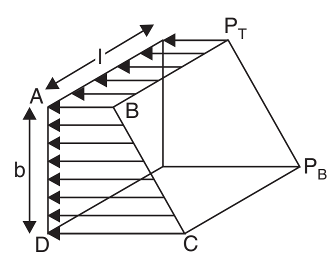
Note:

Magnitude of the resultant force acting on a plane surface of a **completely submerged plate** in a **homogenous** (constant density) fluid is equal to the product of pressure at centroid of surface and Area 'A' of the surface

$F = P_C A$ and this force acts at y_P

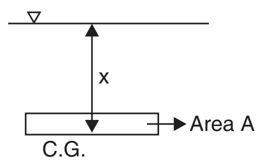
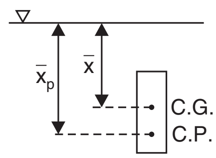
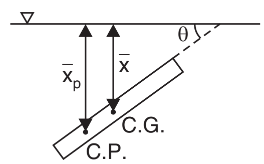
As we go deeper, difference of y_P and y_C will reduce.

Concept of Pressure Prism: Net force acting on a plane area is the product of average pressure acting on that area multiplied by the magnitude of area or it can be said as **Net force** acting on a plane area is equal to the **volume of the pressure prism** formed.

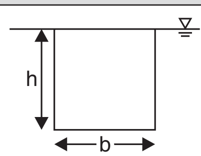
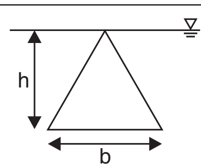
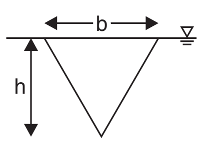
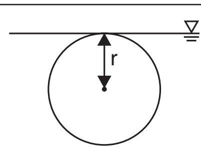


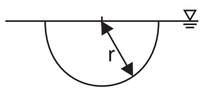
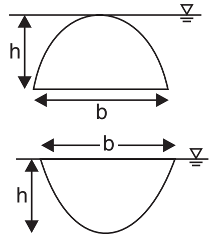
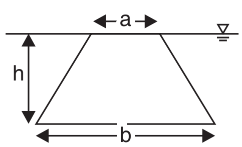
Area of trapezoid (ABCD) = $\frac{1}{2}(P_T + P_B)\,b$ (Force per unit length)

Total Force = $\frac{1}{2}(P_T + P_B)\,bl$ = Volume of prismoid

Horizontal Plane Surface	Vertical Plane Surface	Inclined Plane Surface
 $F = \gamma A\bar{x}$	 $F = \gamma A\bar{x}$ $x_p = \bar{x} + \frac{I_g}{A\bar{x}}$	 $F = \gamma A\bar{x}$ $x_p = \bar{x} + \frac{I_g \sin^2\theta}{A\bar{x}}$

$\bar{x}$ and $\bar{x}_p$ for some vertical plane surface from liquid surface

Surface	C.G.($\bar{x}$)	C.P.($\bar{x}_p$)
1. 	$\frac{h}{2}$	$\frac{2h}{3}$
2. 	$\frac{2h}{3}$	$\frac{3h}{4}$
	$\frac{h}{3}$	$\frac{h}{2}$
3. 	r	$\frac{5}{4}r$

4. 	$\frac{4r}{3\pi}$	$\frac{3\pi r}{16}$
5. 	$\frac{3h}{5}$ $\frac{2h}{5}$	$\frac{5h}{7}$ $\frac{4h}{7}$
6. 	$\left(\frac{a+2b}{a+b}\right)\left(\frac{h}{3}\right)$	$\left(\frac{a+3b}{a+2b}\right)\frac{h}{2}$

Hydrostatic forces on the curved surface:

Horizontal Force (F_H): It is the resultant hydrostatic force 'f_x' of curved surface may be computed by projecting the surface upon a vertical plane and multiplying the projected area by the pressure at its own centre of area.

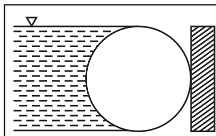
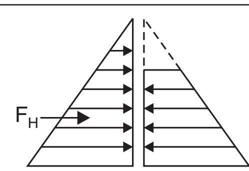
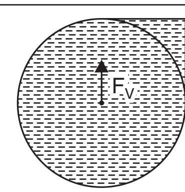
Vertical Fore (F_V): It is the weight of the liquid contained in the zone bounded by two verticals drawn from the two ends of the curved surface, the curved surface and the free surface (which is applying pressure on the curved surface)

Resultant force (F): $F = \sqrt{F_V^2 + F_H^2}$

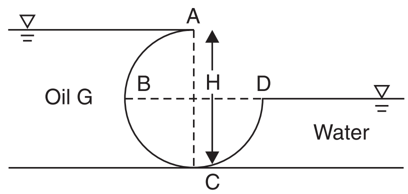
$$\tan\theta = F_y/F_x$$

$\theta \rightarrow$ Angle *b/w* line of action and Horizontal axis

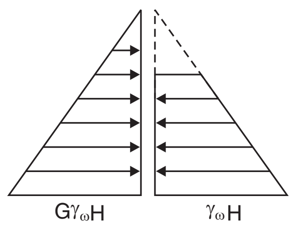
Special case (1)

	 Horizontal force (F_H) / length $\Rightarrow$ Resultant at these two pressure triangles.	 Vertical force (F_v)/length = weight of the liquid contained in the above diagram

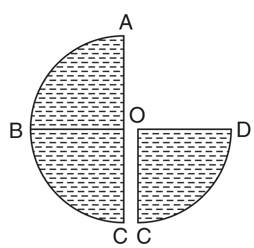
Special Case (2)



Horizontal force/length = Resultant of these two pressure triangles.



Vertical force/length = weight of oil filed in ABCO + weight of water filled in OCD



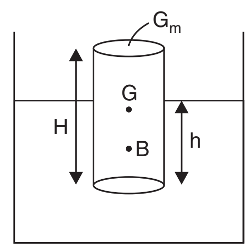
4 Buoyancy and Flotation

Archimedes Principle: When a body is wholly or partially submerged in a liquid then the vertical upward force acting on the body (called Buoyant force) is equal to the weight of the liquid displaced by the immersed part of the body.

Buoyant force = Net upward force = weight of liquid displaced

Note:
Point of application of this force is the C.G of the displaced liquid and it is called centre of buoyancy.

Floatation: A body will float in a liquid, if weight of body = weight of liquid displaced by its immersed part.



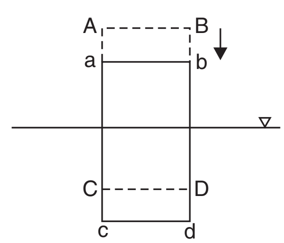
B = Centre of buoyancy at a distance of $h/2$ from base of cylinder.

$$h = G_m H$$

G = Centre of gravity, at a distance of H/2 from base of cylinder.

Where, G_m is specific gravity of material wrt liquid, which should be < 1

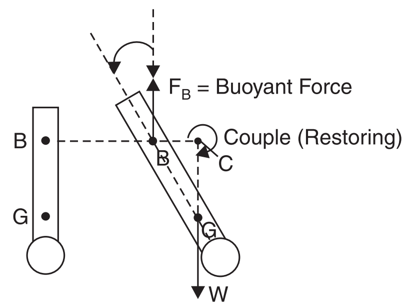
Linear Stability: When a small linear displacement sets up a restoring force, the body is in linear stability



Submerged body
It remains in neutral equilibrium against linear displacement

Floating body
Remains in stable equilibrium against vertical displacement and in neutral equilibrium against horizontal displacement

Rotational Stability: When a small angular displacement sets up a restoring couple, then stability is known as rotational stability.



	Submerged body	Floating body
Stable equilibrium	G below B	M above G BM > BG GM = MB – BG = + Ve
Unstable equilibrium	G above B	M below G BM < MG GM = BM – BG = – Ve
Neutral equilibrium	G and B coincide	M at G GM = 0

Note:

Metacentre (M) is the point of intersection of lines of action of buoyant force before and after rotation.

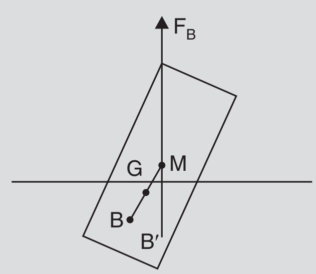
$$GM = \text{metacentre height}$$

$$GM = BM - BG$$

Where $BM = \dfrac{I}{V}$,

V is the volume of liquid displaced

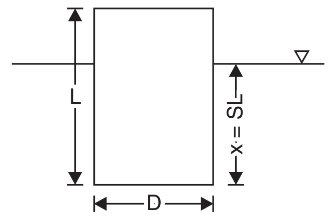
I = MOI of top view of the immersed part of the body about longitudinal axis.



Larger the metacentric height, greater the stability.

Special Case: For cylinder of specific gravity 'S' wrt liquid to be in neutral equilibrium.

$$\frac{L}{D} = \frac{1}{\sqrt{8S(1-S)}}$$



Time period of oscillation: If a floating body oscillates then its time period of transverse oscillation wrt metacentre is given by

I → MOI about axis of rotation.

$$T = 2\neq\sqrt{\frac{I}{W(\overline{GM})}}$$

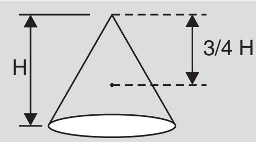
Larger the time period, more will be the comfort of passenger.

For passenger ship, $\overline{GM}$ is less so more comfortable.

For cargo ships $\overline{GM}$ is more so more stability.

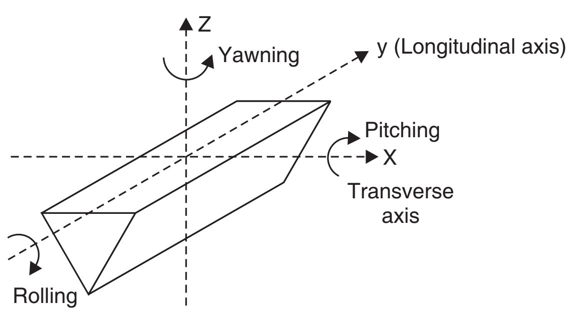
Note:

C.G of cone lies at $\frac{3}{4}$H from the pointed end



Movements of a ship:

If a ship is safe in rolling, it will also be safe in pitching.



5 Liquid in Relative Equilibrium

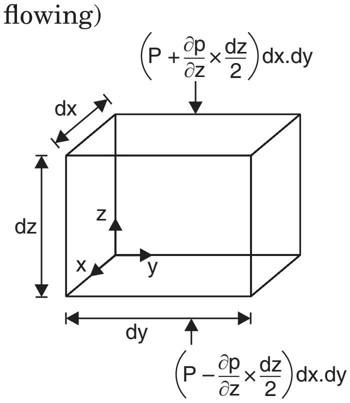
When a liquid is contained in a moving container then it behaves as a rigid body (*i.e.*, liquid is moving but not flowing)

By Newton's low of motion

$$\frac{-\partial P}{\partial x} = \gamma\frac{a_x}{g}$$

$$\frac{-\partial P}{\partial y} = \gamma\frac{a_y}{g}$$

$$\frac{-\partial P}{\partial z} = \gamma\frac{(a_z + g)}{g} \Rightarrow \text{Euler's equation}$$



Adding all three,

$$-\left(\frac{\partial P}{\partial x}\hat{i} + \frac{\partial P}{\partial y}\hat{j} + \frac{\partial P}{\partial z}\hat{k}\right) = \frac{\gamma}{g}(ax\,\hat{i} + ay\,\hat{j} + (a_z + g)\,\hat{k})$$

1. When fluid at rest $a_x = a_y = a_z = 0$

then $\frac{\partial P}{\partial x} = \frac{\partial P}{\partial y} = 0 \qquad \frac{\partial P}{\partial z} = -\delta g \Rightarrow P = -\delta g\, z$

2. When fluid moves in upward direction with constant acceleration $(-a_z)$ then

$$a_x = a_y = 0 \Rightarrow \frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0 \qquad \frac{\partial p}{\partial z} = -\rho(g - q_z)$$

$$\boxed{P = -\rho(g - a_z)z}$$

3. When fluid moves in downward direction with constant acceleration (a_z) then

$$a_x = a_y = 0 \Rightarrow \frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0 \qquad \frac{\partial p}{\partial z} = -\delta(g + a_z)$$

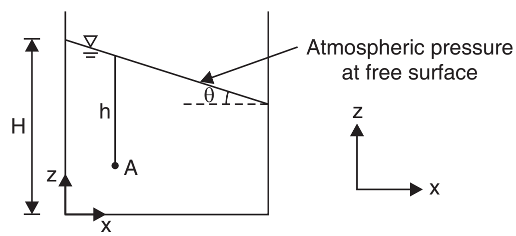
$$\boxed{P = -\rho(g + a_z)z}$$

4. With constant acceleration a_x an x-direction

$$P_A = \delta\, gh$$

$$\tan\theta = \frac{a_x}{g_{eff}}$$

$$Z = H - \frac{a_x\, x}{g_{eff}} \quad \Rightarrow \quad \text{Equation of free surface}$$



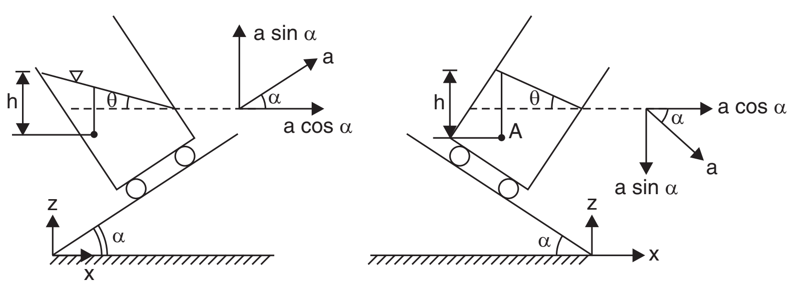
5. Constant acceleration on inclined slope

$$\tan\theta = \frac{a\cos\alpha}{g \pm a\sin\alpha}$$

$$P_A = \delta\, g_{eff}\, h$$

$$g_{eff} = g \pm a\sin\alpha$$

$+ \Rightarrow$ upward $\qquad - \Rightarrow$ downwards



Note:
θ is measured from the horizontal axis.

Rotation in Cylindrical Container.

$$\frac{\partial P}{\partial r} = \frac{\delta V^2}{r} \quad ...(i)$$

$$\frac{\partial P}{\partial z} = -\delta(a_z + g) \quad ...(ii)$$

Combining (*i*) and (*ii*)

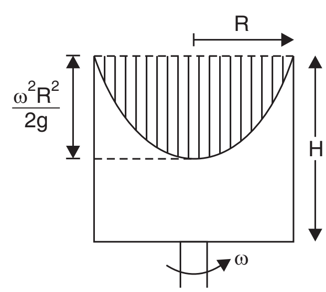
$$dP = \frac{\delta V^2}{r} dr - \delta(a_z + g)d_z$$

- **In free vortex motion**, angular momentum remains conserved as external torque is zero so mvr = constant = C

So $$V \propto \frac{C}{r}$$

Hence from (*i*) as radius increases pressure decreases. Eg: whirling mass of liquid in wash basin.

- **In forced vortex motion**, fluid is rotated about a vertical axis at constant speed such that every particle has the same angular velocity.



$$V = r\,\omega$$

Hence from (*i*) as radius increases pressure increases. Eg: Flow inside centrifugal pump.

Amount of water spilled out = original volume – Remaining volume

Remaining volume = Volume of cylinder – volume of shaded paraboloid.

Volume of cylinder = $\pi R^2 H$

Volume of paraboloid. = $\frac{1}{2}(\pi R^2)\left(\frac{\omega^2 R^2}{2g}\right)$

Note:
For No spilling case, Rise above original water level = Fall below original water level.

6 Fluid Kinematics

Fluid Kinematics: It deals with the motion of the fluids without necessarily considering the forces and moments that cause the motion.

Lagrangian concept study of motion of single fluid particle	Eulerian concept study of motion of fluid through a particular section or a control volume.

Note:
We generally follow Eulerian concept, as its difficult to keep the track of a single fluid particle.

Types of fluid show:

1. **Steady and Unsteady Flow:** At any given location, the flow and fluid properties do not change **with time** then its steady flow otherwise unsteady.

$$\frac{\partial v}{\partial t} = 0, \ \frac{\partial p}{\partial t} = 0, \ \frac{\partial f}{\partial t} = 0 \Rightarrow \text{Steady flow}$$

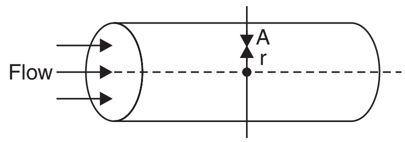
2. **Uniform and Non-Uniform Flow:** At particular instant of time, the flow properties do not change with location then its uniform flow otherwise non-uniform flow $\left.\frac{\partial v}{\partial s}\right|_{t=0} = 0$ uniform flow

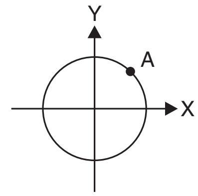
3. **One, two or three Dimensional Flow:** If flow parameters varies in one dimension wrt space only then its one dimensional otherwise its two or three dimension respectively.

$$V = V(x, t) \rightarrow \text{one dimensional}$$
$$V = V(x, y, t) \rightarrow \text{two dimensional}$$
$$V = V(x, y, z, t) \rightarrow \text{three dimensional}$$

In cartesian co-ordinate system point A is represented as $A(x, y)$

While in polar co-ordinate system properties will be same all around the circumference of circle hence will depend only on r.





4. **Laminar and Turbulent Flow:** In Laminar flow, the particles moves in layers sliding smoothly over the adjacent layers while in turbulent flow particles have the random and erratic movement, intermixing in the adjacent layers. Which causes continuos momentum transfer.

5. **Rotational and Irrotational Flow:** When fluid particles rotate about their mass centre during movement. Flow is said to be rotational otherwise irrotational.

   Rotational Flow → Forced Vortex, Flow inside boundary layer.

   Irrotational Flow → Free Vortex, Flow outside boundary layer.

6. **Compressible and In compressible Flow:** In compressible flow density of fluid changes from time to time where an in Incompressible flow it remains constant.

Flow lines

Stream lines	Path line	Streak line
An imaginary line drawn in a flow field such that tangent drawn at any point on this line represents the direction of velocity vector at that point. • Stream lines neither touch or cross each other. • Spacing varies inversely with the flow velocity or mass of flow remains constant.	Actual path travelled by an individual fluid particle over some time period. It' s based on lagrangian concept.	Locus of fluid particles that have passed sequently through a proscribed point in the flow. Formed by continuos introduction of dye or smoke from a point in the flow.

$$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \text{ Equation of stream line}}$$

Continuity Equation: Fluid mass can neither be created or can be destroyed so mass of fluid entering a fixed region should be equal to mass of fluid leaving that fixed region in a particular time. It is based on **principle of conservation of mass.**

Various forms of continuity Equation:

(*a*) **Cartesian co-ordinate System:**

(*i*) 3-D $$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$$

(*ii*) 1-D $$\frac{\partial(\rho A)}{\partial t}+\frac{\partial(\rho A v)}{\partial s}=0$$

(*iii*) Steady Flow in 1-D $\rho\, AV = \text{Constant}$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

(*iv*) Steady Incompressible in 1-D $A_1V_1 = A_2V_2$

(*v*) Steady flow in 3D $$\frac{\partial(\rho u)}{\partial y}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$$

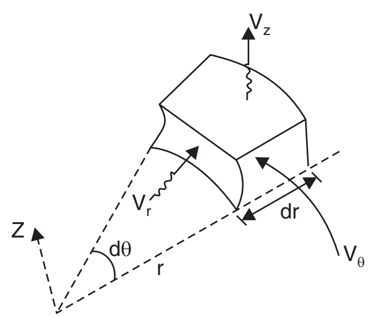
(*vi*) Steady Incompressible in 3D $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Divergence of Velocity = 0 *i.e.*, $\vec{\nabla}\cdot\vec{v} = 0$

(*b*) **Cylindrical Polar considerate System**

(*i*) 3-D $$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial(\rho r V_r)}{\partial r}+\frac{\partial(r V_\theta)}{r d\theta}+\frac{\partial(\rho V_z)}{\partial z}=0$$

(*ii*) Steady Incompressible in 2D $$\frac{\partial(r V_r)}{\partial r}+\frac{\partial V_\theta}{\partial\theta}=0$$



Acceleration of fluid:

$$\vec{V} = u(x, y, z, t)\,\hat{i} + v(x, y, z, t)\,\hat{j} + w(x, y, z, t)\,\hat{k}$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

Local/Temporal acceleration *i.e.*, acceleration component wrt time

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{dt}$$

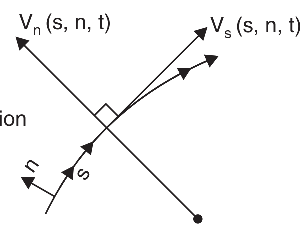
For steady Uniform flow, Total acceleration = 0

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

Convective/Advective acceleration *i.e.*, acceleration wrt place

Acceleration on a stream line

Tangential Acceleration
It is due to change in magnitude of velocity. If spacing b/w stream line changes tangensial acceleration exists



Normal Acceleration
It is due to the change in the direction of fluid moving on a curved path

$$a_s = V_s\frac{\partial v_s}{\partial s} + \frac{\partial v_s}{\partial t}$$

convective tangential acceleration + local tangential acceleration

$$a_n = V_s\frac{\partial v_n}{\partial s} + \frac{\partial v_s}{\partial t}$$

convective normal acceleration + local normal acceleration

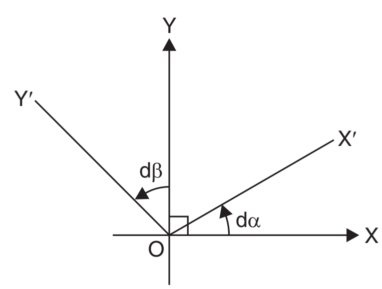
Angular Velocity: It is the average of rotation rate of two initially perpendicular lines that intersect at that point.

$$\frac{d\theta}{dt} = \frac{\left(\frac{d\alpha}{dt} + \frac{d\beta}{dt}\right)}{2}$$

$$\Omega = \omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k}$$

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$



$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\Omega = \frac{1}{2}\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{bmatrix}$$

Note:

It angular velocity is zero, flow will be irrotational.

Vorticity (ξ) = Twice of Angular Velocity

Circulation (Γ) = line integral of tangential component of velocity vector along a closed curve.

Circulation = Vorticity × Area

Velocity Potential or Potential Function (ϕ): It is the scalar function of space and time such that its negative derivative wrt any direction gives velocity of flow in that direction.

In Cartesian co-ordinate System

$$\phi = f(r, y, z, t)$$

$$\frac{-\partial\phi}{\partial x} = u, \frac{-\partial\phi}{\partial y} = v, \frac{-\partial\phi}{\partial z} = w$$

In Cylindrical Polar co-ordinate System

$$\phi = f(r, \theta, z, t)$$

$$V_r = \frac{-\partial\phi}{\partial r}, \ V_\theta = \frac{-1}{r}\frac{d\phi}{d\theta}, \ V_2 = \frac{-\partial\phi}{\partial z}$$

Note:

1. Velocity potential exists only for ideal and irrotational flow.
2. Velocity of flow is in direction of decreasing potential function.
3. Equipotential line is the line joining points having same potential function.

Stream Function (ψ): It is a scalar function of space and time such that its partial derivative wrt. any direction gives the velocity component at right angles (in anti clock wise direction) to this direction.

Cartesian co-ordinate system

$$\frac{\partial \Psi}{\partial x} = v, \quad \frac{\partial \Psi}{\partial y} = -u$$

Polar co-ordinate system

$$\frac{\partial \psi}{\partial r} = V_\theta \qquad \frac{\partial \psi}{r d\theta} = -V_r$$

Note:

If Stream function (ψ) satisfies the Laplace equation, then flow is irrotational otherwise rotational *i.e.*,

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = 0$$

It two points lie on same straight line then ψ will be constant.

Cauchy-Riemann Equation: For incompressible irrotational flow

$$u = \frac{-\partial \phi}{\partial x} = \frac{-\partial \Psi}{\partial y} \qquad v = \frac{-\partial \phi}{\partial y} = \frac{\partial \Psi}{\partial x}$$

7 Fluid Dynamics

It is the study of motion of fluid along with the forces causing the motion.

(*i*) Newton's equation of motion

$$\vec{F}_g + \vec{F}_P + \vec{F}_V + \vec{F}_t + \vec{F}_c + \vec{F}_\sigma = m\vec{a}$$

(*ii*) Reynold's equation of motion

$$\vec{F}_g + \vec{F}_P + \vec{F}_V + \vec{F}_t = m\vec{a}$$

(*iii*) Navier-stock equation of motion

$$\vec{F}_g + \vec{F}_P + \vec{F}_V = m\vec{a}$$

(*iv*) Euler's equation of motion

$$\vec{F}_g + \vec{F}_P = m\vec{a}$$

where, F_g = Gravity force
F_P = Pressure force
F_V = Viscous force
F_t = Turbulence force
F_c = Compressibility force
F_σ = Surface tension force

Note:
Euler equation based on momention conservation while Bernoulli is based on energy conservation.

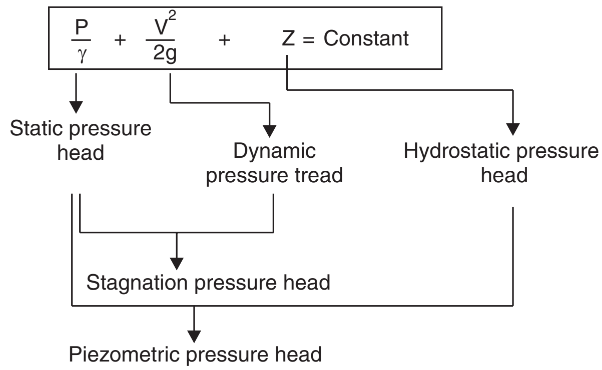
Bernoulli's Equation: It is the integration of Euler's equation of motion along a stream line under steady incompressible flow conditions.

Assumptions:

(*i*) Along Stream line

(*ii*) Ideal flow

(*iii*) Steady and Incompressible



Note:
When normal acceleration is zero *i.e.*, when particles move on a straight line then the piezometric head is a constant.

Kinetic Energy Correction Factor (α)

$$\alpha = \frac{\text{Actual K.E.}}{\text{K.E. Calculated from Average Velocity}}$$

$$\alpha = \frac{\int_A v^3 dA}{A\,V_{avg}^2} \qquad V_{avg} = \int_A v\,dA$$

Momentum Correction Factor (β)

$$\beta = \frac{\text{Actual linear momentum/sec}}{\text{Linear momentum calculated from Average Velocity}}$$

$$\beta = \frac{\int_A v^2 dA}{A\,V_{avg}^2} \qquad V_{avg} = \int_A v\,dA$$

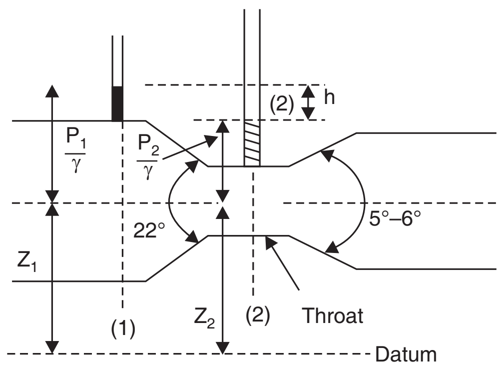
Note:

	α	β
Laminar flow between plates	2	4/3
Laminar flow between plates	1.543	1.2
Turbulent flow in pipes	4/3	1.015

Applications of Energy Equation :-

1. **Venturimeter**

- To find discharge from a large diameter pipe
- Reduction in Area leads to increase in Velocity and decrease in pressure, this pressure decrease is noted and used in Bernoulli to calculate discharge.



Piezometric head difference $h = \dfrac{V_2^2}{2g} - \dfrac{V_1^2}{2g}$

- $Q_{actual} = c_d \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$

$\Rightarrow a_1, a_2$ cross-sectional areas at section 1 and 2

$\Rightarrow c_d \rightarrow$ discharge coefficient

$\Rightarrow \dfrac{a_1}{a_2}$ = area ratio

$\Rightarrow \dfrac{a_1 a_2 \sqrt{2g}}{\sqrt{a_1^2 - a_2^2}}$, as this depends only on dimensions of venturimeter

it is called venturi-constant.

- $d \approx \left(\dfrac{1}{3} \text{ to } \dfrac{3}{4}\right)$D where d = dia of throat

D = dia of pipe

- $c_d = \sqrt{\dfrac{h - h_L}{h}} \approx 0.98$
- If inclined Venturimeter with differential manometer is used then piezometer head difference (h)

$$\left[\begin{matrix} h = \left(\dfrac{G_2}{G_1} - 1\right)x \\ G_2 > G_1 \end{matrix}\right] \text{ or } \left[\begin{matrix} h = \left(1 - \dfrac{G_2}{G_1}\right)x \\ G_2 < G_1 \end{matrix}\right]$$

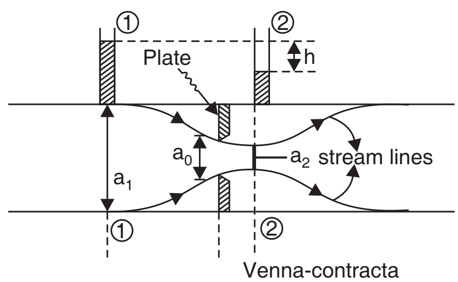
$G_2 \rightarrow$ always of liquid filled in manometer

$G_1 \rightarrow$ of flowing liquid.

$x \rightarrow$ reading on manometer

2. Orificemeter

- Circular plate with concentric sharp edged hole is installed in a pipe such that the plate is perpendicular to the axis of pipe.



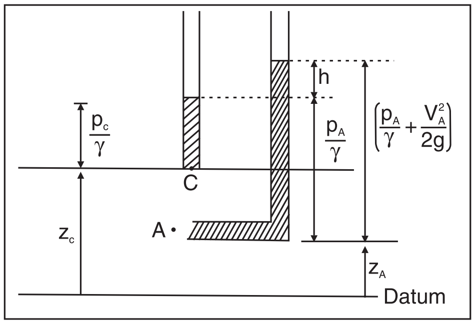
- Cheaper instrument, measures discharge but has more losses hence $c_d = 0.64 - 0.76$
- Region of minimum flow area is called Vena contractra, here stream lines are assumed to be nearly parallel.

$$c_c = \frac{a_2}{a_0} = \frac{\text{Area of Vena contractra}}{\text{Area of opening}}$$

- $Q_{\text{actual}} = c_d \dfrac{a_1 a_0}{\sqrt{a_1^2 - a_0^2}} \sqrt{2gh}$

Note:
It the discharge is changed, then the position of Vena contractra will also change and then stream lineas will not be parallel at sec (2)-(2).

3. Pitot tube:



- Measures the velocity of fluid
- $\frac{V_A^2}{2g} = h$ = measured

$V_a = \sqrt{2\,gh}$

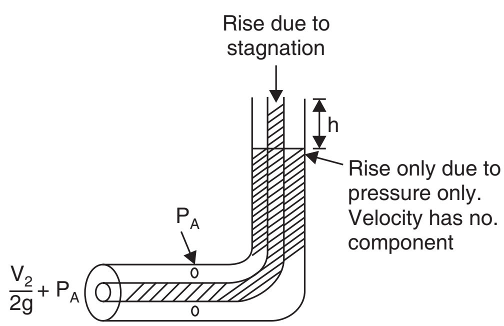
- $V_{a\text{ actual}} = C_V\sqrt{2gh}$

$C_V = 0.98$ (coefficient of velocity)

- **Anemometer** measures gas and air velocity.

4. Pitot Static tube (Prandtl tube)

- Measures the velocity of fluid.
- It measures the piezometric head at the same point where velocity is to be measured.



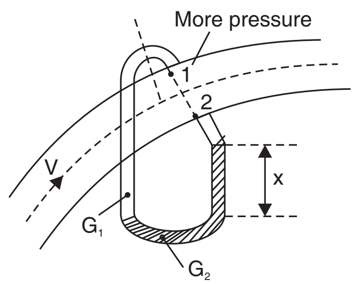
Prandtl tube

$$V_A = C_V\sqrt{2gh}$$

- Velocity head is found out from difference of total head and piezometric head.
- It can also used on rough boundaries.

5. Elbow meter or Bend meter

- Measures discharge
- When liquid moves along a bend pipe in free vortex $\left(V = \frac{c}{r}\right)$ then its pressure increases with radius.



Elbow metere

- More r, less v, more P → at outer surface 1 or compared to 2
- $Q = c_d\ A\sqrt{2g\left(\frac{G_2}{G_1} - 1\right)x}$

Note:
If in venturimeter, the pipe is not contracted such that $c_c = 1$ then it is termed as Nozzle meter and it is also used for calculating discharge.

- **Rotameter** is used to measure discharge while current meter is used to measure velocity in open channel.

Momentum Equation and Application

8

Rate of change of linear momentum in any direction of a body with respect to a fixed frame of reference is equal to external forces acting on the body in that direction.

$$\begin{bmatrix} \Sigma F_{x\text{external}} \\ \text{on control} \\ \text{volume} \end{bmatrix} = \begin{bmatrix} \text{Net rate of flow} \\ \text{of linear} \\ \text{momentum out} \\ \text{of control volume} \\ \text{in x-direction} \end{bmatrix}$$

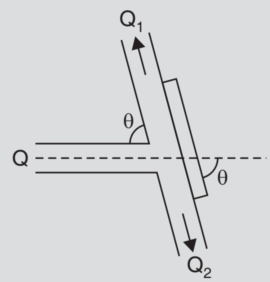
$$= \begin{bmatrix} \text{Rate of flow} \\ \text{of momentum} \\ \text{out of control} \\ \text{volume in} \\ x\text{-direction} \end{bmatrix} - \begin{bmatrix} \text{Rate of flow} \\ \text{of momentum} \\ \text{into the control} \\ \text{volume in} \\ \text{x- direction} \end{bmatrix}$$

Rate of change of Angular momentum in any direction of a body with respect to a fixed frame of reference is equal to torque applied on the body in that direction.

$$\begin{bmatrix} \Sigma M_{z\text{ external}} \\ \text{on control} \\ \text{volume} \end{bmatrix} = \begin{bmatrix} \text{Net torque on} \\ \text{control volume} \end{bmatrix}$$

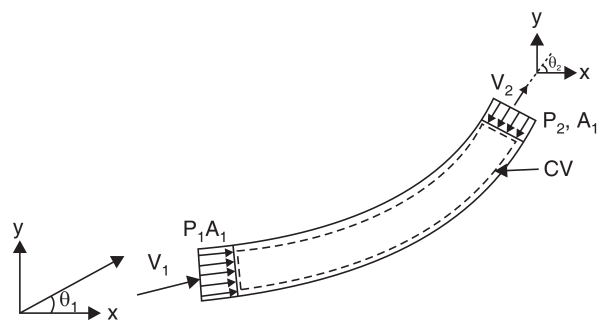
$$= \begin{bmatrix} \text{Rate of change} \\ \text{of angular momentum} \\ \text{of control volume wrt} \\ \text{fixed frame of reference} \\ \text{in } z\text{-direction} \end{bmatrix} + \begin{bmatrix} \text{Net rate of flow of} \\ \text{angular momentum} \\ \text{out of control volume} \\ \text{wrt fixed frame of} \\ \text{reference in } z\text{-direction} \end{bmatrix}$$

Note:



$$Q_1 = \frac{Q(1-\cos\theta)}{2} \qquad Q_2 = \frac{Q(1+\cos\theta)}{2}$$

Force acting on a bend pipe:



In x-direction:

$\Sigma F_x = \rho Q\,(v_2 \cos\theta_2 - v_1 \cos\theta_1)$

$P_1A_1 \cos\theta_1 - P_2A_2 \cos\theta_2 + R_x = \rho Q\,(v_2 \cos\theta_2 - v_1 \cos\theta_1)$

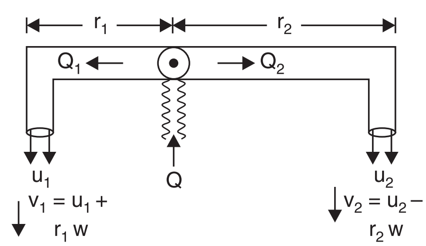
In y-direction: $\Sigma F_y = \rho Q\,(v_2 \sin\theta_2 - v_1 \sin\theta_1)$

$P_1A_1 \sin\theta_1 - P_2A_2 \sin\theta_2 + R_y = \rho Q\,(v_2 \sin\theta_2 - v_1 \sin\theta_1)$

Resultant force $R = \sqrt{R_x^2 + R_y^2}$

Sprinkler:

Torque on jet in z-direction = $\Sigma mvr = 0$



$\rho Q_2 v_2 r_2 - \rho Q_1 v_1 r_1 = 0$

$\text{Torque} = \rho(A_2 u_2) v_2 r_2 - \rho(A_1 u_1) v_1 r_1 = 0$

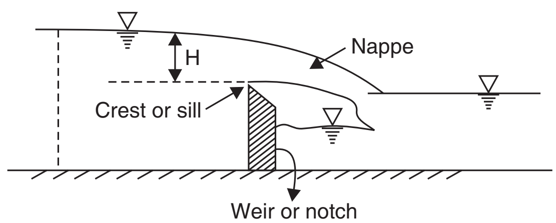
Note:

- Discharge Q is measured wrt relative velocity $Q_1 = A_1 u_1$, $Q_2 = A_2 u_2$
- v_1 and v_2 are absolute velocities of jet wrt ground

If net torque is zero then $\rho Q_1 v_1 r_1 = \rho Q_2 v_2 r_2$

9 Weir and Notches

Weir: It is a concrete or masonary structure, constructed in an open channel to measure its discharge. It is generally in the form of vertical wall, with sharp edge at the top, running all the way across the open channel.



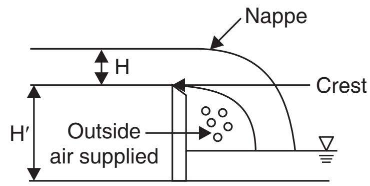
Notch: It is a device (generally metallic plate) used for measuring the discharge through a small channel or a tank.

Types of weirs

- Shape of opening
  - Rectangular
  - Triangular
  - Trapezoidal
  - Cipollettie
- Shape of crest
  - Sharp crested
  - Broad crested
  - Narrow crested
  - Ogee-shaped
- Effect of sides on emerging nappe
  - With end contraction
  - Without end contraction
- Notre of discharge
  - Ordinary weir
  - Submerged weir

1. **Rectangular sharp-crested Suppressed weir:**
   - Suppressed $\Rightarrow$ without end contraction.
   - $Q_{actual} = \frac{2}{3} c_d \, L\sqrt{2}\ g\ H^{2/3} \quad c_d = 0.62$

   H $\rightarrow$ depth of water above crest level



- If velocity of approach (V_a) is also considered then

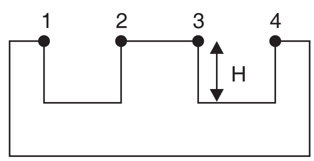
$$V_a = \frac{Q}{(H + H')L}, \quad h_a = \frac{V_a^2}{2g}, \quad Q = \frac{2}{3} c_d \sqrt{2g}\, L\left[(H + h_a)^{3/2} - h_a^{3/2}\right]$$

- Effect of end contraction *i.e.*, if not suppressed L is replaced by Left

$$L_{eff} = L - 0.1\, nH$$

n = Number of end contractions (Here its 4)

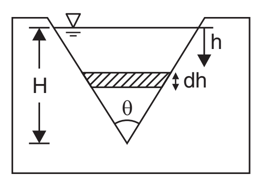
$$Q = \frac{2}{3} c_d \sqrt{2g}\, L_{eff}\, H^{3/2}$$



- When there is no ventilation of Nappe (*i.e.*, air is not supplied from outside) then the discharge will increase or the Nappe will be pulled down due to negative pressure created in the zone below Nappe.

2. Flow over V-Notch or triangular weir:

- $Q = \frac{8}{15} c_d \sqrt{2g} \tan\frac{\theta}{2} H^{5/2}$ $\qquad c_d = 0.52$



- Considering velocity of approach then

$$Q = \frac{8}{15} c_d \sqrt{2g} \tan\frac{\theta}{2}\left[(H + h_a)^{5/2} - h_a^{5/2}\right]$$

- End contraction is not taken into account in this case.

Advantages

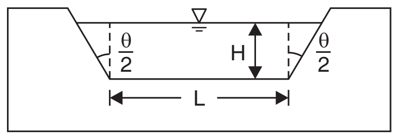
(*a*) Only one dimension is to be measured hence more accurate

(*b*) c_d nearly constant with depth.

(*c*) Even for small discharge, high head is obtained. Hence no effect of viscosity and surface tension.

3. Trapezoidal Notch or weir:

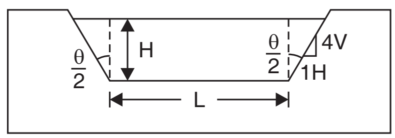
$$Q = \frac{2}{3} c_{d_1} \sqrt{2g}\, L\, H^{3/2} + \frac{8}{15} c_{d_2} \sqrt{2g} \tan\frac{\theta}{2} H^{5/2}$$



4. Cipolletti-Weir:

It is a trapezoidal weir whose slopes are adjusted such that:

Decrease in discharge due to end contraction in rectangular weir = Increase in discharge due to triangular portion.



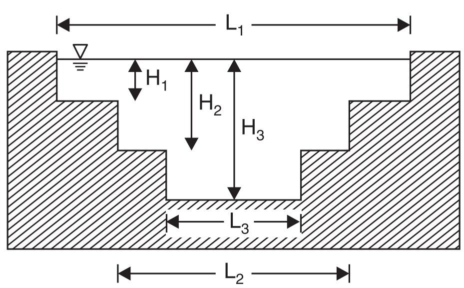
$$\tan\frac{\theta}{2} = \frac{1}{4},\ \theta = 28°$$

$$Q = \frac{2}{3} c_d \sqrt{2g} L H^{3/2}$$

$$c_d = 0.63$$

5. Stepped Notch

$$Q = \frac{2}{3} c_d \sqrt{2g}\left[L_1 H_1^{3/2} + L_2 H_2^{3/2} + L_3 H_3^{3/2}\right]$$

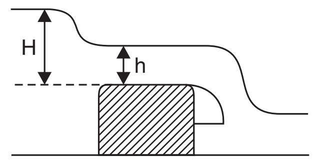


Note:
H_1, H_2, H_3 are measured from the top.

6. Broad crested weir

- Supports a Nappe such that stream lines become straight and pressure variation become hydrostatic over the weir.

- $Q = c_d Lh \sqrt{2g(H - h)}$
- In this, flow adjusts itself to give max. discharge at available head H.
- For max, discharge



$$h = \frac{2}{3}H \qquad Q = 1.7\, c_d\, LH^{3/2}$$

$c_d = 0.85 - 1,$

h = critical depth as discharge is maximum

- If Velocity of approach is also considered

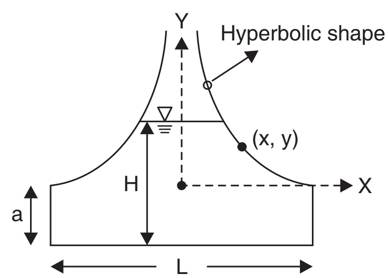
$$Q = 1.7\, c_d\, L\,[(H + h_a)^{3/2} - h_a^{\ 3/2}]$$

7. **Proportional weir:**
   - $Q \,\alpha\, H$

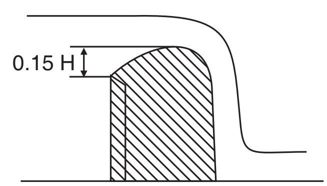
$$Q = K\left(H - \frac{a}{3}\right) \qquad K = c_d\, L\sqrt{2ga}$$

$$c_d = 0.6 - 0.65$$

- Equation of curve $\dfrac{2x}{L} = 1 - \dfrac{2}{\pi}\tan^{-1}\left(\sqrt{\dfrac{y}{a}}\right)$
- If there are fluctuations in discharge, then there will be less fluctuations in 'H' as compared to rectangular and triangular weirs.



8. **Ogee spillway**

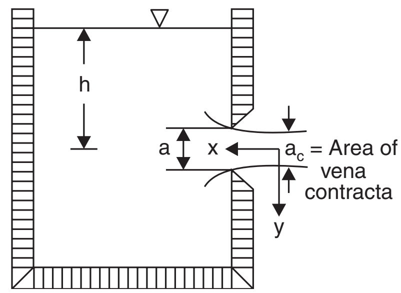


- Profile of the crest is made such that it matches with the shape of water profile over sharp crested weir so as to avoid development of negative pressure below nappe (or Adhering Nappe).

$$Q = \frac{2}{3} c_d\, L\sqrt{2g}\, H^{3/2} \qquad c_0 = 0.62$$

9. Flow through orifice

- Orifice is small opering in tank
- $Q = c_d\, a\sqrt{2gh}$



$$c_c = \frac{a_c}{a} \qquad c_d = c_c \times c_v$$

For equation of flow, eliminate 't' from below equations

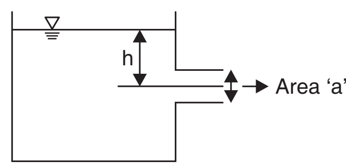
$$x = vt \qquad y = \frac{1}{2}gt^2 \qquad \text{and get } V = \sqrt{\frac{gx^2}{2y}}$$

- For c_v, put $V = c_v\sqrt{2gh}$ and get $c_v = \sqrt{\frac{x^2}{4yh}}$
- Coefficient of resistance

$$c_r = \frac{\text{loss of KE through orifice}}{\text{Actual KE}} = \left(\frac{1}{c_v^2} - 1\right)$$

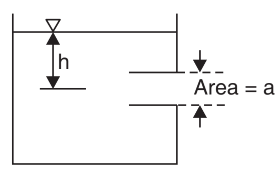
10. Flow through mouth piece

- Mouth piece is short length of tube with length < (2 - 3) diameter
- $Q = c_d\, a\sqrt{2gh} \qquad c_d = 0.82$



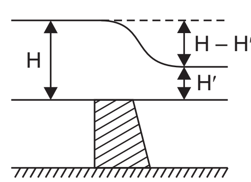
11. Borda's weir

$$Q = c_d\, a\sqrt{2gh}$$



12. Submerged weir

- When downstream water level is above the crest of the weir then it is said to be submerged
- $Q = \frac{2}{3}c_{d_1}\, L\sqrt{2g}\,(H - H')^{3/2} + c_{d_2}\, LH'\sqrt{2g(H - H')}$



- Sharp crested weir is more susceptible to submergence than a broad crested weir

- Modular limit, is the limiting value of submergence ratio upto which submerged weir behaves as free weir.

Submergence ratio = $\left(\dfrac{H'}{H + \dfrac{v_a^{\,2}}{2g}}\right)$ generally = 0.83 to 0.85

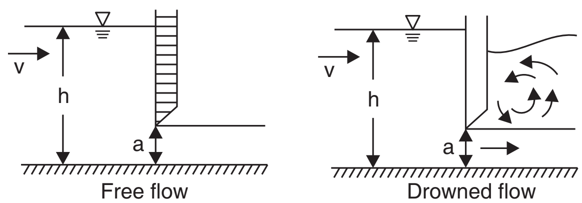
13. Discharge through sluice gate

$$v = \sqrt{2gh}$$

$$Q = c_d\, a\, L\, \sqrt{2gh}$$

L = Inside length

h = Water depth from ground



Effect on discharge due to error in head measurement

(i) For infinitesimal error's in head measurement

$$Q = KH^n$$

$$dQ = Kn\, H^{n-1}\, dH$$

$$\frac{dQ}{Q}\times 100 = n\left(\frac{dH}{H}\times 100\right)$$

% error in discharge = n × % error in head measurement

n = 1 proportional weir,

n = 1.5 rectangular weir,

n = 2.5 triangular weir.

(ii) For Large error's in head measurement

- $\dfrac{Q_2 - Q_1}{Q_1}\times 100 = \dfrac{KH_2^n - KH_1^n}{KH_1^n}\times 100$

- For V-notch $\dfrac{dQ}{Q}\times 100 = \left(\dfrac{\theta}{\sin\theta}\right)\left(\dfrac{d\theta}{\theta}\times 100\right)$

10 Laminar Flow

In Laminar flow fluid particles move along the straight parallel paths in layers. It occurs at a very low velocity, and here **Viscous force predominates the inertial forces**.

Nature of flow according to Reynold's number (R_e)

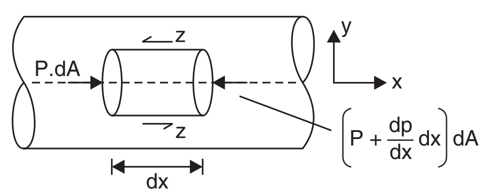
	Laminar	Transition	Turbulent
Flow in pipe	$R_e < 2000$	$2000 < R_e < 4000$	$R_e > 4000$
Flow between parallel plate	$R_e < 1000$	$1000 < R_e < 2000$	$R_e > 2000$
Flow in open channel	$R_e < 500$	$500 < R_e < 2000$	$R_e > 2000$
Flow through soil	$R_e < 1$	$1 < R_e < 2$	$R_e > 2$

Flow through circular pipe (steady uniform flow)

1. $\dfrac{dp}{dx} = \dfrac{dz}{dy}$

x is the direction of flow

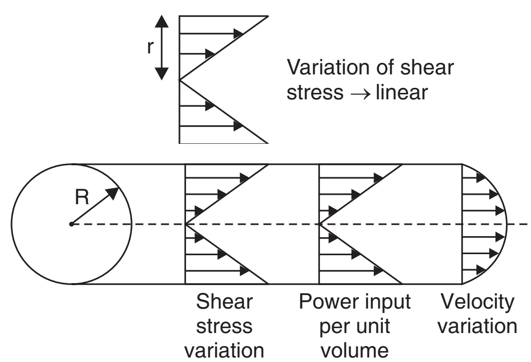
y is perpendicular to x



2. $\tau = z = \dfrac{-dp}{dx}\left(\dfrac{r}{2}\right)$

3. $V = V_{max}\left(1 - \dfrac{r^2}{R^2}\right)$

4. $V_{max} = \dfrac{1}{4\mu}\left(\dfrac{-dp}{dx}\right)R^2$



5. $\tau_{max} = \dfrac{2\mu V_{max}}{R} = \left(\dfrac{P_1 - P_1}{L}\right)\dfrac{R}{2}$

6. $Q = \dfrac{\pi}{128\mu}\left(\dfrac{-dp}{dx}\right)D^4$ Hagen Poiseuille Formula

7. $V_{avg} = \dfrac{Q}{\pi R^2} = \dfrac{1}{32\mu}\left(\dfrac{-\partial p}{\partial x}\right)D^2$

8. $V_{max} = 2\ V_{avg}$

9. $V = V_{avg}$ at $r = \dfrac{R}{\sqrt{2}} = 0.0707\ R$

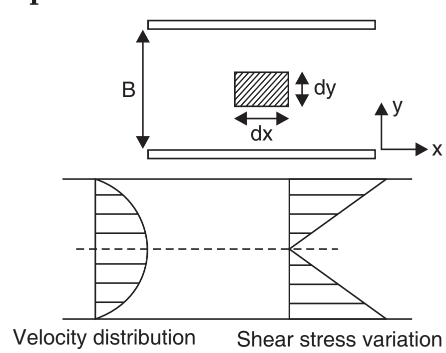
10. $h_L = \dfrac{32\mu V^2 L}{\gamma D^2} = \dfrac{128\mu QL}{\pi\gamma D^4}$

11. $h_L = \dfrac{flV^2}{2gD} = \dfrac{(4f')lV^2}{2gD}$

f = friction factor = $\dfrac{64}{R_c}$

f' = coefficient of friction

Flow between two fixed parallel plates



1. $u = \dfrac{1}{2\mu}\left(\dfrac{-dp}{dx}\right)(By - y^2)$

2. $\tau = \mu\dfrac{du}{dy} = \dfrac{1}{2}\left(\dfrac{-dp}{dx}\right)(B - 2y)$

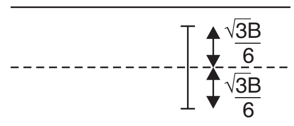
3. $Q = \dfrac{1}{12\mu}\left(\dfrac{-dp}{dx}\right)B^3$

4. $V_{avg} = \dfrac{Q}{A} = \dfrac{1}{12\mu}\left(\dfrac{-dp}{dx}\right)B^2$

5. $V_{max} = \dfrac{1}{8\mu}\left(\dfrac{-dp}{dx}\right)B^2$

6. $V_{max} = \dfrac{3}{2}V_{avg}$

7. $V = V_{avg}$ at $y = \dfrac{B}{2} \pm \dfrac{\sqrt{3}B}{6}$

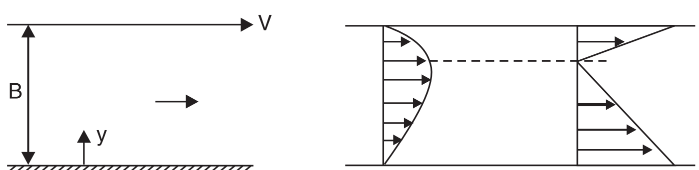


8. $h_L = h_L = \dfrac{12\mu VL}{\gamma B^2}$

For couette flow (one plate moving other at rest)

$$u = \frac{V_y}{B} + \frac{1}{2\mu}\left(\frac{-dp}{dx}\right)(By - y^2)$$

$$\tau = \frac{\mu V}{B} + \left(\frac{-dp}{dx}\right)\left(\frac{B}{2} - y\right)$$



Velocity distribution

Shear stress variation

Entrance length: The length of pipe from its entrance upto the point where flow attains fully developed velocity profile and which remains unaltered beyond that the known as entrance length.

For Laminar Flow $L_e = 0.07\, R_e\, D$

For Turbulent Flow $L_e = 50\, D$

NOTE:

For stability of laminar flow, a dimensionless parameter χ (chi) has been defined where $\chi(\text{Chi}) = \dfrac{y^2\delta}{\mu}\left(\dfrac{du}{dy}\right)$ and if $\chi > 500$, then flow becomes unstable

- At the wall ($y = 0$) and at centre of pipe $\left(\dfrac{du}{dy} = 0\right)$, x is zero.

11 Turbulent Flow

Turbulent flow results from the instability of laminar flow and due to continuous mixing between different layers. Momentum transfer occurs which gives rise to addition shear called Turbulent shear.

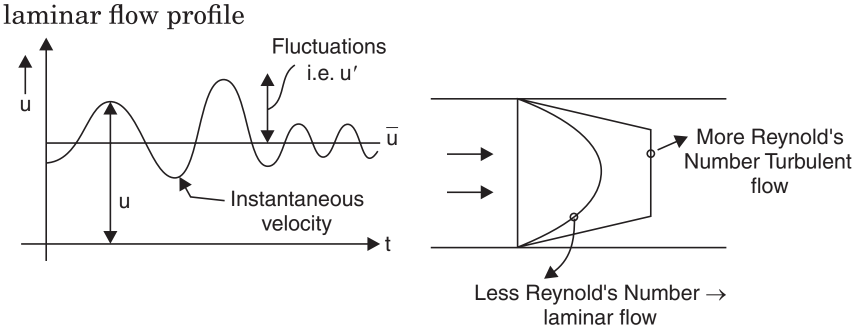
In turbulent flow every flow parameter is a combination of average value and fluctuating Value

i.e. $u = \overline{u} + u'$

Note :-

• Time average of fluctuating component is considered zero.

• As the Reynold's number of the flow increases, velocity profile becomes more flatter i.e. turbulent flow velocity profile is flatter than laminar flow profile



• **Shear stress at boundary** (τ_w) is **much less** in **Laminar flow** as compared to turbulent flow as velocity gradient near the boundary is large in turbulent flow.

Shear stress in turbulent flow

$$\tau_{Total} = \tau_{Laminar} + \tau_{Turbulent}$$

$$\tau_{laminar} = \mu\left(\frac{du}{dy}\right)$$

$\mu \rightarrow$ Dynamic Viscosity (depends on **fluid** characterstic)

Now for τ Turbulent

- **Boussineq's**

  $z = \eta \dfrac{du}{dy}$

  η = eddy viscosity or turbulent mixing coefficient (depends on flow property)

  $\dfrac{\eta}{\rho}$ = eddy kinematic viscosity

- **Reynold's**

  $z = f\, u'\, v'$

  u′, v′ are fluctuating components of velocity in x and y direction

- **Prandtl**

  $z = \rho l^2 \left(\dfrac{du}{dy}\right)$

  l = mixing length, it is the distance covered by a particle before striking any other particle (or before its momentum is changed)

Hydrodynamically smooth and Rough boundary

In hydrodynamically smooth boundries, average height of roughness (K) is much less than the laminar sub layer (δ′) while if average height of roughness (K) is more than the laminar sub layer its hydrodynamically rough boundary.

$$\delta' = 11.6 \frac{v}{u_*}$$

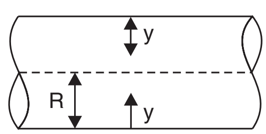
$\nu \rightarrow$ Kinematic Viscosity

u_* = Shear velocity = $\sqrt{\dfrac{\tau_w}{\rho}}$; τ_w = Boundary shear stress

	Smooth	Transition	Rough
As per Nikuradse Experiment	$\dfrac{K}{\delta'} < 0.25$	$0.25 < \dfrac{K}{\delta'} < 6$	$\dfrac{K}{\delta'} > 6$
In terms of Roughness Reynold No. $\left(\text{put } \delta' = \dfrac{11.6v}{u_*}\right)$	$\dfrac{u_* K}{v} < 3$	$3 < \dfrac{u_* K}{v} < 70$	$\dfrac{u_* K}{v} > 70$

Velocity distribution for turbulent flow in smooth as well as Rough pipe

1. $\dfrac{u - u_{avg}}{u_*} = 5.75 \log_{10}\left(\dfrac{y}{R}\right) + 3.75$



Here, y is measured from the boundary surface not from centre.

2. $U = U_{max}$ at $y = R$ so $\dfrac{U_{max} - U_{avg}}{u_*} = 3.75$; $\log_{10} 1 = 0$

3. As $u_* = \sqrt{\dfrac{f}{8}}\, U_{avg}$ then in 2, $\dfrac{U_{max}}{U_{avg}} = 1 + 1.33\sqrt{f}$

4. In pipe flow, $z_w = \dfrac{dp}{dx}\left(\dfrac{R}{2}\right) \Rightarrow \dfrac{R}{2}\left(\dfrac{\Delta P}{L}\right) \Rightarrow \dfrac{R}{2L}(\gamma h_L)$

$$\Rightarrow \quad \frac{z_w}{\rho} = \frac{R}{2L}(gh_L) = \frac{R}{2L} g \frac{flV^2}{2gD} = \frac{D}{4L} g \frac{flV^2}{2gD}$$

$$\Rightarrow \quad \frac{z_w}{\rho} = \frac{f}{8} V^2 \Rightarrow \sqrt{\frac{z_w}{\rho}} = u_* = \sqrt{\frac{f}{8}} V_{avg}$$

5. $\dfrac{1}{7}$ th power law of velocity distribution for smooth pipes

$$\frac{u}{u_{max}} = \left(\frac{y}{R}\right)^{\frac{1}{7}} \text{ (As per Nikuradse)}$$

Friction factor 'f' for Turbulent flow (Artificial Roughnes)

1. **For smooth pipes**

(*a*) $f = \dfrac{0.316}{(R_e)^{1/4}}$, $\quad 4000 < R_e < 10^5$

> **Note:**
>
> For laminar flow, $f = \dfrac{64}{R_e}$ circular pipe

(*b*) $f = 0.0032 + \dfrac{0.221}{(R_e)^{0.237}}$, $\quad 10^5 < R_e < 4 \times 10^7$

(*c*) $\dfrac{1}{\sqrt{f}} = 2\log_{10}(Re\sqrt{f}) - 0.8$, $\quad 5 \times 10^4 < Re < 4 \times 10^7$ (Nikuradse)

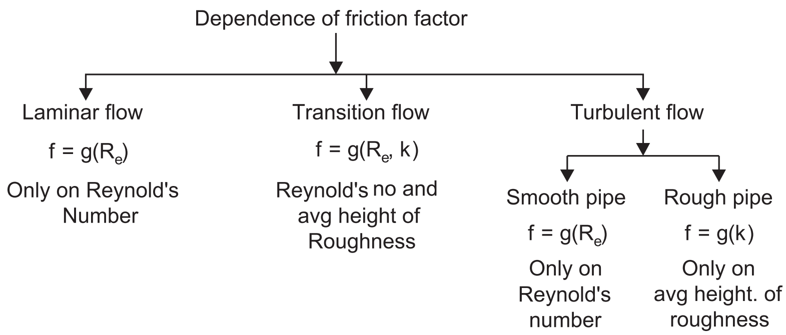
2. **For Rough pipes**

$\dfrac{1}{\sqrt{f}} = 2\log_{10}\left(\dfrac{R}{K}\right) + 1.74$, $\quad R \rightarrow$ Radius of pipe

$\left(\dfrac{R}{K}\right) \rightarrow$ Relative Smoothness

3. For Smooth Commercial pipes

$$\frac{1}{\sqrt{f}} = 2\log_{10}(R_e\sqrt{f}) - 0.8 \quad 5 \times 10^4 < R_e < 4 \times 10^7 \quad \text{(Prandtl)}$$



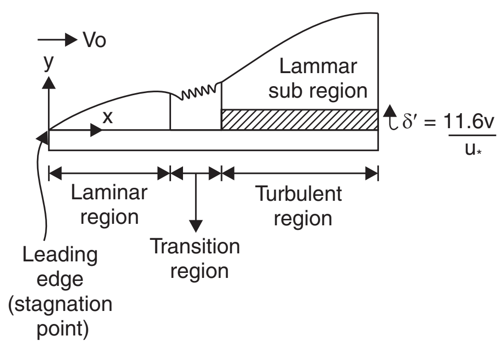
ENTRI

Boundary Layer Thickness

12

It is the region in the immediate Vicinity of the boundary surface in which the velocity of flowing fluid increases gradually from zero at the boundary surface to the velocity of the main stream.

- Developed by Prandtl in 1904 (even before Partition of Bengal)
- Valid for infinitely large medium of real fluid and Not for ideal fluid.



Essential boundary conditions:

1. $x - 0, \delta = 0$ **2.** $y = 0, u = 0$

3. $y = \delta, u = V_0$ **4.** $y = \delta, \dfrac{du}{dy} = 0$

Desirable boundary conditions: At $y = \delta, \dfrac{du}{dy} = 0, \dfrac{d^2u}{dy^2} = 0$

Salient points regarding boundary layer:

1. As the roughness of plate increases, length of laminar region decreases
2. Positive pressure gradient increases boundary layer thickness as well as reduces the length of laminar region.
3. With increase in velocity, boundary layer thickness decreases but with increase in viscosity boundary layer thickness increases.

4. On a smooth plate, in turbulent layer region, there is very thin layer adjacent to the boundary where the flow remains laminar. This region is called laminar sub layer.
5. $R_{ex} = 5 \times 10^5$ is called critical Reynold's number.
   If $R_e < R_{ex}$ then laminar boundary layer region in **flat plates**
   $R_e > R_{ex}$ then Turbulent boundary layer region, in **flat plates**

Velocity profiles
↓
Laminar region | Laminar sublayer | Turbulent Region

Laminar region:

$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^2$$

$$\frac{u}{u_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

Laminar sublayer:

$$\frac{u}{u_*} = \frac{u_* \times y}{v}$$

Turbulent Region:

$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/7}$$

Boundary layer Thickness (δ): It is the distance from the boundary surface in which velocity reaches 99% of the free stream velocity.

At $\qquad y = \delta, \quad V = 0.99\, V_0$

Displacement Thickness (δ^*): Distance by which boundary should be shifted in order to compensate for the reduction in mass flow rate on account of boundry layer formation.

$$\delta^* = \int_0^\delta \left(1 - \frac{V}{V_0}\right) dy$$

For
$$\frac{V}{V_0} = \left(\frac{y}{\delta}\right)^{1/m}$$

$$\Rightarrow \qquad \delta^* = \frac{\delta}{m+1}$$

V = Velocity at any distance y from the boundary
V_0 = Free stream velocity

Reduction in mass flow rate per unit width = $\rho\, \delta^*\, V_0$

Momentum Thickness (θ): Distance by which boundary should be shifted in order to compensate for the loss of momentum due to formation of Boundary layer.

$$\theta = \int_0^{\delta} \frac{V}{V_0}\left(1 - \frac{V}{V_0}\right) dy$$

For $\frac{V}{V_0} - \left(\frac{y}{\delta}\right)^{1/m}$

$$\theta = \frac{\delta m}{(m+1)(m+2)}$$

Loss of momentum per unit width of boundary layer = $\rho\theta V_0^2$

Energy thickness δ_E: Distance by which boundary should be shifted in order to compensate loss of energy due to boundary layer formation.

$$\delta_E = \int_0^{\delta} \frac{V}{V_0}\left(1 - \frac{V^2}{V_0^2}\right) dy$$

Loss of energy due to boundary layer formation = $\frac{\delta V_0^3}{2} E$

Note:

- $\delta^* > \delta_E > \theta$
- Shape factor = $\frac{\delta^*}{\theta}$, For $\frac{V}{V_0} = \left(\frac{y}{\delta}\right)^{1/m}$, Shape factor = $\frac{m+2}{m}$

Equation's in boundary layer

1. Continuously equation, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

2. Constant pressure gradient across boundary layer, $\frac{-1}{\rho}\frac{\partial p}{\partial y} = 0$

3. For steady 2–D Laminar flow, $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial z}{\partial y}$

4. Von-Karman momentum integral equation, $\frac{\tau_0}{\rho V_0^2} = \frac{d\theta}{dx}$

τ_0 = Boundary shear stress
V_0 – Free stream velocity
θ = Momentum thickness

Hardness

Local skin friction coefficient

$$C_{f_x} = \frac{\tau_0}{\left(\frac{\delta V_0^2}{2}\right)} = \frac{\text{Wall shear stress}}{\text{Dynamic pressure}}$$

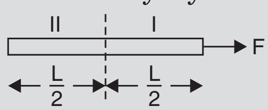
Average drag coefficient

$$C_{f_{avg}} = \frac{(\text{Drag F.M./Area}}{\frac{\delta V_0^2}{2}}$$

$$= \frac{\text{Avg. wall shear stress}}{\text{Dynamic pressure}}$$

Note:

It Boundary layer is laminar through out



$$\frac{\text{Drag force on I half}}{\text{Drag force on II half}} = 1 + \sqrt{2} \ \textit{i.e.}, > 1$$

Blassius solution in absence of velocity profile on smooth plate

	Laminar	Turbulent	
	$R_e < 5 \times 10^5$	$5 \times 10^5 < R_e < 10^7$	$10^7 < R_e < 10^9$
δ	$\frac{5x}{\sqrt{R_{ex}}}$	$\frac{0.376\,x}{(R_{ex})^{1/5}}$	$\frac{0.22\,x}{(R_{ex})^{1/6}}$
C_{fx}	$\frac{0.664}{(R_{ex})^{1/5}}$	$\frac{0.059}{(R_{ex})^{1/5}}$	$\frac{0.37}{(\log_{10} R_{ex})^{2.58}}$
C_{favg}	$\frac{1.328}{\sqrt{R_{ex}}}$	$\frac{0.074}{(R_{eL})}$	$\frac{0.455}{(\log_{10} R_{eL})^{2.58}}$
		Applicable only if boundary layer is Turbulent through out	

Note:

- In Laminar region $\delta \alpha \sqrt{x}$, while in turbulent region $\delta \,\alpha\, x^{4/5}$, Hence δ increases more rapidly in turbulent region than in laminar region
- In Laminar region $C_{fx}\ \alpha\ \frac{1}{\sqrt{x}}$, while in turbulent region $C_{fx}\ \alpha\ \frac{1}{x^{1/5}}$ Hence, τ_0 decreases more rapidly in laminar region than in turbulent region.

Drag force on a plate which has both Laminar and Turbulent regions

$$\text{Drag Force} = C_{favg} \times \frac{\rho V_0^2}{2} \times \text{Area}$$

(*a*) $$5 \times 10^5 < R_e < 10^7,\ C_{favg} = \frac{0.074}{(R_{e_L})^{1/5}} - \frac{1700}{R_{e_L}}$$

(*b*) $$10^7 < R_e < 10^9,\ C_{favg} = \frac{0.455}{(\log_{10} R_{e_L})^{2.58}} - \frac{1700}{R_{e_L}}$$

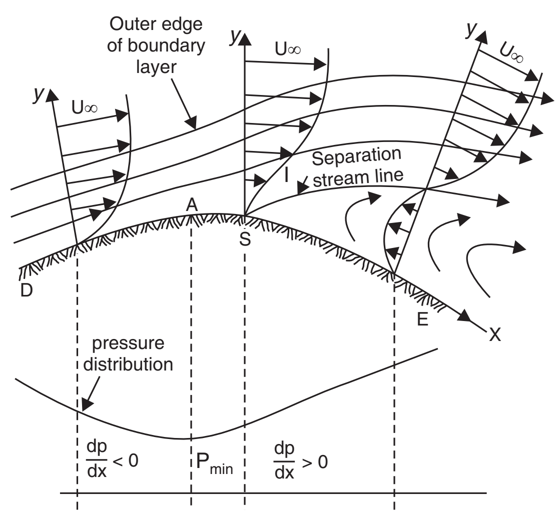
Separation of Boundary layer

Boundary layer thickness increases along the length of the body. So the fluid close to the body surface has to do work against skin friction at the expense of kinetic energy. This leads to decrease in velocity and increase in pressure in the forward direction. And at the point where

$$\left(\frac{du}{dy}\right)_{y=0} = 0 \text{ is called separation point}$$

$$\left(\frac{du}{dy}\right)_{y=0} > 0\text{, Attached flow}$$

$$\left(\frac{du}{dy}\right)_{y=0} < 0\text{, Already separated flow}$$



Separation of flow occurs in Pumps, Turbines, Aerofoils, open channel transitions etc.

Consequences of boundary layer separation

(*a*) There is increase in pressure drag if there is boundary layer separation in case of external flow E.g. cars

(*b*) Separation of boundary layer increases flow losses in case of internal flow like pipes.

Methods to control separation

1. Rotating boundary in flow direction
2. Supplying additional energy from blower
3. Streamlining of body shapes
4. Suction of flow moving fluid by suction slot
5. Injecting fluid into boundary layer
6. Providing guide blades on bends.

Dimensional Analysis and Model Studies 13

It helps in determining a systematic arrangement of the variations in the physical relationship, combining dimensional variable to form non-dimensional parameters. The various physical quantities can be expressed in terms of fundamental quantities of Mass (M). Length (L), Time (T) and Temperature (θ).

Dimensional homogeneity: States that every term in an equation when reduced to its primary (fundamental) dimensions must contain identical powers of each dimension.

Dimensions of Few Physical Quantities

(*a*) Kinematic Quantities:

1. Angular velocity	T^{-1}
2. Angular accelration	T^{-2}
3. Vorticity	T^{-1}
4. Kinematic viscosity	L^2T^{-1}
5. Stream function	L^2T^{-1}
6. Circulation	L^2T^{-1}

(*b*) Dynamic Quantities:

1. Specific weight	$ML^{-2}T^{-2}$
2. Dynamic viscosity	$ML^{-1}T^{-1}$
3. Surface tension	MT^{-2}
4. Modulus of elasticity	$ML^{-1}T^{-2}$
5. Bulk modulus	$ML^{-1}T^{-2}$
6. Angular momentum	ML^2T^{-1}

Methods of Dimensional Analysis

(*a*) **Rayleigh's Method:** It is used for determining the expression for a variable which depends upon maximum of three or four variables. It does not provides any information regarding the number of dimensionless groups to be obtained as a result of dimensional analysis.

(*b*) **Buckingham's π-theorum:** If there are n no. of variables in a dimensionally homogenous equation and these variables contain m fundamental dimensions then the no. of dimensionless groups which can be formed shall be n-m. These dimensionless groups are called π-terms.

Selection of 3 repeating variables should be from **geometry of flow** (size and shape of fluid passage, or diameter or length of moving body), **fluid property** (like density, surface tension, viscosity, elasticity, vapour pressure) and **fluid motion** (like velocity, accelration, discharge, pressure, power, force)

Similitude: To achieve similarity between the flow in the model and its prototype, every dimensionless parameter referring to the conditions in the model must have the same numerical value as the corresponding parameter referring to the prototype

Geometric (similarity of shape)	Kinematic (similarity of motion)	Dynamic (similarity of forces)

Note:
For kinetic similarity, Geometric similarity must exist and for dynamic similarity both Geometric and kinematic similarity must exist. These are necessary conditions but not sufficient conditions i.e. if kinematic similarity exists then geometry similarity will be definitely there but if geometric similarity exists then kinematic similarity may or may not exist. (same for dynamic similarity)

Forces acting on Fluid mass

1. Inertia Force (F_i) = δL^2V^2
2. Viscous Force (F_v) = μVL
3. Gravity Force (F_g) = δL^3g
4. Pressure Force (F_p) = PL^2
5. Surface tension Force (F_σ) = σL
6. Elasticity Force (F_e) = KL^2

Dimension less-Parameters

1. Reynold's Number

$$R_e = \frac{\text{Inertial Force}}{\text{Viscous Force}}$$

$$R_e = \frac{VL}{\nu}$$

2. Froude's Number

$$F_e = \sqrt{\frac{\text{Inertial Force}}{\text{Gravity Force}}}$$

$$F_e = \frac{V}{\sqrt{gL}}$$

3. Euler's Number

$$E_u = \sqrt{\frac{\text{Inertial Force}}{\text{Pressure Force}}}$$

$$E_u = \frac{V}{\sqrt{p/\delta}}$$

4. Weber Number

$$W_e = \sqrt{\frac{\text{Inertial Force}}{\text{Surface tension Force}}}$$

$$W_e = \frac{V}{\sqrt{(\sigma/\delta L)}}$$

5. Mach Number

$$M = \sqrt{\frac{\text{Inertial Force}}{\text{Elastic Force}}}$$

$$M = \frac{V}{\sqrt{K/\delta}} = \frac{V}{C}$$

(1) **Reynold's model law** $\frac{\delta_r V_r L_r}{\mu_r} = 1$

⇒ Pipe flow, Submarines, Aeroplanes, drag on parachutes

(2) **Froude's law** $\frac{V_r}{\sqrt{L_r g_r}} = 1$

⇒ when a free surface is present eg wiers, spillway, channels etc where **gravity** is predominant.

Note: L = Hydraulic depth = $\frac{A}{P}$ (in case of open channel flow)

	Reynold's law	Froude's law
Velocity Ratio (V_r)	$\frac{\mu_r}{L_r \delta_r}$	$\sqrt{L_r}$
Time Ratio (T_r)	$\frac{\delta_r L_r^2}{\mu_r}$	$\sqrt{L_r}$
Acceleration Ratio (a_r)	$\frac{\mu_r^2}{\delta_r^2 \ L_r^2}$	1
Force Ratio (F_r)	$\frac{\mu_r^2}{\delta_r}$	$\delta_r L_r^3$
Power Ratio (P_r)	$\frac{\mu_r^3}{\delta_r^2 L_r}$	$\delta_r L_r^{3.5}$
Discharge Ratio (Q_R)	$\frac{\mu_r L_r}{\delta_r}$	$\delta_r L_r^{2.5}$

3. **Euler's law** $V_r = \sqrt{P_r / \delta_r}$

   ⇒ In case of cavitation, pressure due the sudden closure of value, high pressure flow in pipes.

4. **Weber model law** $V_r = \sqrt{\frac{\sigma_r}{\delta_r L_r}}$

   ⇒ Flow of blood in arteries and veins, seepage through soil capillary rise, rising bubble, flow over weir for small head.

5. **Mach law** $V = \sqrt{k_r / \delta_r}$

   Compressibility force are predominant when mach no ≥ 0.3

   M < 1 → Subsonic M = 1 → Sonic

   M > 1 → Supersonic M >> 1 → Hyper sonic

River model law (Distorted model law)

(*a*) Horizontal scale ratio $L_{rH} = \dfrac{L_p}{L_m} = \dfrac{B_p}{B_m}$

(*b*) Vertical scale ratio $L_{rv} = \dfrac{H_p}{H_m}$

(*c*) Velocity ratio $Vr = \sqrt{L_{rv}}$

(*d*) Area ratio $A_r = L_{rv} \times L_{rH}$

(*e*) Discharge ratio $Q_r = L_{rH} \cdot L_{rv}^{3/2}$

Model Laws for

(*a*) Head ratio	$H_r = N_r^2 \quad D_r^2$	D_r = diameter ratio
(*b*) Discharge ratio	$Q_r = N_r\, D_r^3$	N_r = rotational speed ratio
(*c*) Power ratio	$P_r = \delta_r \quad N_r^3 \quad D_r^5$	

Pipe Flow 14

Practically, all the flow in the pipes is **turbulent** in nature.

Head loss

- Frictional loss
  Major loss → 80 – 90%
- Losses in pipe fillings
  Minor loss → 10 – 20%

Major Losses

(*a*) Darcy's weisbach equation

$$h_f = \frac{f\, l\, Q^2}{12.1\, D^5}$$

(*b*) Chezy's formula

$$V = C\sqrt{RS} \qquad R = \frac{A}{P} = \frac{\frac{\pi}{4} D^2}{\pi D} \Rightarrow R = \frac{D}{4}$$

$$\Rightarrow \qquad S = \text{Slope} = \frac{h_L}{L}$$

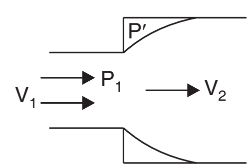
Note:

Equating both the above equations we get $C = \sqrt{\dfrac{8g}{f}}$

Minor losses

(*a*) Due to sudden expansion

$$h_L = \frac{(V_1 - V_2)^2}{2g} = \frac{V_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2$$



A_1 = Area of smaller dia pipe

A_2 = Area of bigger dia pipe

V_1 = Velocity of smaller dia pipe

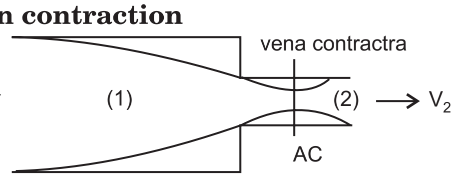
$$h_L = K\frac{V_1^2}{2g} \qquad \text{where } K = \left(1 - \frac{A_1}{A_2}\right)^2$$

Note:

- It is assumed that $P_1 = P'$ during derivation of this head loss
- Momentum equation and Bernouilli's equation are used in derivation of losses
- Losses are always expressed in terms of velocity of smaller diameter pipe.

(*b*) Losses due to sudden contraction

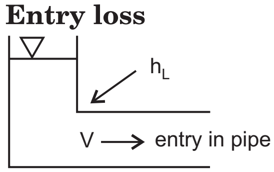
$$h_L = \frac{(V_C - V_2)^2}{2g} = \frac{KV_2^2}{2g}$$



$$\text{where } \quad K = \left(\frac{1}{C_c} - 1\right)^2, \quad C_c = \frac{A_c}{A_2}$$

Note: Loss in expansion is much higher than loss in contraction.

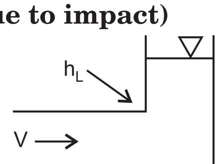
(*c*) Entry loss



$$h_L = \frac{0.5\, V^2}{2g}$$

(*d*) Exist loss (due to impact)

$$h_L = \frac{KV^2}{2g}$$



Note: In exist loss due to impact, K is the kinetic energy correction factor. For Turbulent its K = 1 and for Laminar its K = 2.

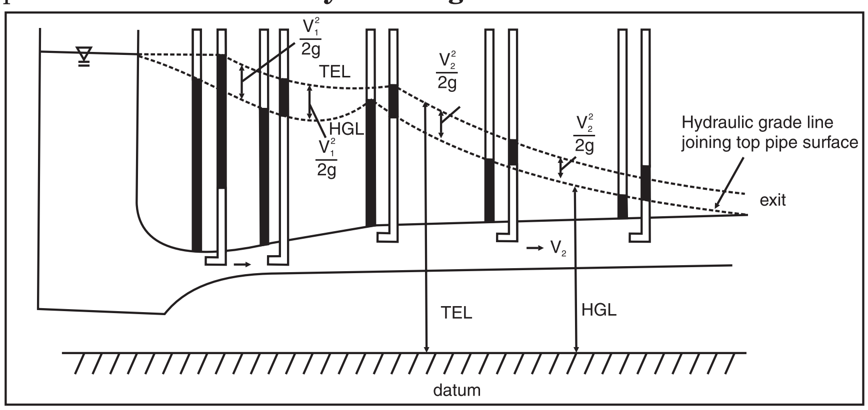
(*e*) Loss due to pipe fittings and bends

$$h_L = \frac{KV^2}{2g}$$

Type of fitting	K
Standard Tee	1.8
Standard Elbow	0.9
45° Elbow	0.4
90° Bend (Sharp)	1.2
Foot value of pump	1.5
Gate value half open	5.6
Angle value	5.0

Hydraulic gradient line and Total energy line

Line joining the points of **piezometric head** $\left(\frac{P}{\gamma} + z\right)$ at various points in a flow is called **hydraulic gradient line**.



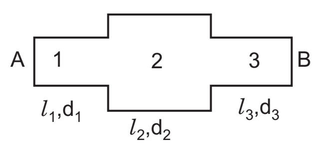
Line joining the points of total energy $\left(\frac{P}{\gamma} + \frac{V^2}{2g} + z\right)$ at various points in a flow is called **total energy line**.

Note:

- Total energy line always fall down. But if there is a pump or turbine placed in the flow there will be sudden rise or fall respectively.
- Hydraulic grade line may rise or fall in the flow direction, depending upon the velocity head (which varies with the area of cross section)
- Total energy line is horizontal in case of idealised Bernoulli's flow because losses are zero.

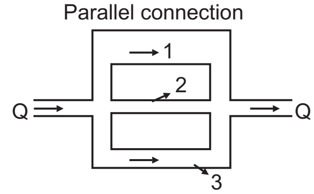
Pipe connection

Series connection



$$h_{L_{AB}} = h_{L_1} + h_{L_2} + h_{L_3}$$
$$Q \quad Q_1 = Q_2 = Q_3$$

Parallel connection

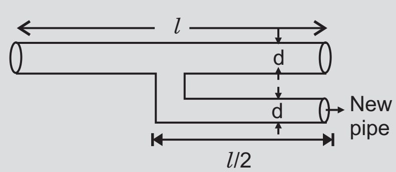


$$Q = Q_1 + Q_2 + Q_3$$
$$h_{L_{AB}} = h_{L_1} = h_{L_2} = h_{L_3}$$

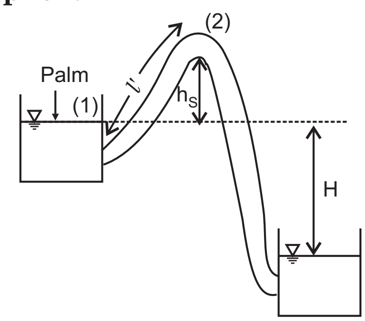
Equivalent pipe: A pipe which can replace existing compound pipe while carrying some discharge under same losses. For **series connection** equivalent pipe of length 'L' and diameter 'D' will be

$$\frac{L}{D^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5}$$

Note: Increase in discharge by adding a pipe of same diameter in mid way of a pipe but keeping the head constant is 26.53%.



Flow through siphon:



Note: For No. vaporisation Ps > Pvaporization, otherwise vapours will form and flow will be stopped.

1. Get velocity of flow in pipe from head difference

$$H = \frac{flV^2}{2gD} + 0.5\frac{V^2}{2g} + \frac{V^2}{2g}$$

[Friction loss] [Entry loss] [Exist loss]

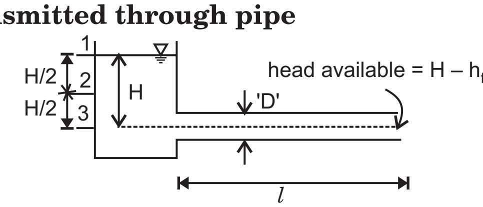
l = Total length of pipe

2. Apply Bernoulli's between 1 and summit (s)

$$\frac{P_{atm1}}{\gamma_w} = \frac{P_s}{\gamma_w} + \frac{V^2}{2g} + h_s + \frac{0.5V^2}{2g} + \frac{fl'V^2}{2gD}$$

l' = length of pipe b/w 1 and (s)

Power transmitted through pipe



Efficiency $(\eta) = \frac{H - h_f}{H}$

Power (P) = $\gamma Q (H - h_f)$

For max power $\frac{dP}{dQ} = 0$ i.e. $h_f = \frac{H}{3}$

η_{max} = 66.67%, max power lost = 33.33%

Note: It Nozzle of area 'a' is attached at exit then for maximum efficiency $\frac{fla^2}{DA^2} = \frac{1}{2}$ where A corresponds to area of diameter D.

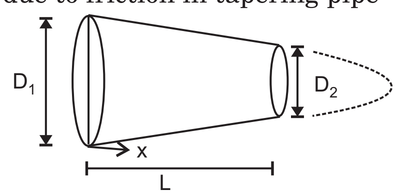
- Time required to empty the reservoir $\int_0^H \frac{dh}{\sqrt{h}} = \int_0^T \sqrt{\frac{2g}{K}} \times \frac{a}{A} dt$ where

K is the head loss constant.

- Time required to empty the top half of tank from 1 to 2 be t_1 and for bottom half from 2 to 3 be t_2 then $t_1 = 0.414\, t_2$

Special cases of head loss

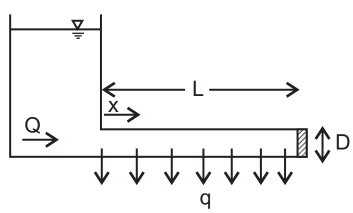
(*a*) Loss of head due to friction in tapering pipe



$$h_f = \int_0^L \frac{f\,Q^2\,dx}{12.1(D_1 - kx)^5}$$

$$K = \left(\frac{D_1 - D_2}{L}\right)$$

(*b*) Head loss due to uniform discharge at regular interval from a closed end pipe



$$q' = \frac{Q}{L}$$

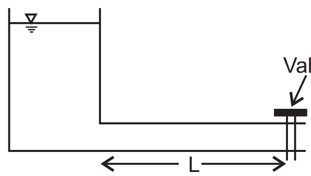
$$h_f = \frac{1}{3}\left(\frac{flQ^2}{12.1\,D^5}\right)$$

Head loss is one-third the head loss of constant discharge.

Water-hammer Pressure: Sudden closure of valve in a pipe carying flowing liquid destroys the momentum of flowing liquid and sets up a high pressure wave. This pressure wave travels with the speed of sound and causes hammering action in pipe called Knocking or water hammer.

Velocity of Pressure Wave (c)

- In Rigid Pipe
- In Elastic Pipe



$$C = \sqrt{\frac{K}{\delta}} \qquad\qquad C = \sqrt{\frac{K}{\delta}}\left(\frac{1}{1+\frac{KD}{tE}}\right)^{1/2}$$

K = Bulk density of liquid

δ = mass density of liquid

E = modulas of elasticity of material

t = thickness of pipe

D = diameter of pipe

- Time period for complete cycle of water hammer pressure $= \dfrac{4L}{C}$

Critical time $T_0 = \dfrac{2L}{C}$

Water hammer pressure (ph)

- Rapid closure $T \le T_o$
- Slow closure
  - $T_o < T \le 1.5\,T_o$
  - $T_o << T$

$P_{hr} = \delta\, C\, V \quad \dfrac{P_{hs}}{P_{hr}} = \dfrac{T_0}{T} \qquad\qquad p = \delta\left(\dfrac{L}{T}\right)V$

V = Initial velocity of flow

> **Note:** Equations used in solving branching of pipes connecting reservoirs at different levels are (*a*) Continuity equation (*b*) Bernoulli's equation (*c*) Darcy weis bach equation.

Hardy-Cross method of solving closed loop pipe networks.

1. Flow into any junction must be equal to flow out of each junction.
2. Loss of head due to flow in clockwise direction must be equal to loss of head due to flow in anti clockwise direction.

Modification in discharge $\Delta Q = \dfrac{-\Sigma\, r\, Q^n}{\left|r\, n\, Q^{n-1}\right|}$

where $h_f = rQ^n$ and ΔQ is algebrically added.