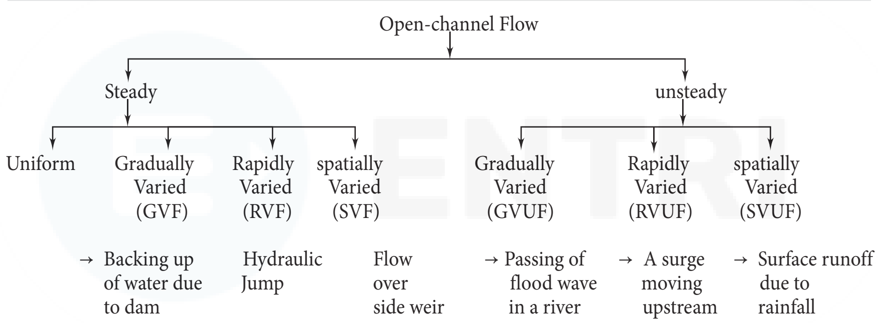
1 Introduction

It's a natural or manmade structure in which liquid flows with a free surface at atmospheric pressure.

Prismatic channels are those in which X-sectional shape, size and bed slope is constant. While all natural channels having varying X-section are consequently known as Non-Prismatic channels.

Rigid channels are those in which the boundary is not deformable. Here shape and roughness factors is not a function of flow parameter while mobile channels are those in which boundaries undergo deformation due to continuous process of erosion and deposition due to flow.

Note: Rigid channels have only 1 degree of freedom (i.e. depth of flow) while mobile channels have 4 degree's of freedom.



Uniform flow: If the **flow** properties like **depth of flow and discharge** in an open channel remains constant along the length of the channel. Then its Uniform flow. The constant depth of flow is called the **normal depth**.

Note: → In Uniform flow, Bed Slope = Energy line Slope = Water Surface Slope

→ In GVF, loss of energy is mainly due to boundry friction.

→ In GVF, pressure distribution in Vertical direction is taken as hydrostatic.

Critical, Sub-Critical and Super-critical Flow

Froude Number $(f_r) = \dfrac{v}{\sqrt{gL_e}}$ $\boxed{L_e = \dfrac{\text{Area of flow}}{\text{Top width of flow}}}$

where L_c = characterstic length

for rectangular section $L_c = y$ (depth of flow)

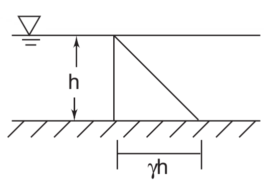
Note: $\sqrt{gy_e}$ represents the speed at which the disturbance wave travels in still water condition, denoted by celerity c_o

Type of flow	Depth of flow	Velocity of flow	Froude No	Comments
Subcritical	$y > y_c$	$v < v_c$	$F_r < 1$	Also known as streaming or tranquil flow
Critical	$y = y_c$	$v = v_c$	$F_r = 1$	Also known as shooting flow, rapid flow, torrential flow
Super critical	$y < y_c$	$v > v_c$	$F_r > 1$	

Control Section: A Section in which fixed relationship exists between **depth and discharge**. For sub-critical flow, downstream section is a control section. (i.e computation of water surface profile must start from down stream location and proceed upstream location). For super-critical flow control section is at upstream section.

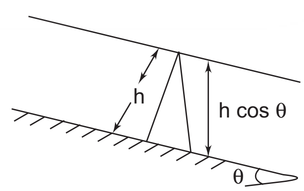
Note: In case of critical condition, disturbance wave will not travel at all.

Pressure distribution



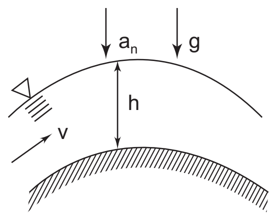
Small slope

$\boxed{P = \gamma h}$



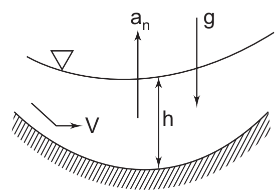
Large slope

$\boxed{P = \gamma h \cos\theta}$



Upward convex surface

$$\boxed{P = \delta(g - a_n)h}$$



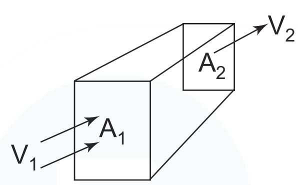
Upward concave surface

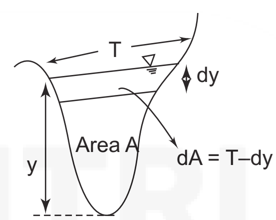
$$\boxed{P = \delta(g - a_n)h}$$

Continuity equation in open channel flow

Contunity equation in open channel flow

- Steady State Flow: $Q = AV = A_1V_1 = A_2V_2$
- Unsteady State Flow: $\frac{\partial Q}{\partial x} = -T\frac{\partial y}{\partial t}$

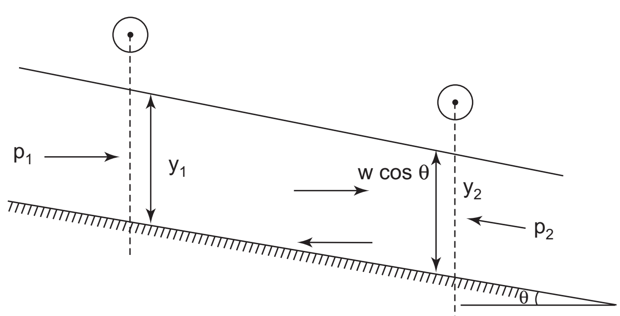




Momentum equation in open channel flow:

$$\boxed{M_2 - M_1 = P_1 - P_2 + w\sin\theta - F_t}$$

$$-\rho A_1 V_1^2 + \rho A_1 V_1^2 = \gamma A_1 \bar{y}_1 - \gamma A_1 \bar{y}_2 + w\sin\theta - F_f$$



M_1, M_2 = Momentum per unit time

P_1, P_2 = Pressure force

F_f = Friction force along the surface of contact b/w water and channel

W = Weight of water enclosed between sections.

θ = Bed slope.

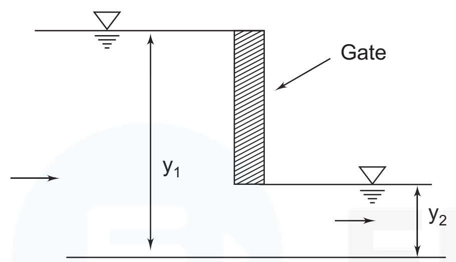
Note: Momentum equation is generally used where the loss of internal energy is high, because it deals with external forces only. For eg: Hydraulic Jump.

Specific force: Sum of pressure force and momentum flux per unit "unit weight" of the fluid at a section.

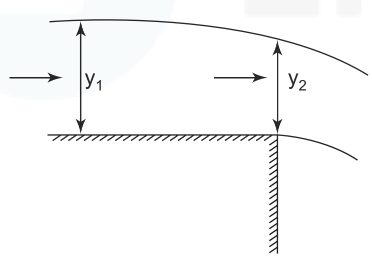
$$\text{Specific force} = \frac{P + M}{r}$$

It is assumed to be constant if the channel is horizontal and frictionless.

Note:



$$\frac{F_{\text{on gate}}}{\text{Width}} = \frac{\gamma}{2}\frac{(y_1 - y_2)^3}{y_1 + y_2}$$



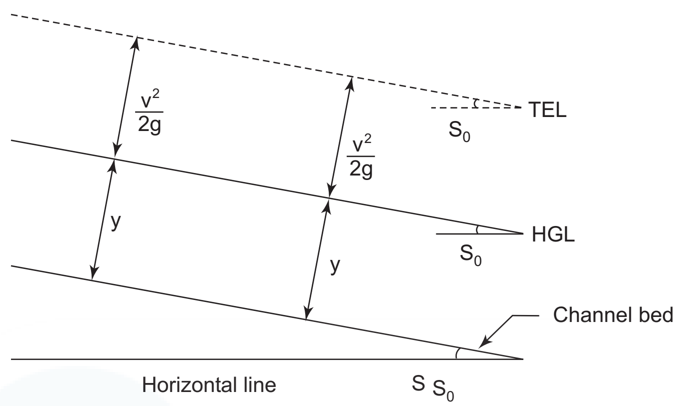
$$\frac{y_2}{y_1} = \frac{2F_1^2}{2F_1^2 + 1}$$

F_1 = Froude no at section 1

2 Uniform– Flow

In Uniform flow, frictional resistance between the fluid and channel boundary are balanced by the gravity forces.

Depth of flow corresponding to the slope of bed will be Normal depth.



Measurement of Velocity:

(1) Chezy's equation

$$\tau_0 = \gamma R S_0 \quad \tau_0 = K\left(\frac{\rho V^2}{2}\right)$$

Note: τ_0 is independent of Viscosity.

where τ_0 = Average shear stress on the wetted area under uniform flow condition.

R = Hydraulic radius (A/P) of the channel section.

S_0 = Slope of channel bed = Slope of HGL = Slope of TEL

K = Constant depending on roughness

ρ = Density of water.

Equating both equations, $\boxed{v = C\sqrt{RS}}$ → Chezy's equation

$\boxed{C = \sqrt{\frac{2r}{\delta K}}}$ → Chezy's coefficient

(2) Manning's equation

$$V = \frac{1}{n} R^{2/3} S_0^{1/2}$$

- By Robert manning in 1889

n = Roughness coefficient, commonly known as manning's "n".

- Strikler's Formula $n = \frac{d_{50}^{1/6}}{21.10}$, $d_{50} \rightarrow$ in meters

Meyer's Formula $n = \frac{d_{90}^{1/6}}{26}$, $d_{90} \rightarrow$ in meters

Note: Relationship between c, n, f

$$c = \sqrt{\frac{8g}{f}} \qquad f = \left(\frac{n^2}{R^{1/3}}\right) 8g$$ f = Darcy's friction factor

Economical channel section: A section whose construction cost is minimum for a given discharge. While the efficient section is the one who's discharge carrying capacity is maximum for a given cross-sectional area.

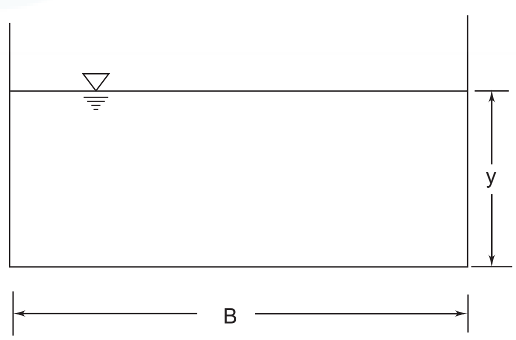
Note: Semi-Circular shape has a least amount of perimeter for a given area.

(1) Rectangular section

$$A = By \qquad P = B + 2y$$

Keeping A constant, $P = \frac{A}{y} + 2y$ put $\frac{dP}{dy} = 0$

Then, $y = \frac{B}{2}$ $R = \frac{y}{2}$



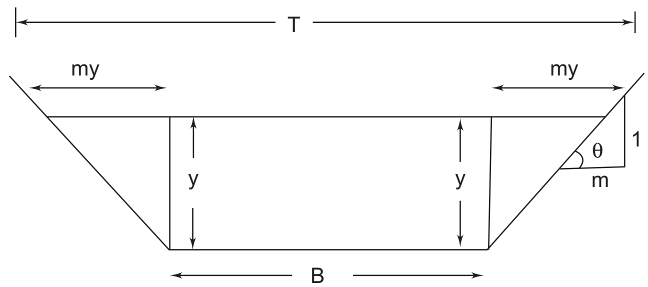
(2) **Trapezoidal section**

(*a*) Side slope is fixed i.e. $\tan\theta = \dfrac{1}{m}$

$$A = (B + my)\,y \qquad P = B + 2y\sqrt{m^2 + 1}$$

$$P = \frac{A}{y} - my + 2y\sqrt{m^2+1}.\ \text{Put } \frac{dP}{dy} = 0$$

Then, $\boxed{\text{Length of side slope} = \dfrac{T}{2} = y\sqrt{m^2+1}}$



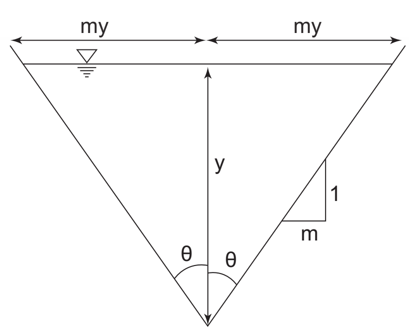
$\boxed{R = \dfrac{y}{2}}$ Circle of radius y can be inscribed in trapezoidal section

(*b*) When side slope is variable.

$$P = 4y\sqrt{m^2+1} - 2my,\ \text{Put } \frac{dp}{dm} = 0\ \boxed{m = \frac{1}{\sqrt{3}}}\ i.e.\ \boxed{\theta = 60°}$$

Most economical trapeziodal section should be half of regular hexagon.

(3) **Triangular section**

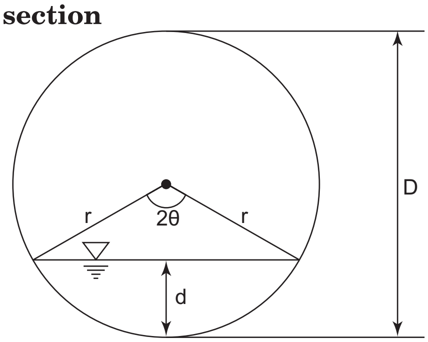


$$A = my^2 \qquad P = 2y\sqrt{m^2+1}$$

$$P = 2\sqrt{A}\sqrt{m+\frac{1}{m}}, \text{ Put } \frac{dp}{dm} = 0$$

$$m = 1 \Rightarrow \boxed{\theta = 45°} \boxed{r = \frac{y}{2\sqrt{2}}}$$

(4) Circular section



$$A = \frac{r^2}{2}(2\theta - \sin 2\theta) \quad P = 2r\theta$$

$$\frac{dA}{d\theta} = r^2(1-\cos 2\theta) \quad \frac{dp}{d\theta} = 2r$$

(*a*) For maximum discharge

$$Q = \frac{1}{n}\frac{A^{5/3}}{P^{2/3}}S^{1/2}, \text{ Put } \frac{dQ}{d\theta} = 0$$

$$\boxed{2\theta = 302°22'} \boxed{d = 0.938 \text{ D}}$$

(*b*) For maximum velocity

$$v = \frac{1}{n}\frac{A^{2/3}}{P^{2/3}}S^{1/2}, \text{ put } \frac{dv}{d\theta} = 0$$

$$\tan 2\theta = 2\theta$$

$$\boxed{2\theta = 257°27'56''} \boxed{d = 0.81\,D}$$

Energy-Depth Relationship 3

Specific energy: Total energy at a section with respect to the channel bed as datum.

For small slopes

$$E = y + \alpha \frac{V^2}{2g}$$

α = K.E correction Factor

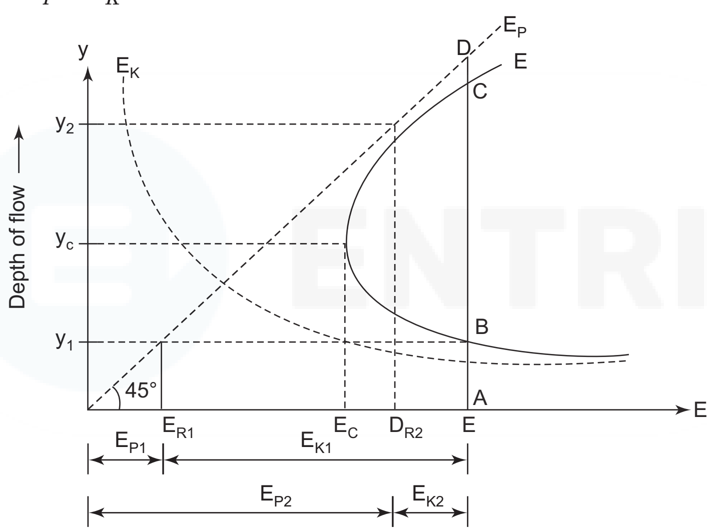
Generally taken as 1

For large slope

$$E = y \cos \theta + \frac{V^2}{2g}$$

θ = Bed slope of channel Bottom.

$E = E_P + E_K$



Energy-Depth Relationship

E_P = Potential energy = y

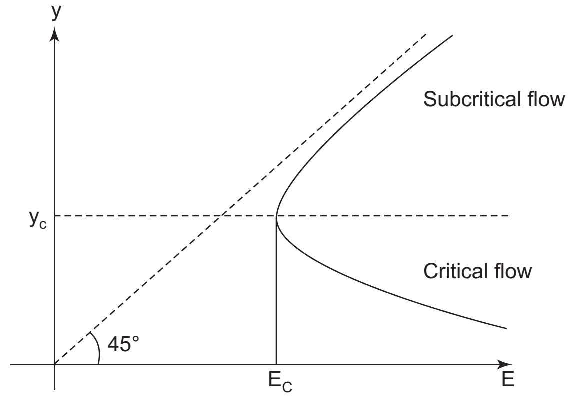
E_K = Kinetic energy = $\dfrac{V^2}{2g}$

For a given Kinetic energy(E),

there are two possible depth of flow y_1 and y_2. These are called **Alternate depth**.

For y_1 $\quad E_{p1} = AB \quad\quad E_{K1} = BD$

For y_2 $\quad E_{p2} = AC \quad\quad E_{K2} = CD$



Energy-Depth relationship

At the critical state of flow, two alternate depths become one, called as **Critical depth** (y_c). For a particular discharge, **specific energy is minimum** at the critical depth.

Note: At critical depth, water surface becomes wavy and unstable as the curve is nearly vertical. A slight change in energy, will give two alternate depths with greater difference

Note: At Critical Flow condition,

$$\boxed{\frac{Q^2T}{gA^3} = 1} \qquad T = \text{Top width}, \quad A = \text{Area of } x\text{-section}$$

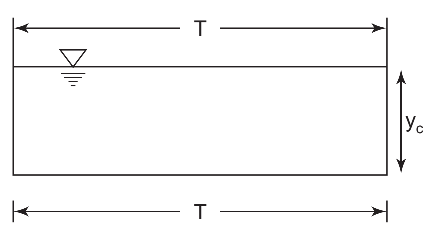
(*a*) For a given discharge, specific energy is minimum

(*b*) For a given specific energy, discharge will be maximum.

(*c*) Specific Force is also minimum for a given discharge.

Calculation of critical depth

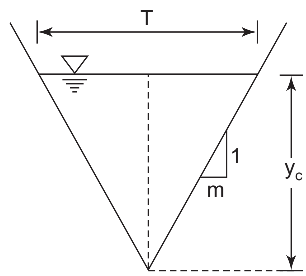
(1) Rectangular Channel Section



$$y_c = \left(\frac{q^2}{g}\right)^{1/3} \quad q = \text{discharge per unit width (B)} = \frac{Q}{B}$$

$$E_c = \frac{3}{2}y_c \quad F_r = \frac{V}{\sqrt{gy_a}}$$

(2) Triangular Channel Section



$$y_c = \left(\frac{2Q^2}{gm^2}\right)^{1/3} \quad Q = \text{Total discharge}$$

$$E_c = \frac{5}{4}y_c \quad F_r = \frac{\sqrt{2}V}{\sqrt{gy_e}} \quad F_r = \left(\frac{2Q^2}{gm^2\ y_c^2}\right)^{1/2}$$

Note: For Parabolic Section $E_c = \frac{4}{3}y_c$.

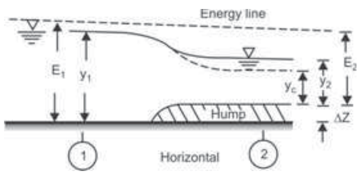
Channel Transition: A transition is the portion of a channel with varying cross-section which connects one uniform flow channel to another uniform flow channel. Use of channel transition are metering of flow, dissipation of channel energy, reduction or increase of velocities, change in channel Section or alignment with minimum of energy dissipation. Assumptions made are

(1) Channel bed is horizontal and frictionless ie TEL will be parallel to the channel bed

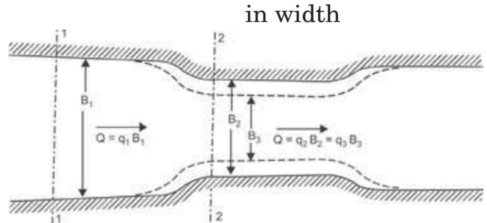
(2) There is no loss of energy between two sections of the channel.

Transition in Rectangular channel

With raised bottom (making hump)



With reduction in width



Subcritical flow at section 1

Supercritical flow at section 1

Subcritical flow at section 1

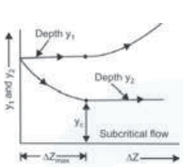
Supercritical flow at section 1

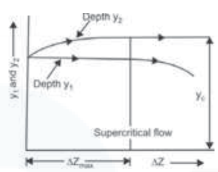
⇒Free surface drops down

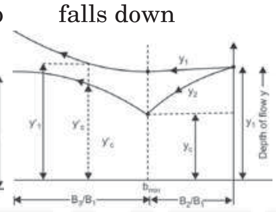
⇒Free surface rises at hump

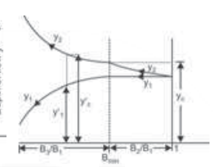
⇒Free surface falls down

⇒Free surface rises up









Venturi Flume: It is a structure in a channel which has a **Contracted Section** called throat, downstream of which follows a flared transition section designed to restore the stream to its original width.

Standing Wave flume or critical depth flume: It is a structure in a channel which has a **narrowed throat with a humpat** bottom. Downstream of which follows a flared section designed to restore the stream to original depth.

Parshall Flume: The converging section of the flume has a level floor, **the throat section has a downward sloping floor**, while the floor in the diverging section slopes upwards

> **Note:** If y_1 and y_2 are alternate depths of horizontal frictionless rectangular channel then
>
> $$\boxed{y_c = \frac{2y_1^2 y_2^2}{y_1 + y_2}} \text{ and } \boxed{E = \frac{y_1^2 + y_1 y_2 + y_2^2}{y_1 + y_2}}$$

Gradually Varied Flow

4

It's a **Steady**, Non uniform flow, where depth of flow varies gradually from section to section along the length of the channel.

Assumptions in GVF:

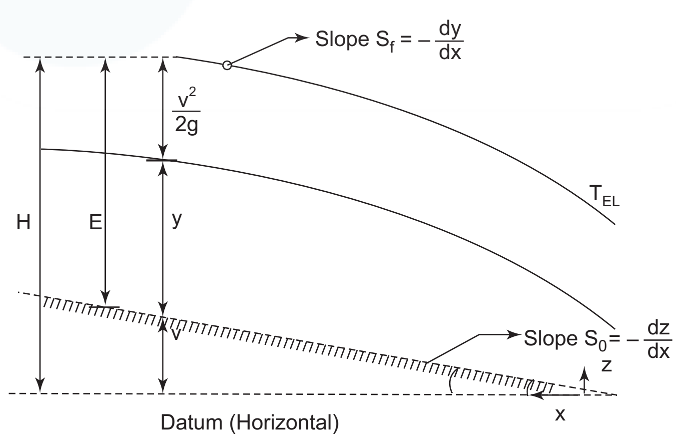
1. Energy Correction Factor (α) = 1
2. Pressure distribution is hydrostatic
3. Flow is Steady i.e, Constant discharge
4. Prismatic channel i.e, Slope and Shape constant
5. Bottom Slope of channel is very small i,e, HGL will lie at free surface.
6. Roughness coefficient is independent of depth.
7. Chezy's and Manning's equations are used to determine energy slope.

Differential Equation for GVF

(1) $$H = Z + E$$

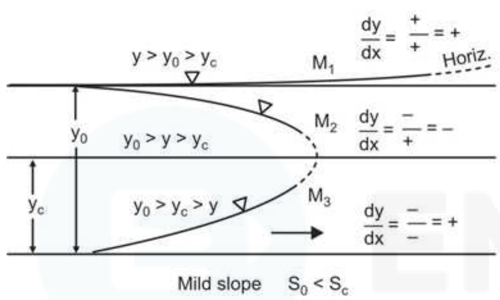
$$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dE}{dx} \Rightarrow \boxed{\frac{dE}{dx} = S_0 - S_1}$$

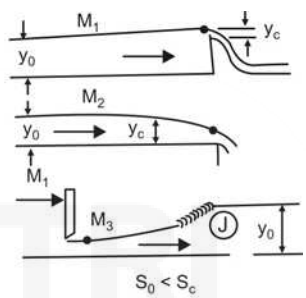
(2) $$H = Z + y + \frac{V^2}{2g}$$

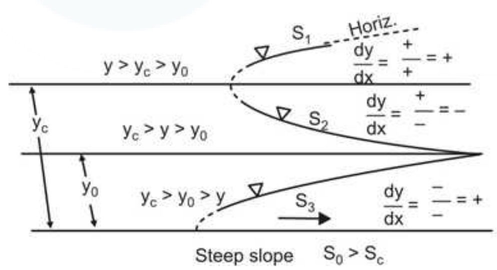


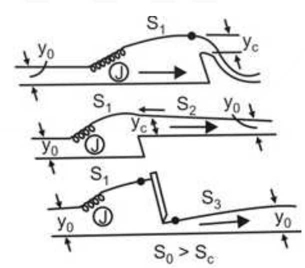
Again differentiating wrt x $\boxed{\dfrac{dy}{dx} = \dfrac{S_0 - S_F}{1 - F_1^2}}$ $F_r = \dfrac{Q^2T}{gA^3}$

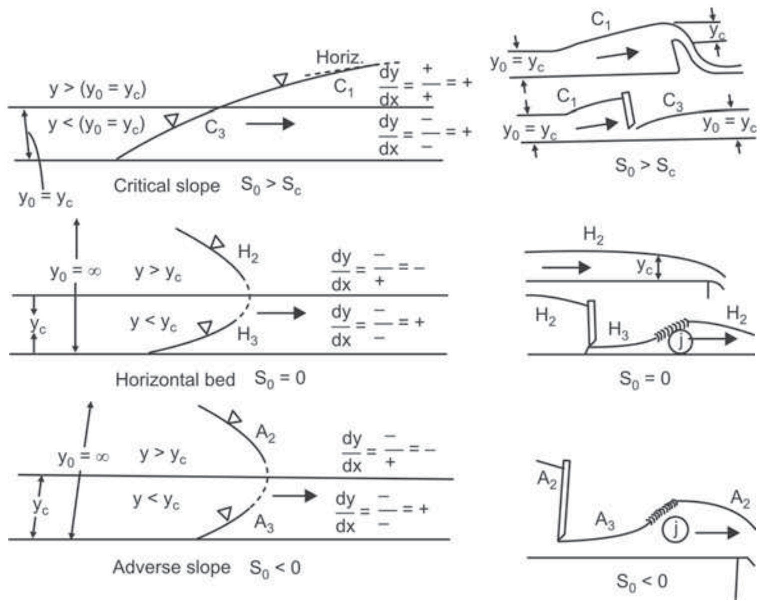
- $\boxed{\dfrac{dy}{dx} = +ve}$ (Region 1 and 3)
  - $\dfrac{(+)}{(+)}\ \dfrac{S_0 > S_F}{1 > F_1^2}$ $\dfrac{(-)}{(-)}\ \dfrac{S_0 < S_F}{1 < F_r^2}$
    - $m_1 (y > y_0 > y_c)$
    - $s_1 (y > y_c > y_0)$
    - $c_1 (y > (y_0 = y_c))$
  - $\dfrac{(-)}{(+)}\ \dfrac{S_0 < S_F}{1 > F_r^2}$ $\dfrac{(+)}{(-)}\ \dfrac{S_0 > S_F}{1 < F_r^2}$
    - $m_3 (y_0 > y_c > y)$
    - $s_3 (y_c > y_0 > y)$
    - $c_3 ((y_0 > y_c) > y)$
    - $H_3 (y_c > y, y_0 \to \infty)$
    - $A_3 (y_c > y, y_0 \to \infty)$
- (Region 2) $\boxed{\dfrac{dy}{dx} = -ve}$
  - $m_2 (y_0 > y > y_c)$
  - $H_2 (y > y_c)$
  - $A_2 (y > y_c)$
  - $s_2 (y_c > y > y_0)$











Note: Surface profile will approach normal depth asymtotically if depth of flow approaches to normal depth of flow. ($dy/dx = 0$)

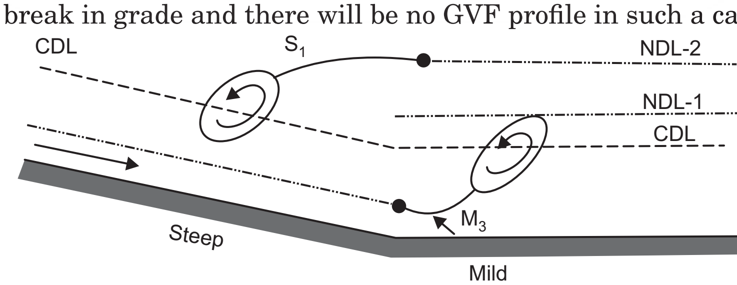
⇒ Surface profile will approach the critical depth line vertically if depth of flow approaches to critical depth of flow. ($dy/dx = S_0$)

⇒ Surface profile becomes horizontal as depth of flow becomes large ($dy/dx = 30$)

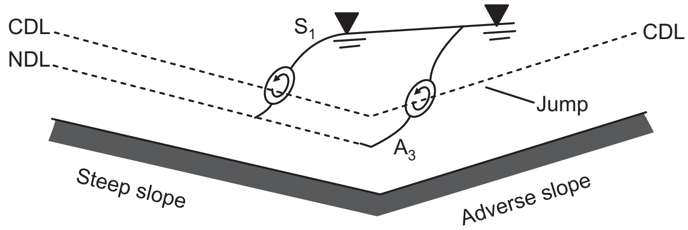
⇒ Surface profile meets the bed vertically if flow depth is reduced to zero ($dy/dx = 0$)

Break-in-Grade: when two channel sections have different bed slope, the section is called break in grade.

Steep to mild : In a limiting case of (i) and (ii) the jump may occur at the break in grade and there will be no GVF profile in such a case.



Either jump - S_1 or M_3 - jump
depending upon sequent depth requirement



Steep to Adverse

> **Note:** CDL is at constant height above the channel bed in both the slopes as critical depth y_c is independent of slope of channel.

$\Rightarrow \dfrac{dy}{dx}$ = +ve means depth of flow increases in flow direction ie Back water curve profile.

$\Rightarrow \dfrac{dy}{dx}$ = –ve means depth of flow decreases in flow direction ie Drow down water curve profile.

Computation of length of gradually varied flow profile:

(1) Direct integration method – chow's method

(2) Direct step method – Numerical method

(3) Graphical method.

Direct step method:

$$\frac{dE}{dX} = s_0 - s_r \quad \boxed{s_r = \frac{n^2Q^2}{A^2R^{4/3}}}$$

$$\boxed{\Delta X = \frac{\Delta E}{s_0 - f_f}}$$

if y_1 and y_2 be depth of flow at section (1) and section (2) respectively then ΔX between them will be calculated as

Y(m)	**Area (m²)**	**R=A/P (m)**	**Velocity (m/sec)**	**Energy (m)**	**ΔE**	**S_f**	$\overline{s_f}$	$s_0 - \overline{s_f}$	**ΔX**
y_1 y_2	A_1 A_2	R_1 R_2	V_1 V_2	E_1 E_2	E_2–E_1	S_{f1} S_{f2}	$\overline{s_f} = \dfrac{s_{t1} + s_{t2}}{2}$	$s_0 - \overline{s_f}$	$\Delta X = \dfrac{\Delta E}{s_0 - s_f}$

5 Rapidly Varied Flow: Hydraulic Jump

In rapidly varied flow, a sudden change in depth occurs at a particular stretch of a channel and the change from one depth to another takes place in a distance of very short length.

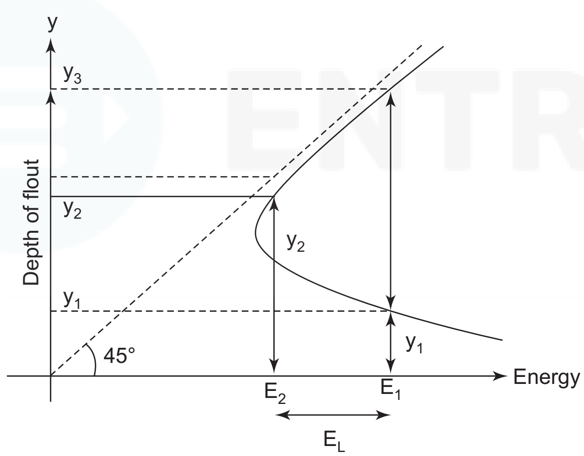
Hydraulic Jump is a formed when water moving at super critical velocity in a relatively shallow stream strikes water having large depth and **sub critical velocity**.

Applications of hydraulic jump are as energy dissipator, to mix chemicals, desalination of sea water, to aerate polluted stream, to reduce the uplift pressure.

Sequent depth or conjugate depth:

During hydraulic jump, water generates considerable disturbances in the form of eddies and reverse flow rollers.

Because of this jump falls shorts of attaining alternate depth (y_3) and attain's another depth ($y_2 < y_3$) called conjugate depth.



Momentum equation for the jump:

$$A\bar{y} + \frac{Q^2}{Ag} = \text{constant} \qquad \frac{P + M}{\gamma} = \text{constant}$$

Hydraulic jump in horizontal frictionless rectangular channel

(1) $\frac{y^2}{2} + \frac{q^2}{gy} = \text{constant}$

(2) $\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$

(3) $\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1 + 8F_1^2} - 1\right)$ where $F_1 = \frac{q^2}{g y_1^3}$

(4) Energy loss $E_L = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$

(5) Relative Energy loss

$$\frac{E_L}{E_1} = \frac{\left(-3 + \sqrt{1 + 8F_1^2}\right)^3}{2\left(2 + F_1^2\right)\left(\sqrt{1 + 8F_1^2} - 1\right)}$$

$$\frac{E_L}{y_1} = \frac{\left(\frac{y_2}{y_1} - 1\right)^2}{4\left(\frac{y_2}{y_1}\right)} \quad \frac{E_1}{y_1} = 1 + \frac{F_1^2}{2}$$

(6) Efficiency of jump $\eta_{jump} = \frac{E_2}{E_1}$

$\eta_{jump} = 1 -$ Relative energy loss

(7) Length of jump $L_j = 6.9\ (y_2 - y_1)$

(8) $F_2^2 = \frac{8F_1^2}{\left(\sqrt{1 + 8F_1^2} - 1\right)^3}$

Types of Jump: It depends on the Froude's number of the **incoming flow** i.e. of upstream end

	Fr	E_L/E_1	Water surface
Undular	1-1.7	≈ 0	Undulating
Weak	1.7-2.5	5 –18%	Small rollers form
Oscillating	2.5-4.5	18–45%	Water oscillates in random manner
Steady	4.5-9	45–70%	Roller and jump action
strong	≥ 9	≥ 70%	Very rough and choppy

Jump on sloping floor:

(1) It requires more tail water depth than corresponding horizontal floor jumps.

(2) Length of jump on sloping floor is longer

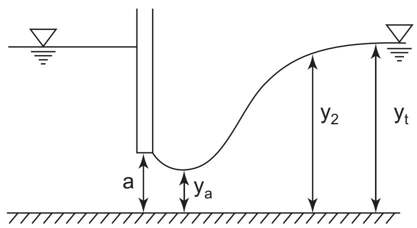
(3) Energy loss will decrease on the sloping floor.

Location of jump:

y_0 = depth of flow at vena-contracta

y_2 = conjugate depth

y_t = tail water depth



(a) $y_t = y_2$ Free hydraulic jump.

(b) $y_1 < y_2$ Free repelled jump.

(c) $y_t > y_2$ Submerged or drowned jump.

Note: Energy loss is less in submerged jump.