MACHINE DESIGNING

Gear

Gears are used with parallel shafts, intersecting shafts or skew shafts (non-parallel, non intersecting).

Gears used with parallel shafts are called **spur gears.** In spur gears, generally the teeth are parallel to the axis of shaft. Helical gears also belong to the family of spur gears. In **helical gears,** the teeth are at some angle with respect to the axis of the shaft.

When the axes of the shafts are intersecting, **bevel gears** are used. Bevel gear can be straight or spiral depending upon the inclination of the teeth. Generally, straight bevel gears are used to connect shafts at low speeds at right angles.

Gear Ratio (G) : The ratio of the number of teeth of the gear to the number of the pinion is called gear ratio.

$$G = \frac{T}{t}$$

Where T - number of teeth on gear and
t = number of teeth on pinion

Velocity Ratio (VR) : It is the ratio of angular velocities of driven to driving gear.

$$\text{or} \quad VR = \frac{\omega_2}{\omega_1} = \frac{2\pi N_2}{2\pi N_1} = \frac{N_2}{N_1}$$

where ω = angular velocity in rad/s
N = angular in revolution per minute
As linear velocity along pitch circle is same for both gears.

$$\pi d_1 N_1 = \pi d_2 N_2$$

$$\therefore VR = \frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{T_1}{T_2}$$

as $d \propto T$ for same module

Arc of Contact

It is the locus of a point on the pitch circle from the beginning to the end of the engagement of a pair of teeth.

Similarly to the path of contact it consists of arc of approach and arc of recess.

$$\text{Length of the arc of contact} = \frac{\text{Length of path of contact}}{\cos\phi}$$

Angle of Action (δ)

The angle turned by a gear from the beginning of engagement to the end of engagement with a pair teeth is called the angle of action. Similarly, the angle of approach (α) and the angle of recess (β) can be defined.

$$\therefore \delta = \alpha + \beta$$

Contact Ratio

It is the ratio of the length of the arc of contact to the circular pitch and represents the pairs of teeth in contact at a time.

There should be at least one pair of teeth in contact always for continuous action. The larger the contact ratio is, the more quietly the gears will operate.

Law of Gearing

According to the law of gearing, the common normal at the point of contact between two teeth always pass through the pitch point at all positions of the gears.

Velocity of Sliding

If the curved surfaces of the two teeth are to remain in contact, one can have a sliding motion relative to the other along the common tangent T - T.

Velocity of sliding = sum of angular velocities × distance between the pitch point and the point of contact.

Conjugate Profile of Gears

When the profile of gear teeth is such that a constant angular velocity ratio is maintained, such profiles are called conjugate profiles. The commonly used tooth profiles are

1. Cycloidal profile
2. Involute profile

In involute gears, the pressure angle remains constant from the start to the end of engagement, but in cycloidal teeth the pressure angle varies.

Cycloidal teeth are free from interference problem, while interference problem occurs in involute gears.

Interference

In involute gears, the involute profile of the teeth are formed from the base circle. The radius of the base circle should be less than the dedendum circle for the smooth working of the gears. The relative position of the base circle and the dedendum circle depends upon the number of teeth, module and pressure angle. If radius of the base circle is less than the dedendum circle, the tip of the mating gear digs into the flank of the gear. this phenomenon is known as **interference.**

Minimum Number of Teeth to Avoid Interference

To avoid interference the **minimum number of teeth required on the pinion** is given by

$$t = \frac{2A_p}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}$$

where A_p = Addentum coefficient of pinion

$$= \frac{\text{Addentum}}{\text{Module}}$$

G = gear ratio

$$= \frac{T}{t} = \frac{D}{d} = \frac{N_p}{N_G}$$

Permissible Working Stress for gear Teeth in the Lewis Equation

The permissible working stress is arrived at using the following formula

$$\sigma_w = \sigma_0 \times C_v$$

Where σ_0 = allowable static stress

and C_v = velocity factor

The values of velocity factor for various working conditions are as follows.

1. For ordinary cut gears, with operating velocity upto 12.5 m/s

$$C_v = \frac{3}{3 + v}$$

2. For carefully cut gears with operating velocities upto 12.5 m/s

$$C_v = \frac{4.5}{4.5 + v}$$

3. For very accurately cut and ground metallic gears with operating velocity upto 20 m/s

$$C_v = \frac{6}{6 + v}$$

(v = Pitch line velocity in m/s)

Dynamic Tooth Load (W_D)

Dynamic tooth load is obtained using the following formula.

$$W_D = W_T + W_I = W_T + \frac{2lv(bc + W_T)}{2\,lv + \sqrt{bc + W_T}}$$

Where W_D = Total dynamic load

W_T = Steady load due to the torque transmitted.

W_I = Increment load due to dynamic action.

v = Pitch line velocity (m/s)

b = Face width of gear tooth (mm)

c = Deformation or dynamic factor (N/mm)

$$c = \frac{k.e}{\frac{1}{E_p} + \frac{1}{E_g}}$$

Where E_p and E_g are Young's modulus of pinion and gear materials, respectively

Bearings

The purpose of a bearing is to support rotating axles and shaft, with parts fitted on them, ensuring free rotation. Bearings also transmit the force acting on them to the machine frame or a foundation. Depending upon he nature of friction in the bearings they are classified as sliding contact plain bearings or rolling contact antifriction bearings.

Bearing Characteristic Number

Bearing characteristic number is a dimensionless number given by $\frac{\mu N}{P}$ where,

μ = Absolute viscosity of the lubricant

N = Speed of journal.

P = Unit bearing pressure

$= \frac{W}{\ell d}$ where W = radial load,

ℓ = Length of bearing

d = Diameter of bearing

Coefficient of friction

Experimentally, it has been shown that coefficient of friction for full lubricated journal bearing is a function of three variables.

$$\frac{\mu N}{P}, \frac{d}{c} \text{ and } \frac{\ell}{d}$$

Therefore, coefficient of friction is expressed as

$$f = \phi\left(\frac{\mu N}{P}, \frac{d}{c}, \frac{\ell}{d}\right)$$

Where f = Coefficient of friction

μ = Absolute viscosity of lubricant in Ns/m^2

n = Speed of journal in rpm

p = Bearing pressure in N/mm^2

d = Diameter of journal

c = Diameter clearance

$\frac{\mu N}{P}$ is bearing characteristic number as already mentioned.

Coefficient of Friction for Journal Bearing

Based on experimental investigation, McKee arrived at the following empirical relation for coefficient of friction.

$$f = \frac{0.326}{10^6}\left(\frac{\mu N}{P}\right)\left(\frac{d}{c}\right) + K$$

Where K = a factor to correct for end leakage

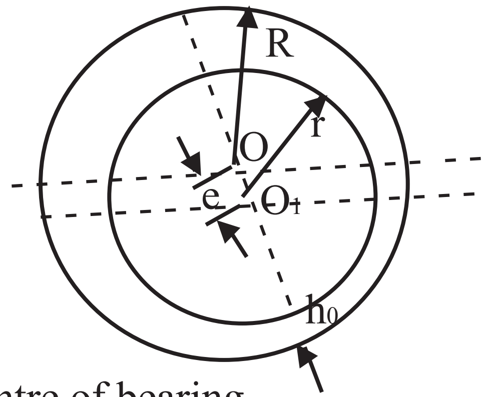
Value of K depends on length to diameter ratio $\left(\frac{\ell}{d}\right)$ of the bearing

For $\frac{\ell}{d}$ ratio 0.75 to 2.8

K is taken as 0.002

Eccentricity Ratio

The ratio of eccentricity to radial clearance is called eccentricity ratio or attitude.



O = Centre of bearing

O_1 = Displaced centre of the journal under load.

e = eccentricity

= Displacement between O and O_1

h_0 = Minimum oil film thickness

$= R - r - e$

$= c_1 - e$

where c_1 = radial clearance

$= R - r$

(Value of h_0 is taken as $\frac{c_1}{2}$)

Eccentricity ratio, $\varepsilon = \frac{e}{c_1}$

Diameter clearance

$c = D - d = 2(R - r) = 2c_1$

Diametral Clearance Ratio

Ratio of diameter clearance to diameter of the journal is called the diameter clearance ratio.

Diametral clearance ratio $= \frac{D - d}{d} = \frac{c}{d}$

Critical Pressure

It is the pressure at which oil film breaks and metal to metal contact begins. It is the minimum operating pressure. It is given by the following empirical relation

$$p = \frac{\mu N}{4.75 \times 10^6}\left(\frac{d}{c}\right)^2\left(\frac{\ell}{\ell + d}\right) \text{ N/mm}^2$$

Sommerfield Number

Sommerfield number is a dimensionless parameter used in the design journal bearings. It is given by

$$\frac{\mu N_S}{p}\left(\frac{d}{c}\right)^2$$

Where μ = absolute viscosity in $\frac{kg}{ms}$ or $\frac{Ns}{m^2}$

c = diameteral clearance

N_S = revolution per second (rps)

p = Bearing pressure in N/m^2

Dynamic Load Carrying Capacity

Dynamic load carrying capacity of a bearing is the load the bearing can carry (radial load for radial bearings) for a minimum life of one million revolutions. It is denoted by the letter C.

Rating Life or Minimum Life

It is the life that 90% of a group of bearings can attain before fatigue failure. Rating life is denoted as L_{10}..

The life that 50% of the group can attain is the average life. It has been found that the average life is 5 times the rating life. The maximum life of a single life of a single bearing is about 30 to 50 times the minimum life.

Dynamic Equivalent Bearing Load

Dynamic equivalent load is the constant radial load in the radial bearing (or thrust load in the thrust bearing) which gives the same bearing life under actual load condition.

Equivalent dynamic load $W = XV\,W_R + YW_A$

Where W_R= radial load in newton

W_A = axial load in newton

V = race rotation factor

X and Y are radial and axial load factors

When outer race is stationary and inner race rotates,

V = 1

When inner race is stationary and outer race rotates, V = 1.2

Load Life relationship

Rating life $L_{10} = \left(\frac{C}{W}\right)^P$ million revolutions (MR)

or $C = W(L_{10})^{1/e}$

Where C = dynamic load capacity (netwon)

W = equivalent dynamic load.

$p = 3$ for ball bearings $= \dfrac{10}{3}$ for roller bearings

Rated Bearing Life in Hours (L_{10h})

$$L_{10h} = \frac{L_{10} \times 10^6}{N \times 60}$$

or $$L_{10} = \frac{60N\,L_{10h}}{10}$$

Where N = speed of rotation (rpm)

Shafts

Shafts are rotating machine elements, usually of circular cross section, solid or hollow, transmitting power. They support transmission elements such as gears, pulleys and sprockets and are supported in bearings. Shafts are subjected to tensile, bending or torsional shear stresses or a combination of these. Shafts are designed on the basis of strength or rigidity or both strength and rigidity.

Shafts Subjected to Axial Force

Tensile stress $\sigma_t = \dfrac{P}{\frac{\pi}{4} d^2} = \dfrac{4P}{\pi d^2}$

Where P = tensile force, d = diameter

Shafts Subjected to Pure Bending Moment

Bending stress $\sigma_b = \dfrac{M_b\, y}{I}$

Where M_b = Bending moment

y = Distance from neutral axis

I = Moment of inertia

$= \dfrac{\pi d^4}{64}$ for solid shafts

$\therefore \sigma_b = \dfrac{M_b}{\pi d^3}$ where $y = \dfrac{d}{2}$

$= \dfrac{M_b}{Z}$ where $Z = \dfrac{\pi d^3}{32}$

= Section modulus

Equivalent Bending Moment (M_e) and Equivalent Torsional Moment (T_e)

$$\sigma_{max} = \frac{32M_e}{\pi d^3}$$

$$\tau_{max} = \frac{16T_e}{\pi d^3}$$

So we can write

$$M_e = \frac{1}{2}\left[M_b + \sqrt{M_b^2 + M_t^2}\right]$$

$$T_e = \sqrt{M_b^2 + M_t^2}$$

Brakes

Brake is a device used for retarding or stopping the motion of a machine by means of frictional resistance. In the braking process, either the kinetic energy or potential energy is absorbed and the energy is dissipated in the form of heat.

Types of Brakes

According to the means used for transforming energy, brakes may be classified as

1. Hydraulic brakes
2. Electric brakes
3. Mechanical brakes

Hydraulic and electric brakes are used where large amount of energy is to be transformed. They cannot bring the member to rest and are used for controlling the speed.

Mechanical brakes may be classified into radial brakes or axial brakes according to the direction of acting force.

In axial brakes, the force acting on the brake drum is in the axial direction.

Disc brakes and cone brakes are examples of axial brakes. Analysis of these brakes are similar to that of clutches.

Radial brakes include block or shoe brakes, band brakes, internal expanding brakes, etc.

Pivoted Block or Shoe Brake

When the angle of contact 2θ is less than 60° as in the case of single block or shoe brake, the normal pressure between block and wheel can be assumed to be uniform. But when 2θ is greater than 60°, the normal pressure is less towards the ends of the shoe. In such cases, pivoted shoes or brakes are used. This results in uniform wear of the shoe in the direction of the applied force. Breaking torque is given by,

$$T_B = F_t \times r = \mu' R_N \times r$$

Where $$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta}$$

Clutches

Clutches may be divided into two types - positive clutches and friction clutches. Jaw and claw clutches are examples of positive clutches. Electrical, electro magnetic and hydraulic clutches are also used.

A clutch is primarily used to engage and disengage driver and driven shafts according to requirements. Its principal application is in power transmission where shafts and machines are to be started and stopped frequently.

A brake is used for stopping or controlling the speed of a running machine, whereas a clutch is used to disengege a machine before stoppage or engage the driven shaft from rest to proper speed.

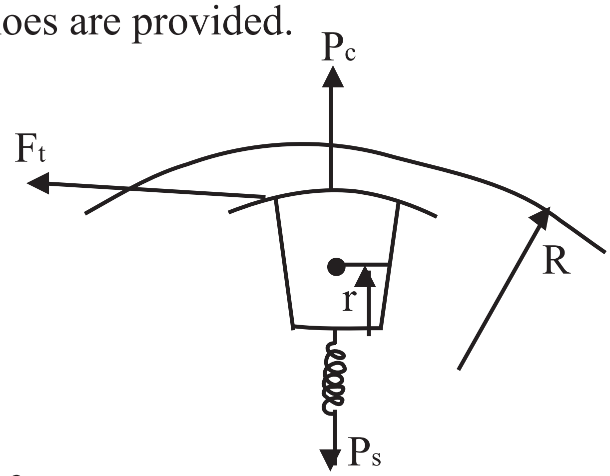
In friction clutches, plates or cones with friction lining are used to engage or disengage the driver and driven shafts.

The main types of friction clutches are

1. Disc or plate clutch (single or multiple plate clutches)
2. Cone clutch
3. Centrifugal clutches

Centrifugal Clutch

In a centrifugal clutch held by springs move radially outward due to centrifugal force and engage the inside surface of the cylindrical drum, in which the shoes are provided.



P_s = Spring force = s × a

where s = stiffness of springs

a = amount of compression of spring

R = radius of friction surface

r = Radial distance of centre of gravity of shoe

m = mass of each shoe

n = number of each shoes.

When engagement starts the centrifugal force is equal to the spring force.

Centrifugal force $P_c = m\omega^2 r$

where ω = angular velocity

Net outward force = $P_c - p_s$

Frictional force on each shoe = $\mu(P_c - P_s)$

Frictional torque on each shoe = $\mu(P_c - P_s) \times R$

Total torque transmitted = $n\mu(P_c - P_s)R$

Force exerted on each shoe = $P_c - P_s = p\ell b$

Where p = pressure intensity on each shoe

ℓ = contact length of the shoe
b = breadth of the shoe
Contact length $\ell = R\theta$
where θ = the subtensing angle of the shoe

Bolted Joints
Bolted joint is a separable joint of two or more components held together by means of a threaded fastening such as bolt and nut.

Minor Diameter
It is the smallest diameter of an external or internal thread. It is also known as **core diameter or root diameter** for external threads.

Pitch Diameter (d_p)
It is defined as the diameter of an imaginary cylinder that pass through pitch points of the thread. **Pitch points** are points on the surface of thread such that width of a thread is equal to the space between the threads.

Pitch (p)
Pitch is the distance between two similar points on adjacent threads, measured parallel to the axis of the thread.

$$\text{Pitch} = \frac{1}{\text{Number of threads per unit length}}$$

Lead (L)
Lead is the distance between two corresponding points on the same helix. It is important in the case of multi start threads.

Lead is equal to pitch in the case of single start threads. It is the distance travelled by the nut in one turn.

$$L = n \times p$$

Where n = number of starts

Thread angle
It is included angle of flanks of two adjacent threads.

Torque Required for Bolt Tightening
two factors considered for tightening the bolt are :
1. Torque to overcome thread friction and induce the pre-load.
2. Torque to overcome collar friction between the nut and the washer.

Torque required to overcome thread friction,

$$M_t = \frac{P_i\, d_m}{2} \times \frac{(\mu \sec\theta + \tan\theta)}{(1 - \mu \sec\theta + \tan\alpha)}$$

For ISO metric screw threads,
$\theta = 30^o$, $\alpha = 25^o$ and $d_m = 0.9\, d$

d = nominal or major diameter of the bolt

$\mu = 0.12 - 0.2$

Taking $\mu = 0.15$

$M_t = 0.098\, P_i\, d$

Collar friction torque

$$(M_t)_C = \left(\frac{\mu P_i}{2}\right)\left(\frac{D_0 + D_i}{2}\right)$$

Taking $\left(\frac{D_0 + D_i}{0}\right) = 1.44$ and $\mu = 0.15$ for ISO threads

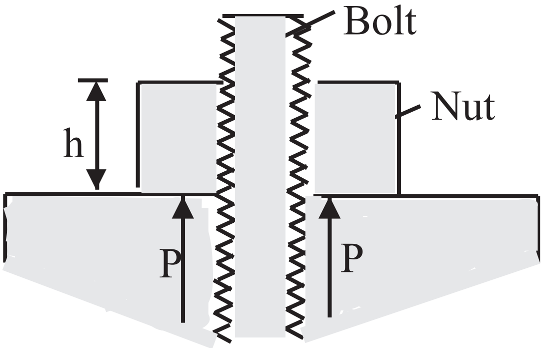
$$(M_t)_C = 0.105\, P_i\, d$$

Total torque (M_t) is given by

$(M_t) = M_t + (M_t)_C = (0.098 + 0.105)P_i\, d = 0.2\, P_i\, d$

The above equation gives the wrench torque (M_t) required to create the required pre-load P_i .

Height of Nut (h)



Permissible tensile stress of bolt

$$\frac{S_{yt}}{FOS} = \frac{P}{\frac{\pi}{4}(d_c)^2}$$

where d_c = core diameter

P = axial load

$$\therefore P = \left(\frac{S_{yt}}{FOS}\right)\frac{\pi}{4}(d_c)^2 \rightarrow (1)$$

Permissible shear stress on bolt threads

$$\frac{S_{yt}}{FOS} = \frac{P}{\pi d_c h}$$

$$\therefore P = \left(\frac{S_{yt}}{FOS}\right) \times \pi d_c h$$

$$= \left(\frac{S_{yt}}{2FOS}\right) \times \pi d_c h \quad \rightarrow \quad (2)$$

As $S_{sy} = \frac{S_{yt}}{2}$ by maximum shear stress theory

$\therefore$ From (1) and (2)

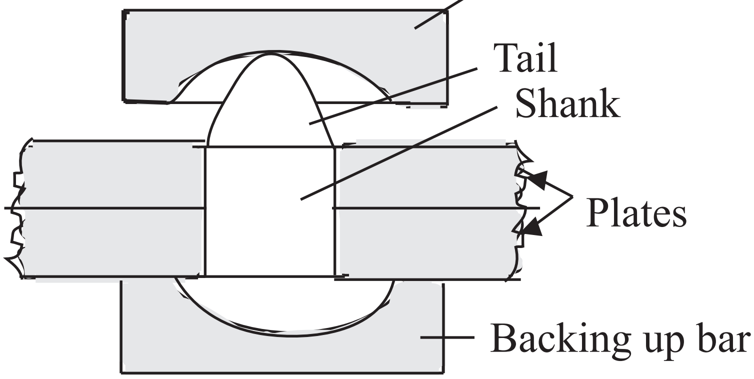
$$\frac{d_c^2}{4} = \frac{d_c h}{2}$$

$$\Rightarrow h = \frac{d_c}{2} \quad \Rightarrow h = 0.5\, d_c$$

$$d = \frac{d_c}{0.8} = \frac{11.1}{0.8} = 13.9 \text{ mm} = 14 \text{ mm}$$

Riveted Joints

A rivet is a short cylindrical shank with a head used for making permanent joints of metal plates. The shank is inserted into the drilled holes of the plates to be joined and the tail at the end of shank is hammered to form the closing head with the help of a die or snap.



Riverting may be done by hand or riveting machine.

Tough and ductile materials are used for riveting. They are usually made of low carbon steel or nickel steel. brass, aluminium or copper rivets are also used. For fluid tight joints steel rivets are used.

Important Terminology Used

Pitch (p) : It is the centre distance between two adjacent rivets measured parallel to the seam of the joint.

Back pitch : It is the perpendicular distance between centre lines of successive rows of rivets.

Diagonal Pitch (P_d) : It is the centre distance between rivets in adjacent rows in zigzag riveting.

Margin or marginal pitch (m) : It is the distance between centre of the rivet hole to the nearest edge of the plate.

Failure of Riveted Joint

Failure of riverted joint may occur in the following manner.

1. Tearing of the plate at the edge
2. Tearing of the plate across a row of rivets
3. Shearing of the rivets
4. Crushing of the plate or rivet

Tearing of plate at the edge can be avoided using sufficient marginal pitch. Usually a marginal pitch m = 1.5 d is provided.
Where d = diameter of hole
Tearing resistance across a row per pitch length

$$P_t = (p - d)t\sigma_t$$

where σ_t = permissible tensile stress
t = thickness of plate
Shearing resistance of rivets per pitch length

$$P_s = n \times \frac{\pi d^2}{4} \times \tau$$

Where τ = permissible shear stress
n = number of rivets per pitch length
In the case of double shear where cover plates are used,

$$P_s = n \times 2 \times \frac{d^2}{4} \times \tau$$

According to Indian Boilre Regulations (IBR)

$$P_s = n \times 1.875 \frac{\pi d^2}{4} \times \tau \text{ for double shear}$$

Crushing resistance per pitch length

$$P_c = ndt \times \sigma_C$$

Where σ_C = permissible crushing stress for rivet or plate materials.

Strength of a Riveted Joint

A riveted joint will fail if the applied force P is greater than P_t , P_s or P_c

The strength of a riveted joint is the maximum force. It can transmit without failing. So, the strength is the least of P_t , P_s or P_c .

For continuous joints, the strength is calculated per pitch length, but for small joints strength is calculated for the whole width of the plates.

Efficiency of a Riveted Joint

Efficiency of a riveted joint is the ratio of the strength of the joint to the strength of the unriveted or solid plate.

i.e. $$\eta = \frac{\text{Least pf } P_t , P_s \text{ or } P_c}{P \times t \times \sigma_t}$$

Welded Joints

Welded joints are permanent joints formed by welding of the metal parts.

Welding is the process of joining metal parts by heating to plastic or liquid state at the regions where they are to be joined. In the first case, compressive forces are required for welding.

Depending upon the relative position of the parts to be welded, the welded joints can be classified as

1. Butt joint
2. Lap joint

3. T-joint
4. Corner joint
5. Edge joint etc

Butt joint is the joint between two components or plates lying in the same plane. In butt welding of thick plates (thickness above 5 mm), the ends of the plates are to be beveled to 'U' or 'V' shape. Simgle or double butt joints are used depending upon the thickness of plates. Square butt joints are used for plates of thickness less than 5 mm.

Strength of Parallel Fillet Welds

Parallel fillet welds are designed for shear strength

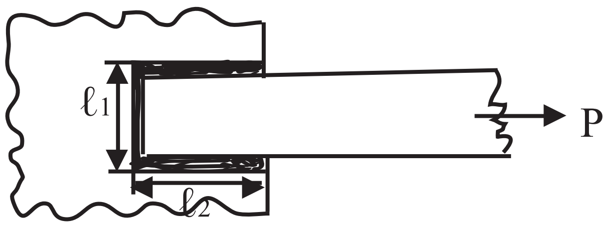
Shear strength = throat area × allowable shear stress

$$= 0.707\ s\ \ell_1\ \tau$$

Shear strength for double parallel fillet weld

$$= 2 \times 0.707\ s\ \ell_1\ \tau$$
$$= 1.414\ s\ \ell_1\ \tau$$

Combination of Parallel and Transverse Fillet Welds



Strength of a combination of transverse and parallel fillet welds as shown above is

$$P = 0.707\ S\ell_n\ \sigma_t + 2 \times 0.707\ S\ \ell_2\ \tau$$

Sometimes, transverse fillet welds are also treated as parallel fillet welds and designed for shear strength. In transverse fillet welds, normal, bending and shear stresses are acting. It is proved that the maximum shear stresses are acting. It is proved that the maximum shear stress is induced in a plane inclined 67.5° to the leg dimension. For any direction of the applied load, shear stress on the throat area can be assumed as the stress for design and the parallel fillet formula can be used. So $P = 0.70\ s \times (\ell_{11} + 2\ell_{12})\tau$

Welds Subjected to Bending

For welds subjected to bending, stress is calculated using the relation

$\sigma = \dfrac{M}{Z}$ where M = bending moment, Z = section modulus about neutral axis

Shear stress

$$\tau = \frac{P}{A}$$

Maximum shear stress $\tau_{max} = \sqrt{\left(\dfrac{\sigma}{2}\right)^2 + \tau^2}$