ENTRI

Civil Engineering

1

Strength of Materials

CONTENTS

Properties of Metals, Stress and Strain

1

Rigid and Deformable Material:

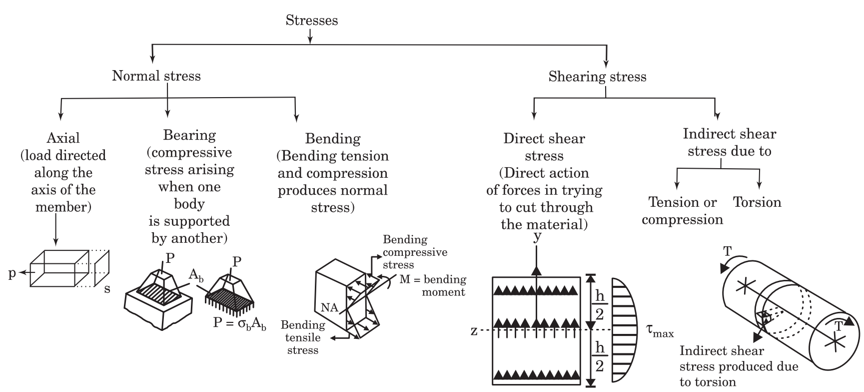
Rigid material is one which does not undergo any change in its geometry, size or shape. On the other hand, a deformable material is the one in which change in size, shape or both will occur when it is subjected to force/moment.

Stresses and strain:

Stresses (Force/Area) are generated as a resistance to the applied external forces or as a result of restrained deformations.

$$\text{Nominal stress (Engineering stress)} = \frac{\text{Load}}{\text{Original Area}}$$

$$\text{Actual/Truestress} = \frac{\text{Load}}{\text{Original (Actual) Area}}$$

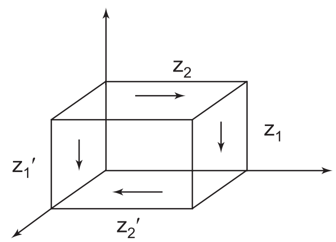


$$\text{Normal stress} = \frac{\partial P}{\partial A} = \sigma \Rightarrow P = \int \sigma dA$$

Equality of shear stress on perpendicular planes

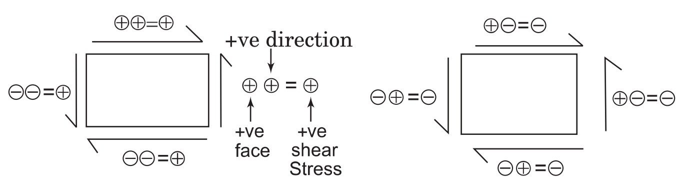
(1) Shear stress on opposite faces of an element are equal in magnitude and opposite in direction.

(2) Shear stress on adjacent and perpendicular faces of an element are equal in magnitude and have directions such that both stresses point towards or both point away from the line of intersection of the faces. These are called Complimentary shear stresses.



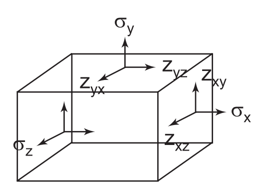
(Shear stress on opposite face are equal and opposite)

Sign convention for shear stress



Stresses under general loading conditions

1. Stress is NOT a Vector
2. **Stress** is a **2nd** order Tensor.
3. σ (Stress tensor) = $\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$



4. Magnitude has only one dimension Hence it a 3° = zero order tensor
5. Direction has three dimension. Hence it is $3^2 = 1^{st}$ order tensor
6. Stress has 9-dimension ($3^2 = 2^{nd}$ order tensor)
7. At any point in 3D condition 9 stress elements are there.
   3 Normal stress components (σ_{xx}, σ_{yy}, σ_{zz})
   6 shear stress components (τ_{xy}, τ_{yx}, τ_{xz}, τ_{zx}, τ_{yz}, τ_{zy})

ONLY **6-stress** components are required to define conditions of stress at a point.

8. In 2-D condition, 4 stress elements exist (σ_x, σ_y, σ_{xy}, σ_{zy}) but ONLY 3-stress components are required to define conditions of stress at a point.

Design of members:

$$\text{Allowable stress} = \frac{\text{yield stress}}{\text{F.O.S}}$$

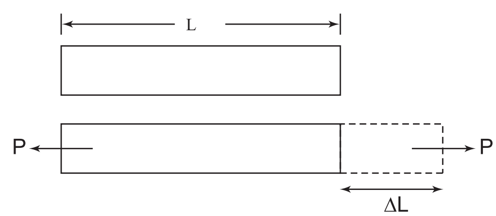
$$\text{Margin of safety} = \text{FOS-1}$$

For **Ductile material:** FOS is applied on **yield stress**

For **Brittle material:** FOS is applied on **Ultimate stress**.

Normal Strain:

1. Deformation per unit length
2. Strain = $\dfrac{\Delta L}{L}$ or $\dfrac{\delta L}{\delta L}$



3. Measured by **EXTENSOMETER** It is a **dimensionless** quanity

Mathematical definition of strain

$\varepsilon_x = \dfrac{\partial u}{\partial x}$ Normal strain in x-direction

$\varepsilon_y = \dfrac{\partial v}{\partial y}$ Normal strain in y-direction

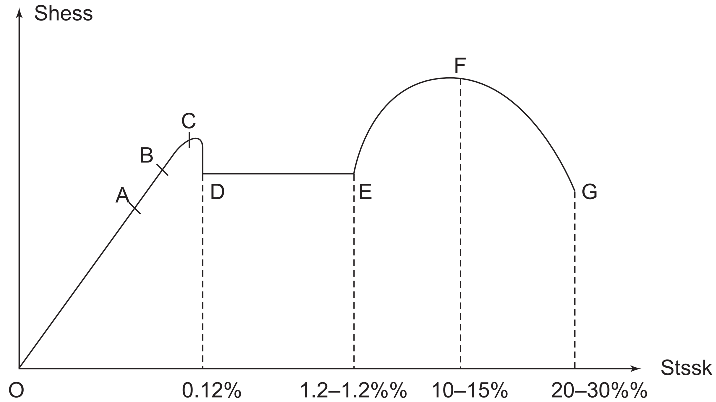
$\varepsilon_z = \dfrac{\partial \omega}{\partial z}$ Normal strain in y-direction

$\gamma_{xy} = \dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y}$ = Shearing strain in xy plane

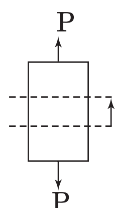
$\gamma_{xz} = \dfrac{\partial \omega}{\partial x} + \dfrac{\partial u}{\partial z}$ = Shearing strain in xz plane

$\gamma_{yz} = \dfrac{\partial \omega}{\partial y} + \dfrac{\partial v}{\partial z}$ = Shearing strain in yz plane

Stress-strain Curve of mild steel (Low carbon steel-Ductile Steel) in Tension



OA = Linear curve
A = Proportional limit
B = Elastic limit
C = upper yield point
D = lower yield point
DE = plastic region
EF = strain hardening region
FG = Neeking region
F = ultimate stress point
G = Fracture point.



L_0 = Gauge length
= lnitiai length

stress = $\frac{p}{A_0}$ strain = $\frac{p}{A_0}$

Salient points:

(1) Volume of specimen increases from O to D

(2) Lower yield point should be used to determine the yield strength of material

(3) From D to E, large deformations but volume of specimen does not changes.

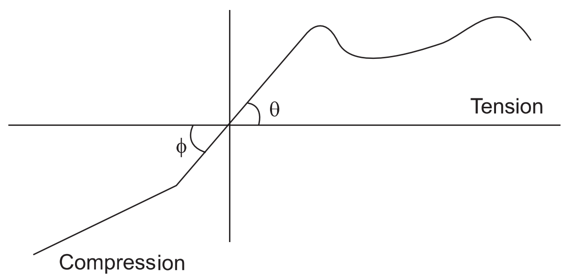
(4) From E to F, its strain hardening, i.e material undergoes changes in its crystalline structure.

(5) F to G, diameter of portion decreases due to instability called Necking.

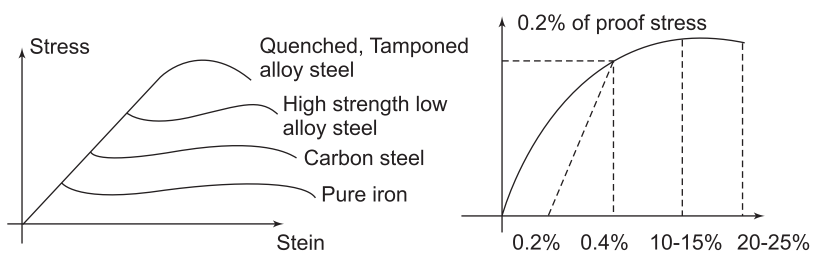
(6) Cup cone failure occurs at 45° with the load in ductile material.

Mild steel in compression

(1) The stress strain curve will eventually be same through its initial straight line portion and through the beginning of the portion corresponding to yield and strain hardening



(2) Modular of Elasticity in Tension= Modular of Elasticity in compression $\theta = \phi$



Stress-strain curves for other materials

(1) All of them possess some Modulas of Elasticity.

(2) As yield strength increases, Ductility falls.

(3) For ductile materials like Aluminium and Copper, do not have defined yield point. Yield strength is defined by offset method.

(4) $E_{AL} = \frac{1}{3} E_{st}$

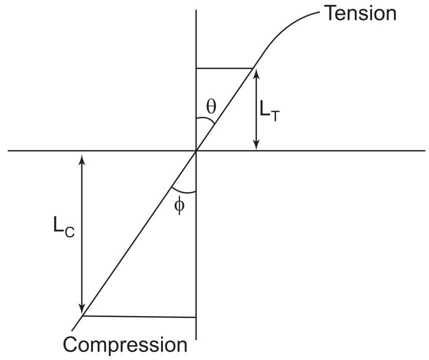
Stress-strain diagram for Brittle material

(1) $\phi = \theta$

(2) Linear Elastic range in compression is more than Tension

(3) Rupture stress = Ultimate stress

(4) No Necking occurs.



TRICK: to Remember failure surface:- Remember any one of the 4 given below and change at least two columns every time keeping the one constant.

(1) Ductile	Tension	45°
(2) Ductile	Torsion	90°
(3) Brittle	Tension	90°
(4) Brittle	Torsion	45°

Eg. Remember

Ductile Tension 45°

Same ↕ ↓ Change ↓

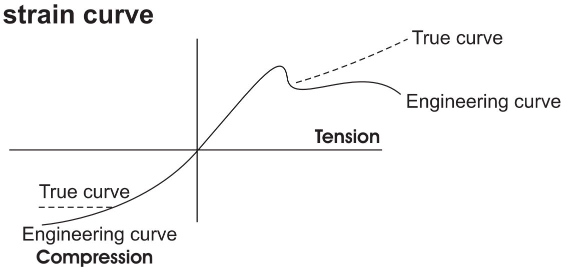
Ductile Torsion 90°

Brittle fracture:

(1) Ductile material at normal temp. may become brittle at very low temp.

(2) A Brittle material at low temp. may become ductile at very high temp.

True stress strain curve



(1) True stress curve is below Engineering stress in compression because resisting area in compression increases

(2) Engineering stress = $\frac{P}{A_0}$ True stress = $\frac{P}{A}$

Engineering stress = $\frac{\delta}{L_0}$ True stress = $\frac{\Delta L}{L}$

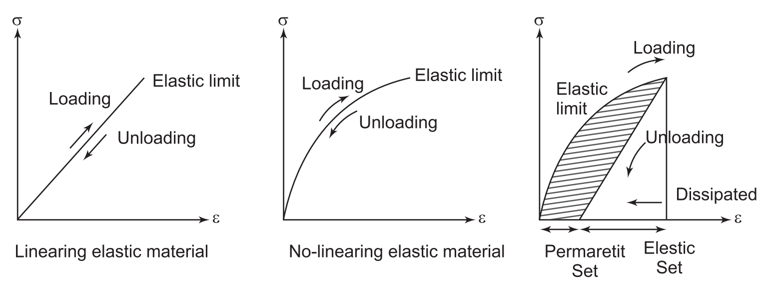
A_0, L_0 → Original Area & length

Relation between True stress and Engineering stress

In Tension: $A = \frac{A_o}{1+\xi}$, $\sigma = \sigma_o(1+\xi)$ In compression $A = \frac{A_o}{1-\xi}$, $\sigma = \sigma_o(1-\xi)$

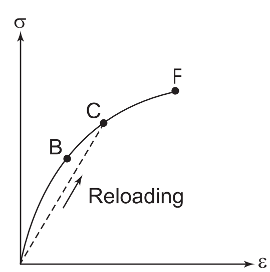
Properties of Materials

Elasticity:- Property by virtue of which material deformed under the load is enabled to **return** to its original dimension when the load is removed.



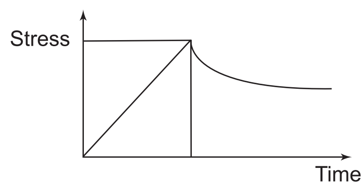
Plasticity:- The characteristics of material by which it undergoes **inelastic strain** beyond those at the **elastic limit**.

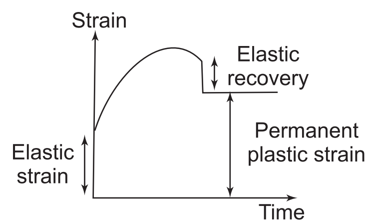
Reloading:- Proportional limit increases from B to C but ductility decreases from 'B to F' to 'C to F'



Creep: Property by virtue of which a material undergoes **additional deformation** (over and above due to applied load) with passage of time under sustained loading with in **elastic limit**

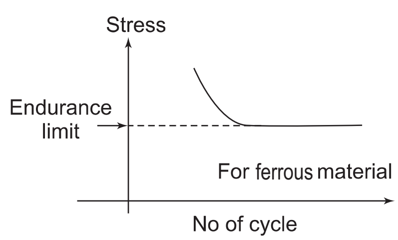
Relaxation:- The decrease in stress in steel as a result of creep with in steel under prolonged strain

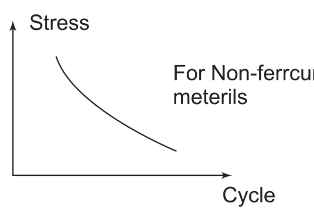




Fatigue:- Deterioration of a material under repeated cycles of stress or strain resulting in progressive cracking that eventually produces fracture.

Endurance limit:- Stress level below which even large number of stress cycle cannot produce fatigue failure.

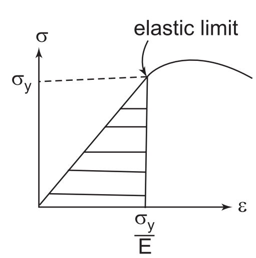




For structural steel, Endurance limit = $\frac{1}{2} \times$ ultimate strength

Resilience:- Property of material to absorb energy when it is deformed elastically and then upon unloading to have **this energy recovered.**

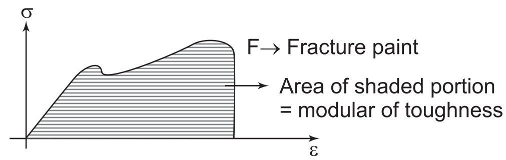
Modular of Resilience: Elastic strain energy stored **per unit volume**



$$= \frac{1}{2} \times \sigma_y \times \frac{\sigma_y}{E}$$

$$= \frac{\sigma_y^2}{2E}$$

Toughness:- Ability to absorb mechanical energy upto **failure**.



Toughness → Resists fracture

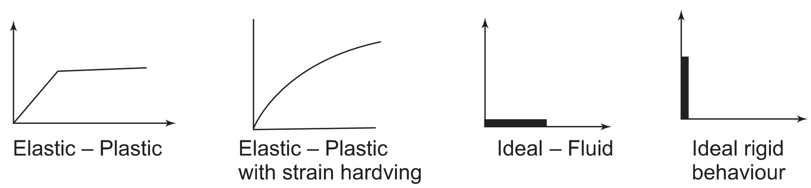
Hardness → Resists scratch or abration

Tenacity:- Property of material to resist fracture under the action of tensile load

Visco-Elastic material

Materials having both Viscous and Elastic properties and exhibit time dependent strain.

Approximate stress-strain curves



Hooke's law:-

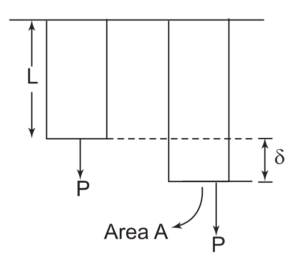
(a) Homogenous $\boxed{\sigma = E.\xi}$

(b) Isotropic

(c) Linearly elastic materials

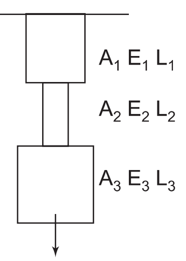
Deformation of member under axial load

(1)



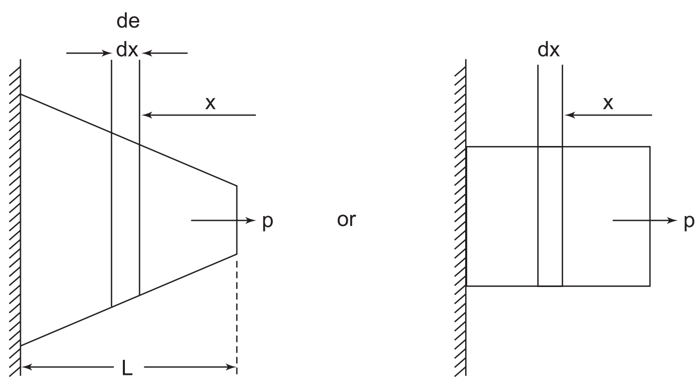
Load P is acting then $\delta = \dfrac{PL}{AE}$

(2)



Load P is acting then $\boxed{\delta = \dfrac{P_1L_1}{A_1E_1} + \dfrac{P_2L_2}{A_2E_2} + \dfrac{P_3L_3}{A_3E_3}}$

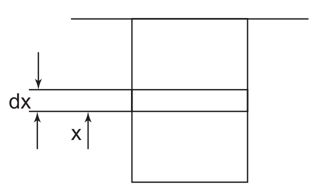
(3) $\boxed{\delta = \int_0^L \dfrac{P(x)dx}{A(x)E}}$ then



Fore = P(x)

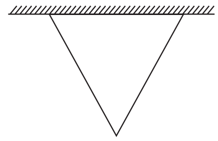
$\boxed{\delta = \int_0^L \dfrac{Pdx}{A(x)E}}$ $\boxed{\delta = \int_0^L \dfrac{P(x)dx}{AE}}$

(4) (a) In prismatic bar due to self weight



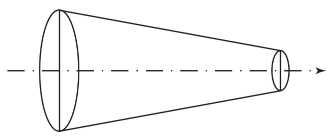
$\boxed{\delta = \dfrac{\gamma L^2}{2E} \text{ or } \dfrac{\left(W/2\right)L}{AE}}$

(b) Conical bar due to self weight



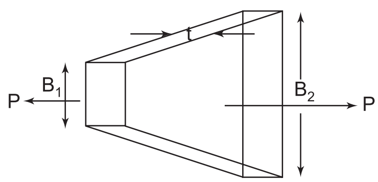
$$\boxed{\delta = \frac{\gamma L^2}{6E}} = \frac{1}{3}\text{(deflection of prismatic bar of same length and same density)}$$

5.



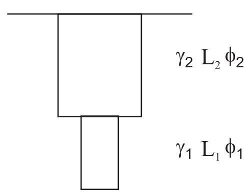
$$\boxed{\Delta = \frac{4PL}{\pi D_1 D_2 E}}$$

6.



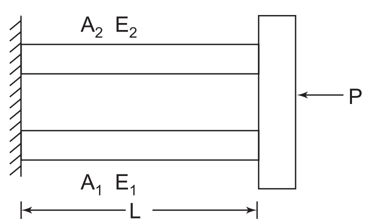
$$\boxed{\Delta = \frac{PL\log_e\left(B_2/B_1\right)}{Et(B_2\text{-}B_1)}}$$

7.



$$\boxed{\Delta = \frac{\gamma L_1^2}{2E} + \frac{\gamma L_2^2}{2E} + \frac{\gamma \phi_1^2 L_1 L_2}{\phi_2^2 E}}$$

Composite Bars



$$P = P_1 = P_2$$

$$\delta_1 = \delta_2 = \frac{P_1 L}{A_1 E_1} = \frac{P_2 L}{A_2 E_2}$$

$$P_1 = \frac{A_1 E_1 P}{A_1 E_1 + A_2 E_2} \qquad P_2 = \frac{A_2 E_2 P}{A_1 E_1 + A_2 E_2}$$

(1) Principle of superposition is applicable only when stress is within proportional limit

(2) If temperature is increased and member is restrained, then force produced is compressive. If temperature is decreased the force produced is tensile.

(3) Temp ↑→more value of α → compression
Temp ↓→ more value of α → Tension

Nut and Bolt problem:

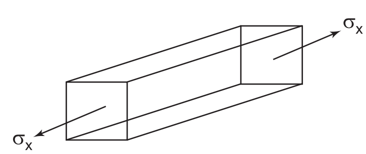
Extension of Bolt + Contraction of Tube = Movement of nut

$$\boxed{\frac{\sigma_b L}{E_b} + \frac{\sigma_T L}{E_c} = np}$$

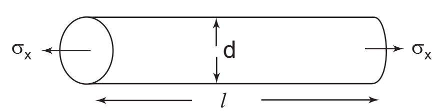
n = no. of rotations of bolt

p = pitch of thread.

Poisson's Ratio:- For **Homogenous** and **isotropic** material, Elongation (or contraction) produced by any Axial Force in the direction of force is accompanied by contraction (or elongation) in all transverse directions and all such contractions (or elongations) are same.



$$\boxed{\mu = -\left(\frac{\text{Lateral Strain}}{\text{Axial Strain}}\right)} = \frac{-\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_Z}{\varepsilon_x}$$



$$\boxed{\mu = \frac{-\Delta d / d}{\Delta l / l}}$$

$\mu = 0$ for cork
$\mu = 0.1 - 0.2$ for concrete
$\mu = 0.5$ Perfectly elastic rubber

Volume of rod remains unchanged as a result of combined effect of elongation and transverse condition.

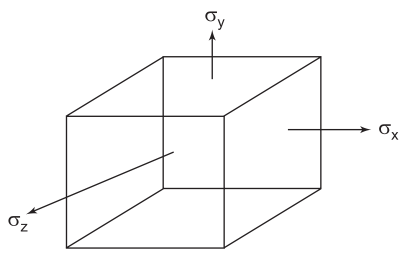
Dilation, Bulk modulas:-

$$\varepsilon_v = \frac{(\sigma_x + \sigma_y + \sigma_z)}{E}(1 - 2\mu)$$

If $\sigma_x = \sigma_y = \sigma_z = p$ than

$$\varepsilon_v = \frac{3p}{E}(1 - 2\mu)$$

$$K = \frac{p}{\varepsilon_v} = \frac{E}{3(1 - 2\mu)}$$



$$\text{Hydrostatic Pressure} = \frac{\sigma_x + \sigma_y + \sigma_z}{3}$$

(1) Stretching of material in one direction i.e due to □x will lead to increase in volume

(2) During plastic deformation, volume of specimen remains constant.

Shearing Strain:-

(1) Hooke's law for shearing stress and strain

$$\tau_{xy} = G\gamma_{xy}$$

(2) Modulas of rigidity or shear modulas G, $G = \frac{E}{2(1 + \mu)}$

as $0 < \mu < 0.5$ then $\frac{E}{3} < G < \frac{E}{2}$

(3) If only shearing stresses are acting then volume of the specimen does not change.

Relationship between Elastic Constants

$$G = \frac{E}{2(1 + \mu)} \qquad K = \frac{E}{3(1 - 2\mu)} \qquad E = \frac{9KG}{3K + G} \qquad \mu = \frac{3K - 2G}{6K + 2G}$$

No. of Independent Elastic Constants

(1) Homogenous and Isotropic $\rightarrow 2$

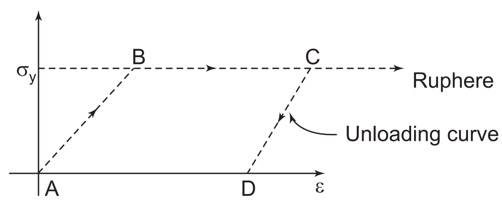
(2) Orthotropic (wood) $\rightarrow 9$

(3) Anisotropic $\rightarrow 21$

Saint-Venant Principle: Except in the immediate vicinity of application of loads, the stress distribution may be assumed independent of the actual mode of application of loads.

Plastic deformation:-

When yield stress of material is exceeded, plastic flow occurs.



Idealised curve for elasto plastic material

Residual stress:-When some part of an indeterminate structure undergoes plastic deformation, or different part undergoes different plastic deformation the stress in various parts of the structure will not return to zero after the load has been removed. These stresses are called Residual stresses

Thermal Stress and Strain:-

$$\sigma = E\alpha\Delta T$$

$$\Delta = L\alpha\Delta t$$

$$\text{Strain} = \frac{L\alpha\Delta t}{L} = \alpha\Delta t$$

$$\alpha_{Aluminum} > \alpha_{Brass} > \alpha_{Copper} > \alpha_{Steel}$$

$$\boxed{\text{TRICK} \quad A > B > C > S}$$

When bar is not restrained, then there will be no induced temperature stresses due to change in temperature.

2 Shear Force and Bending Moment

Span of a beam:-

(i) The clear horizontal distance between the supports is called clear span of the beam.

(ii) The horizontal distance between the centres of the end bearings is called the effective span of the beam.

Types of Support:-

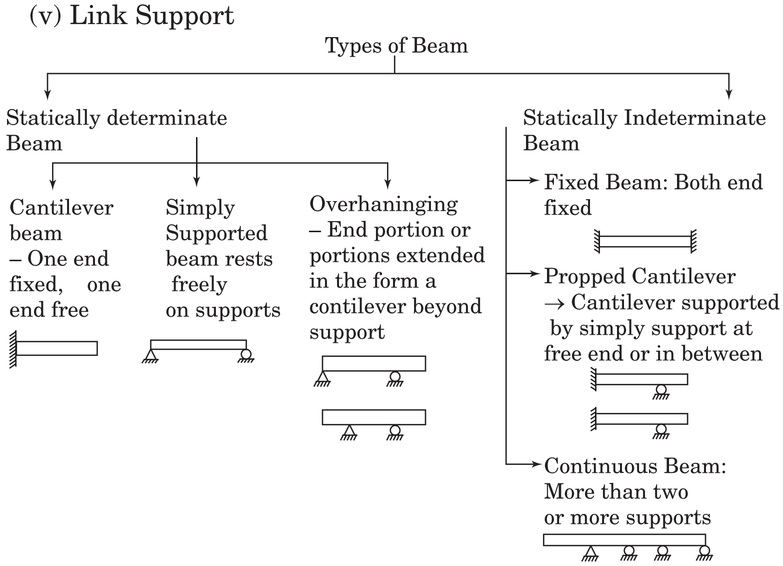
(i) A Simple or free support/Roller Support/Rocker support

(ii) Hinged or pinned support.

(iii) A built in or fixed or encastre support

(iv) Slider support

(v) Link Support



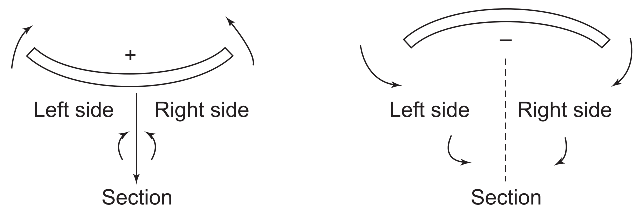
Note:- A continuous beam may or may not be an overhanging beam.

Shear force:- It is the resultant of all transverse forces to the right or left of the section.

S.F at a Section is +ve if the resultant of all transverse forces to the right of the section is downward or resultant of transverse forces to the left of section is upwards.

Positive shear force   Negative shear force

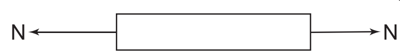
Bending moment:- It is the resultant moment at a section due to all the transverse forces either to the left or right of the section.



Positive Bending moment = Sagging   Negative Bending moment = Hogging

Note:- Bending moment is the algebraic sum of moments at that section while moment at a point is the summation of moment due to all loading on the beam produced at that point.

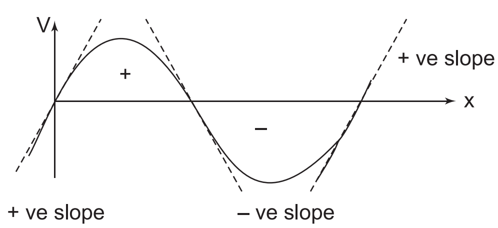
Axial Thrust → Force acting along the longitudinal axis of the members. Axial thrust is +ve if it tries to elongate the members.



Relationship between Bending moment, Shear force and Loading

(i) Slope of the shear force diagram = Intensity of distributed load

$$\frac{dV}{dx} = W_x$$



If the slope of SFD is positive, this implies the load intensity at that point is +ve i.e upwards and if the slope of SFD is negative, this implies the load intensity at that point is –ve ie downwards.

(ii) Slope of Bending moment diagram = Shear force at that section.

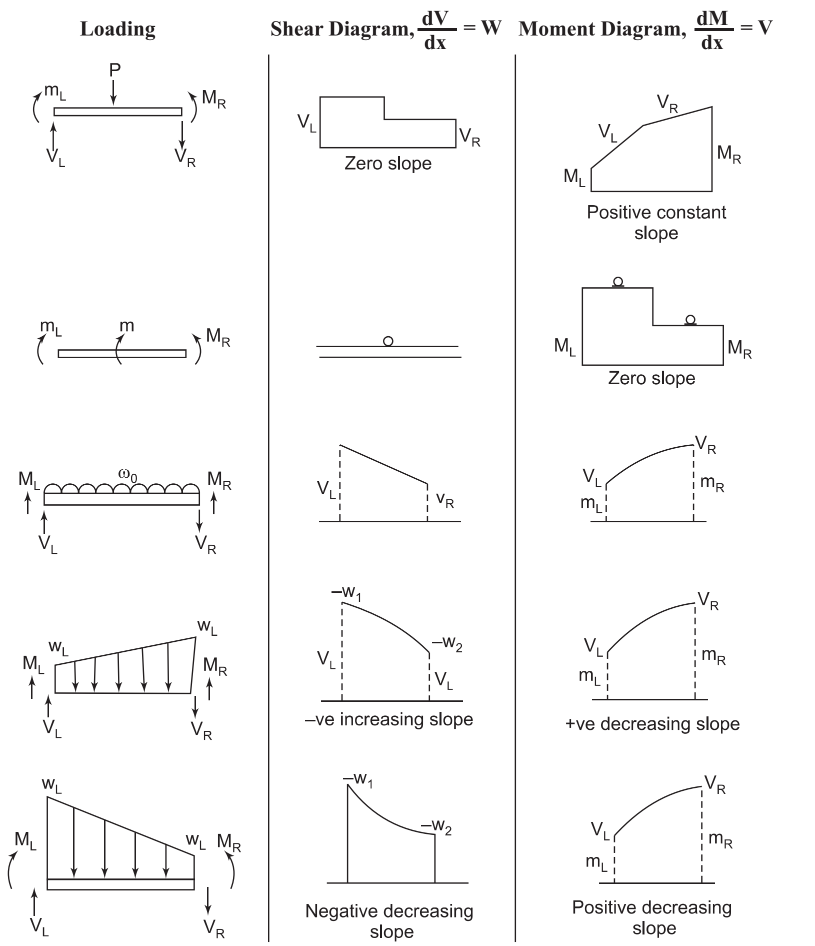
$$\boxed{\frac{dM}{dx} = V}$$

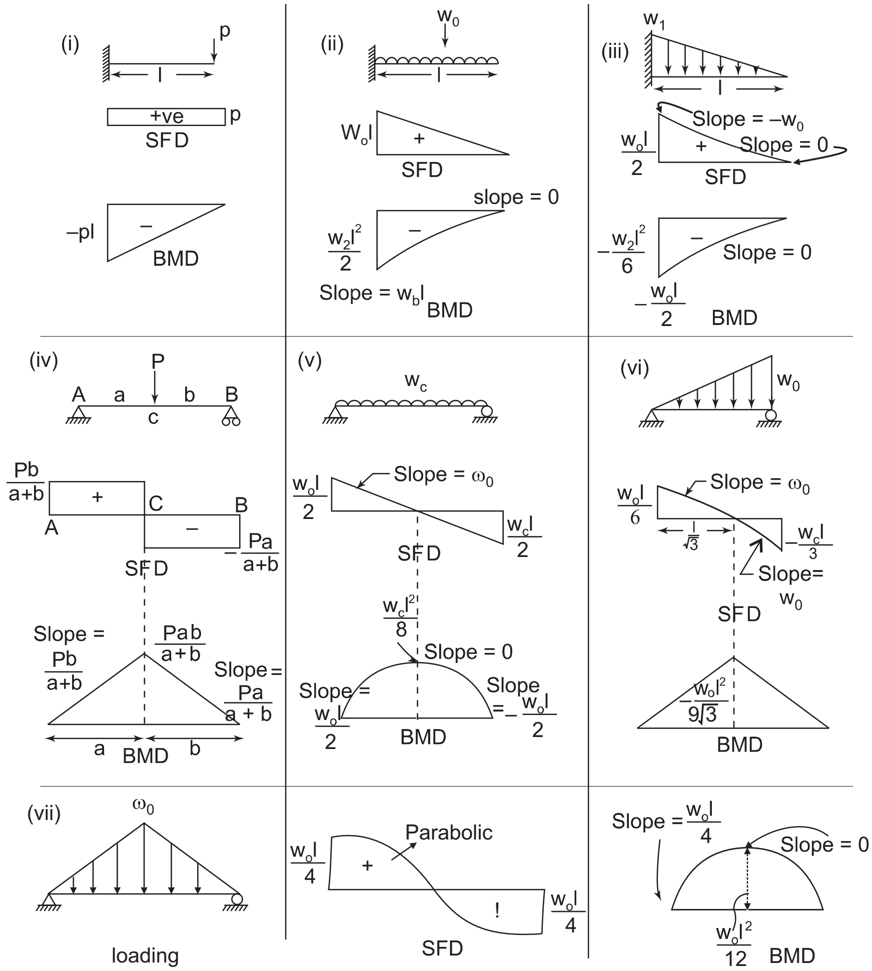
NOTE:-

$$\Delta V = \int W_x dx$$

$$\Delta M = \int V dx$$

$M_{final} - M_{initial}$ = Area under the Shear force diagram between those two sections.





(i)
p
l
+ve
p
SFD
–pl
–
BMD
(ii)
w0
l
Wol
+
SFD
slope = 0
w2l2
2
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Slope = wbl
BMD
(iii)
w1
l
Slope = –w0
wol
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Slope = 0
SFD
w2l2
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Slope = 0
wol
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BMD
(iv)
P
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Pa
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(v)
wc
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wcl2
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Slope = 0
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BMD
(vi)
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SFD
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BMD
(vii)
ω0
loading
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4
Parabolic
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wol
4
SFD
Slope =
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4
Slope = 0
wol2
12
BMD

3 Principal Stress and Principal Strain

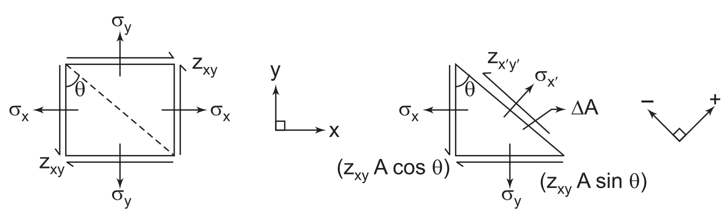
Whenever any structural component is under equilibrium due to external forces then each and every point inside the volume of the structural component must be in equilibrium and must have stress less than the permissible stress.

Plane stress:- When two faces of cubic elements are free of any stress, the stress condition is termed as plane stress condition

$$\boxed{\sigma_z = \tau_{zx} = \tau_{zy} = 0}$$

So plane stress components are and σ_x, σ_y and τ_{xy}

Transformation of Plane Stress



$$\sigma'_x = = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + Z_{xy}\sin 2\theta$$

$$Z_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + Z_{xy}\cos 2\theta$$

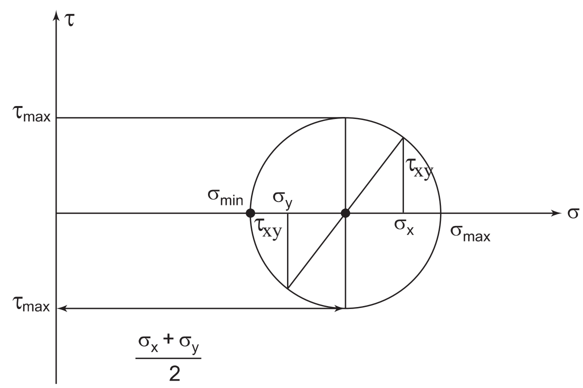
$$\boxed{\sigma_x + \sigma_y = \sigma_x{}' + \sigma_y{}'}$$

NOTE:- The sum of normal stresses exerted on a cubic element of a material is independent of the orientation of element.

Principal Stress and maximum shear stress:- It is the maximum or minimum normal stress which may be developed on a loaded body. The plane of principal stress does not carry any shear stress.

$$\sigma_{max/min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



Mohr's Circle for plane stress → It is the locus of points representing the magnitude of normal and shear stress at various plane in a given stress element.

1. $\sigma_{max/min}$ = Principal stresses, end points of diameter on σ–axis
2. τ_{max} = max shear stress, whose magnitude is equal to radius of mohr's circle
3. $\tan 2\theta_s = \frac{-(\sigma_x + \sigma_y)}{2\tau_{xy}}$ $\quad\quad$ $\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

   So $\quad$ $\tan 2\theta_s \times \tan 2\theta_p = -1$

   ⇒ $\quad$ $2\theta_s$ and $2\theta_p$ are 90° apart.

   Hence, plane of Max. Shear stress are 45° to the principal planes (i.e. θ_s and θ_p are 45° apart)

4. Normal stress on a plane of maximum shear stress is represented by co-ordinates of centre of Mohr's circle.
5. In hydrostatic loading → Mohr circle reduces to a point.
6. In Pure Shear case → centre of mohr circle will fall at origion.

Strain Energy per unit Volume:-

1. **Plane Stress Condition:-**

$$U = \frac{1}{2}\left(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + t_{xy}\gamma_{xy}\right)$$

When σ_1 and σ_2 principal stresses then

$$V = \frac{1}{2E}\left(\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2\right)$$

2. **Under Triaxial Stress Condition:**

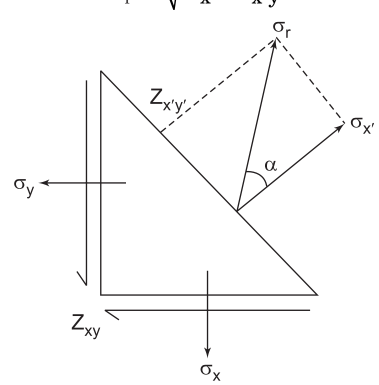
$$V = \frac{1}{2}\left(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{zx}\gamma_{zx}\right)$$

$$V = \frac{1}{2E}\left(\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - 2\mu\left(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x\right)\right)$$

Angle of obliquity:- Angle that line of action of resultant stress on a plane makes with the normal to the plane is called angle of obliquity.

α = Angle of obliquity

$$\sigma_r = \sqrt{\sigma_x^{1^2} + \tau_{x'y'}^2}$$



Plane Strain:- If the deformations are those in x-y plane only then only 3-strain components exist ε_x, ε_y, γ_{xy}

NOTE:- Strain energy only leads to distortion of element. It does not lead to change in volume. Normal stresses on the other hand leads to change in volume.

Transformation of Plane Strain

$$\varepsilon_{x^1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon_{y^1} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon_{x1} + \varepsilon_{y1} = \varepsilon_x + \varepsilon_y$$

$$\frac{\gamma_{x^1y^1}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

Comparison of Plane Stress and Plane Strain

	Plane Stress	Plain Strain
Stress	$\sigma_z = 0$ $\tau_{xz} = 0$ $\tau_{yz} = 0$ σ_x, σ_y, and $\tau_{xy} \rightarrow$ non zero	$\tau_{xz} = 0$, $\tau_{yz} = 0$ σ_x, σ_y, σ_z, $\tau_{xy} \rightarrow$ Non-zero
Strain	$\gamma_{xz} = 0$ $\gamma_{yz} = 0$ ε_x, ε_y, ε_z, ε_{xy} ε Non zero	$\varepsilon_z = 0$ $\gamma_{xz} = 0$ $\gamma_{yz} = 0$ ε_x, ε_y, ε_{xy} ε Non-zero

Mohr circle for Plane Strain:-

1. Principal Strains

$$\varepsilon_{max/min} = \frac{\varepsilon_x + \varepsilon_y}{2} \sqrt{\frac{(\varepsilon_x - \varepsilon_y)^2}{2} + \left(\frac{\tau_{xy}}{2}\right)^2}$$

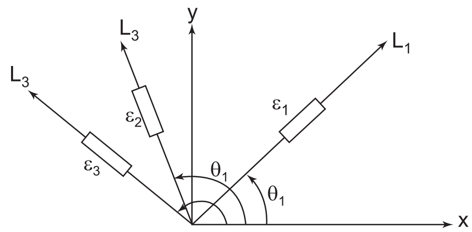
2. Maximum In plane shearing strain = Radius of Mohr's circle (R)

$$R = \left(\frac{\gamma_{max}}{2}\right)_{\text{in plane}} = \sqrt{\frac{(\varepsilon_x - \varepsilon_y)^2}{2} + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

Strain Rosette $\rightarrow$ A group of three gauges arranged in a particular pattern such that it can measure **normal strain** in three different directions on the surface of a structural element.

$$\varepsilon_1 = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta_1 + \frac{\gamma_{xy}}{2}\sin 2\theta_1$$

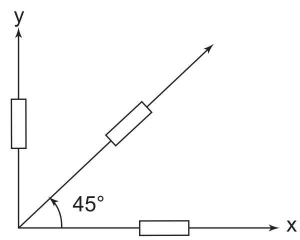
$$\varepsilon_2 = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta_2 + \frac{\gamma_{xy}}{2}\sin 2\theta_2$$



Special Case:- when $\varepsilon_1 \rightarrow$ along x-axis ie ε_x

$\theta_2 \rightarrow 45^\circ$

$\varepsilon_3 \rightarrow$ along y-axis i.e. ε_y



then, $$\varepsilon_2 = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\gamma_{xy}}{2}$$

or $$\gamma_{xy} = \varepsilon_2 - \left(\varepsilon_x + \varepsilon_y\right)$$

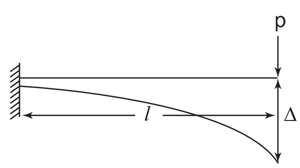
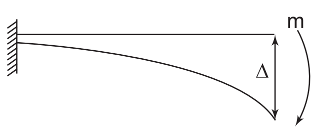
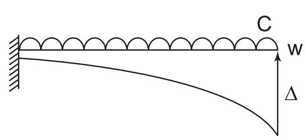
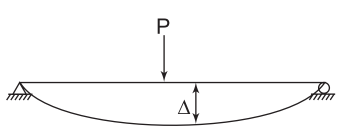
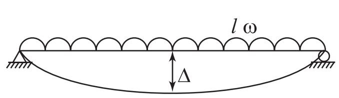
4 Deflection of Beams

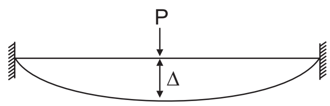
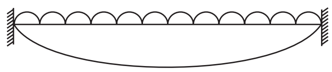
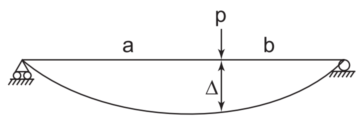
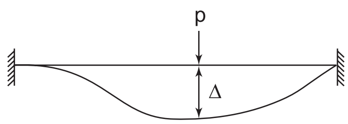
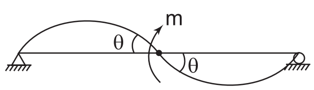
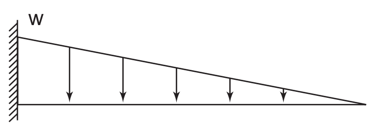
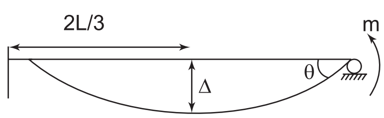
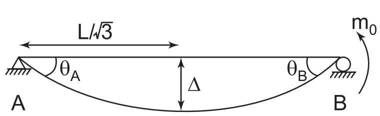
Deflection of structure is caused by its internal loadings such as Normal force, Shear force, Bending Moment, Torsion.

For **Beams** and **Frames**, major deflection is due to **Bending**

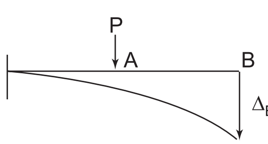
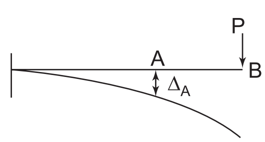
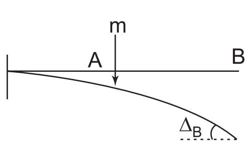
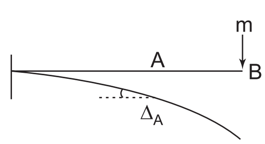
For **Trusses**, deflection is caused by internal Axial Forces

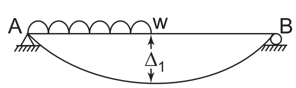
Some standard results of deflection and slopes.

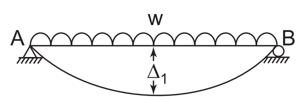
Loading	Deflection	Slopes
	$\Delta = \frac{Pl^3}{3EI}$	$\theta = \frac{Pl^2}{2EI}$
	$\Delta = \frac{ML^2}{2EI}$	$\theta = \frac{ML}{EI}$
	$\Delta = \frac{wL^4}{8EI}$	$\theta = \frac{wL^3}{6EI}$
	$\Delta = \frac{PL^3}{48EI}$	$\theta = \frac{PL^2}{16EI}$
	$\Delta = \frac{5}{384}\frac{wL^4}{EI}$	$\theta = \frac{wL^3}{24EI}$

	$\Delta=\frac{1}{4}\left(\frac{PL^3}{48EI}\right)$	
	$\Delta=\frac{1}{5}\left(\frac{5}{384}\frac{wL^4}{EI}\right)$	
	$\Delta=\frac{Pa^2b^2}{3EIL}$	
	$\Delta=\frac{Pa^3b^3}{3EIL^3}$	
		$\theta=\frac{ML}{24EI}$
	$\Delta=\frac{wL^4}{30EI}$	$\theta=\frac{wL^3}{24EI}$
	$\Delta=\frac{ML^2}{27EI}$	$\theta=\frac{ML}{4EI}$
	$\Delta=\frac{ML^2}{9\sqrt{3}EI}$	$\theta_A=\frac{ML}{6EI}$ $\theta_B=\frac{ML}{3EI}$

Maxwell's Reciprocal Theorem:- In any beam, frame or truss, the deflection at any point due to load P at any point A is equal to deflection at any point A due to load P at any point B.

		$\Delta_A = \Delta_B$
		$\theta_A = \theta_B$





$\Delta_2 = 2\,\Delta_1$
C→ mind
point of AB

Methods of determining slope and deflection at a point

(1) **Double Integration Method** ⇒ Gives deflection only due to bending

$$\boxed{\frac{d^2y}{dx^2} = \frac{M}{EI}} \Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{M}{EI} \Rightarrow \boxed{\frac{d\grave{e}}{dx} = \frac{M}{EI}}$$

where, EI = Flexural rigidity

$$\text{Flexural Stiffness} = \frac{\text{Flexural rigidity}}{\text{Length}}$$

$$\frac{d^3y}{dx^3} = \frac{dM/dx}{EI} \Rightarrow \boxed{\frac{d^3y}{dx^3} = \frac{V}{EI}} \Rightarrow \frac{d^4y}{dx^4} = \frac{dV/dx}{EI} \Rightarrow \boxed{\frac{d^4y}{dx^4} = \frac{W}{EI}}$$

(2) **Macaulay's Method** ⇒ Modification is done in loading pattern so that udl or uvl becomes continuous up to last segment.

$$EI\frac{d^2y}{dx^2} = M = \frac{-P}{2}x + \frac{3P}{2}(x - 2a)$$

Integrate and put boundary conditions

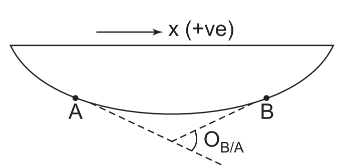
(a) $x = 0, y = 0$ (b) $x = 2a, y = 0$

Then, Deflection at $x = 3a$, $y = \frac{-Pa^3}{Ei}$

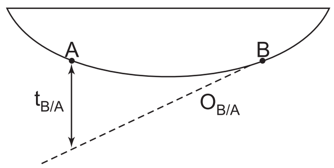
(3) Moment Area Method – Mohr's Method

Theorem 1:- The change in slope between two points on elastic curve equals the area of $\frac{M}{EI}$ diagram between these two points.

$\theta_{B/A}$ = Slope of B with respect to tangent drawn on elastic curve at A = Area of $\frac{M}{EI}$ diagram between A and B.



Theorem 2:→ Deflection of any point A on elastic curve with respect to tangent drawn at another point B (tAB) equals the moment of area under diagram between A and B about point A.



$$t_{A/B} = \bar{x}\int_A^B \frac{M}{EI}dx$$

(4) **Moment diagram by parts:-** The resultant BM at any section is the algebraic sum of bending moments at that section caused by each loading separately (either from left or right of that section). So the effect of individual load can be considered instead of taking effects of all the loads together for drawing BMD.

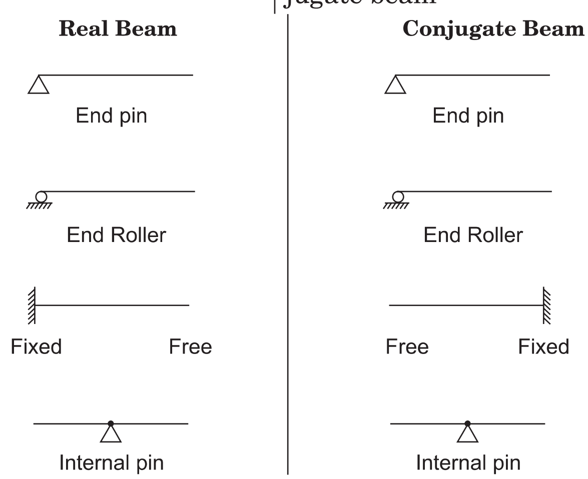
(5) **Conjugate beam method:-**

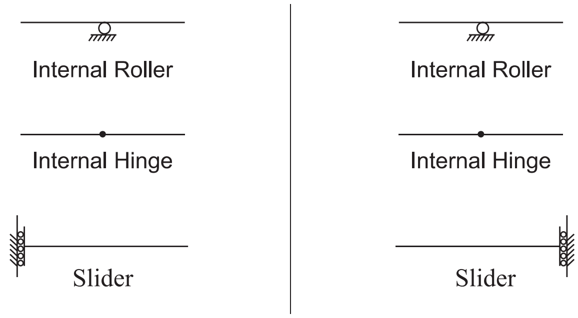
$$V = \int w\,dx$$
↓ ↑
$$\theta = \int \frac{M}{EI}\,dx$$

Slope at any point in real beam = Shear at that point in conjugate beam

$$M = \int\left(\int w\,dx\right)dx$$
↓ ↑
$$y = \int\left(\int \frac{M}{EI}dx\right)dx$$

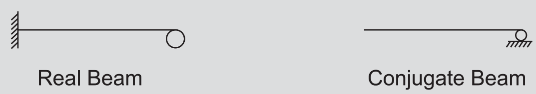
Deflection at any point in real beam = BM at that point in conjugate beam





NOTE:- (1) Area moment theorem requires understanding of geometry of deflected shape and applicable only when deflected shape is continuous, while in conjugate beam method, principle of statics is used. Hence, this method can also be used when deflected shape is not continuous i.e, Internal Hinge case.

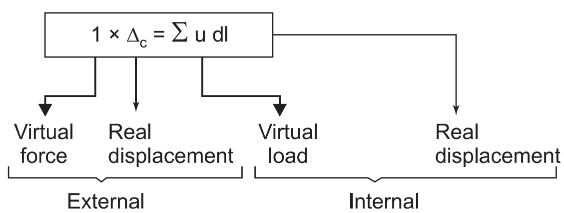
(2)



A **statically intermediate** real beam can have **unstable** conjugate beam.

(6) **Method of virtual work (Unit load method)**

External virtual work = Internal virtual work



So,

$$1 \times \Delta = \int_0^L \frac{m\ M\ dx}{EI}$$

$$1 \times \theta = \int_0^L \frac{m_\theta\ M\ dx}{EI}$$

Note:- Unit load method can be applied to plastic range of stress-strain also, but $d\theta$ will not be equal to $\frac{M}{EI}dx$

(7) **Castigliano's theorem (Method of least work – its 2nd theorem)**

$$\Delta = \frac{\partial U}{\partial P} \qquad \theta = \frac{\partial U}{\partial M}$$

For beam's and frames $U = \frac{M^2\,dx}{2EI}$

So,

$$\Delta = \int_0^L \frac{M\frac{\partial M}{\partial P}\,dx}{EI} \qquad \theta = \int_0^L \frac{M\frac{\partial M}{\partial m}\,dx}{EI}$$

Note:- This theorem is applicable only when there is constant temperature, unyielding support and linear elastic material response.

5

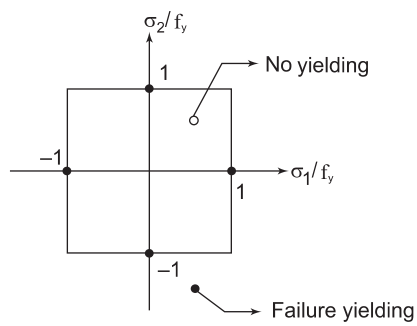
Theories of Failure

Under uniaxial tension or compression practically, yielding begins at the yield strength at which plastic deformation is significant. But when several components exist, the yielding depends on some combination of these components. The theory of failure is used to establish, the behaviour of material subjected to **simple tension** or **compression**, the point at which **failure** will occur under any type of combined loading. These theories are applicable to **static loading** only.

(1) **Maximum principal stress theory (Rankine's theory, Lame's theory are max stress theory)**

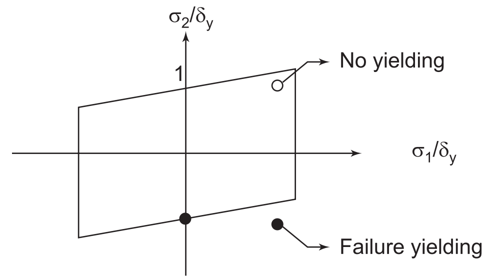
$$\theta \leq \frac{fy}{FOS}$$

Applicable for brittle material



(2) **Max principal strain theory (St. venant theory)**

$$\sigma_1 - \mu(\sigma_2 + \sigma_3) \leq \frac{fy}{FOS}$$

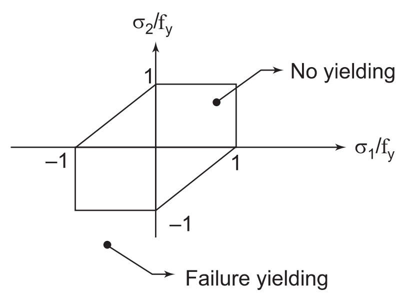


Satisfactorily applicable to brittle material, but over estimates the strength of ductile material. Even not suitable for pure shear case.

(3) **Max shear stress theory (Tresca, Guest, coulomb theory)**

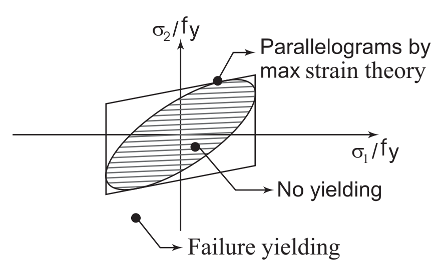
$$\text{Max of}\left[\left|\frac{\sigma_{max}-\sigma_{min}}{2}\right|, \left|\frac{\sigma_{max}}{2}\right|, \left|\frac{\sigma_{min}}{2}\right|\right] \leq \frac{fy}{2(FOS)}$$

Applicable for ductile material and gives the most conservative design out of various other theories of failure



(4) **Maximum strain energy theory (Beltrami - Haigh theory)**

$$\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu\left(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1\right)\right] \leq \left(\frac{fy}{FOS}\right)^2$$

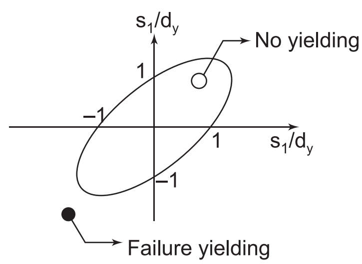


Applicable for ductile material and not suitable for brittle material or pure shear case.

(5) **Max shear strain energy theory (Distortion energy theory) – Huber-Hencky von mises theory**

$$\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]\le\left(\frac{fy}{FOS}\right)^2$$

Applicable in pure shear case



(6) **Octahedral shear stress theory**

$$\sigma_1^2+\sigma_2^2+\sigma_1\sigma_2\left(\frac{fy}{FOS}\right)^2$$

Applicable to ductile material in pure shear case

NOTE:

Total strain energy = volumetric strain energy + Distortion energy.

Volumetric strain energy = $\frac{1}{2}\times$ volumetric stress $\times$ volumetric strain

Volumetric strain energy $=\frac{1}{2}\left(\frac{\sigma_1+\sigma_2+\sigma_3}{3}\right)\left(\frac{(\sigma_1+\sigma_2+\sigma_3)(1-2\mu)}{E}\right)$

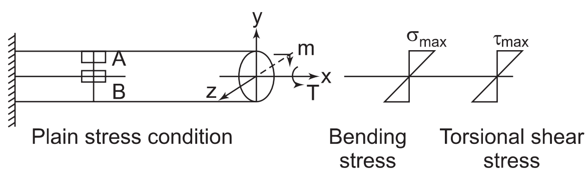
Total strain energy $=\frac{1}{2E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right]$ ..(1)

Hence, (2)-(1) =

$$\text{Distortion Energy}=\frac{1}{12G}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]$$

6 Combined Stress

Combined Bending and Torsion



For point A:-

$$\tau_{max} = \frac{16T}{\pi D^3} \qquad \sigma_{max} = \frac{32M}{\pi D^3}$$

For point B:-

$$\tau_{xy} = \frac{-16T}{\pi D^3}$$

Principal Stresses at A:-

$$\sigma_{max/min} = \frac{16T}{\pi D^3}\left[M \pm \sqrt{M^2 + T^2}\right]$$

$$\tau_{max} = \frac{16T}{\pi D^3}\left(\sqrt{M^2 + T^2}\right)$$

Principal Stresses at B:- $\boxed{\sigma_{max/min} = \pm\frac{16T}{\pi D^3}}$

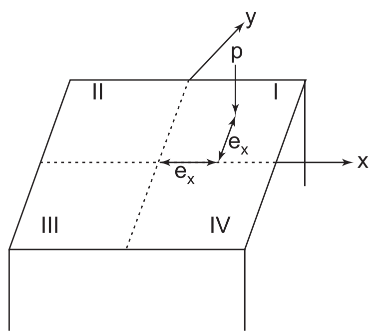
Equivalent Moment $\boxed{M_e = \frac{1}{2}\left(M \pm \sqrt{M^2 + T^2}\right)}$

Equivalent Torque $\boxed{T_e = \sqrt{M^2 + T^2}}$

Combined Bending and Axial Force

$$\sigma = \frac{-P}{A} - \frac{(Pe_x)x}{I_y} - \frac{(Pe_y)y}{I_x}$$

(–ve means compressive)

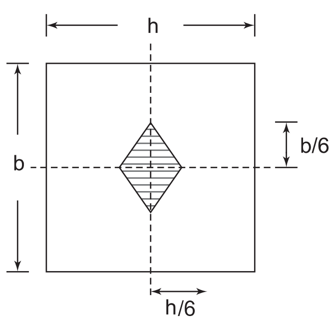


Equation of Neutral axis (put $\sigma = 0$)

$$\left(\frac{e_x}{r_y^2}\right)x + \left(\frac{e_y}{r_x^2}\right)y = 1$$

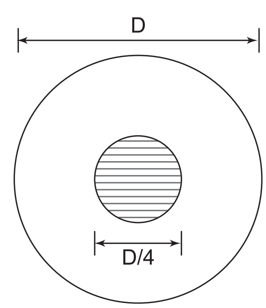
Kern:- It is the area of the x-section on which if compressive loading occurs then there will be no tension anywhere on the entire x-section (when bending occurs due to axial force)

Kern for Rectangular Section



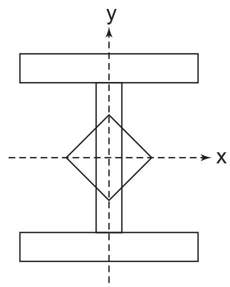
Rhombus shape

Kern for Circular Section



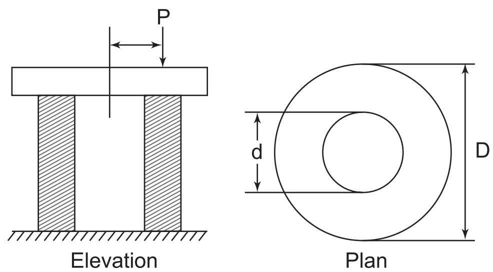
Circular shape of diameter=D/4

Kern for I-section



Rhombus shape

Kern for hallow circular section



$$\text{Dia of Kern} = \frac{D^2 + d^2}{4D}$$

7 Bending Stress in Beam

Symmetric Bending:- When member possess a **Plane of symmetry** and **loading acts in the plane of symmetry** then bending is called symmetric sending.

Unsymmetric Bending:- When bending couple does not acts in the plane of symmetry of member either because they act in different plane or because the member does not possesses a plane of symmetry

Pure Bending:- Bending of beam under constant Bending moment.

Non-uniform Bending:- Bending in presence of **Shear force**.

Flexure Formula: $\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$

where I = MOI about C.G axis about which bending occurs

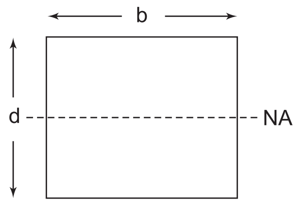
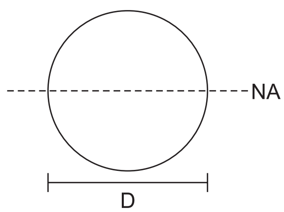
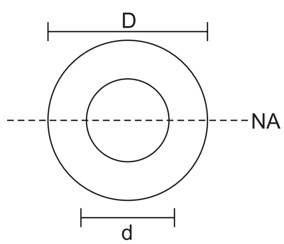
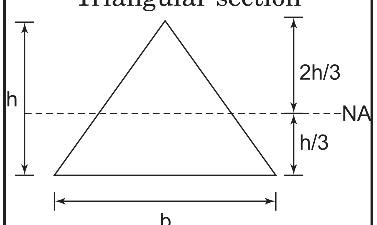
Here, $\boxed{\sigma_{max} = \frac{My}{I}}$

Where, $\boxed{\frac{I}{y} = Z}$ = Section modulas about bending axis.

Moment of resistant (MOR) = $\sigma_{max}\left(\frac{I}{y}\right)$

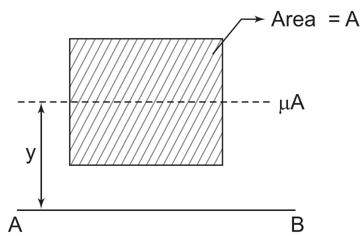
$$\boxed{MOR = \sigma_{max} \times Z}$$

Section modulas (Z)

	I about N.A	ymax	$Z = I/y_{max}$
Rectangular section 	$\frac{bd^3}{12}$	$\frac{d}{2}$	$\frac{bd^2}{6}$
Solid circular section 	$\frac{\pi D^4}{64}$	$\frac{D}{2}$	$\frac{\pi D^3}{32}$
Hollow circular section 	$\frac{\pi\left(D^4 - d^4\right)}{64}$	$\frac{D}{2}$	$\frac{\pi\left(D^4 - d^4\right)}{32D}$
Triangular section 	$\frac{bh^3}{36}$	$\frac{2h}{3}$	$\frac{bh^2}{24}$

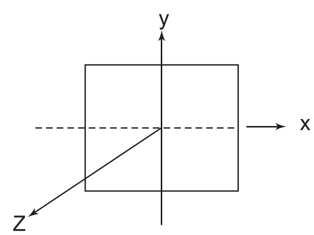
Parallel axis theorem

$\Rightarrow I_{AB} = I_{NA} + AY^2$

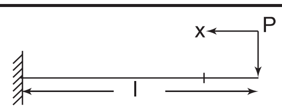
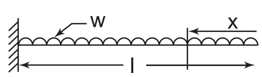
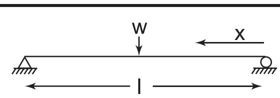


Perpendicular axis theorem

$I_{ZZ} = I_{XX} + I_{YY}$



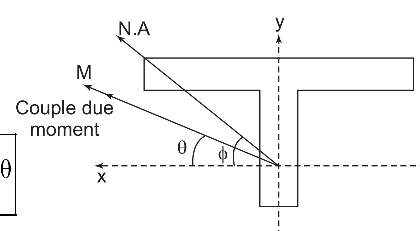
Beam of constant strength or fully stressed beam:- It is the beam in which max. stress at every X-Section of the beam is equal to the maximum allowable bending stress in the beam.

Rectangular beam loading	Max. bending stress	b= constant	d= constant
	$\frac{6Px}{b_x d_x^2}$	$d_x \alpha \sqrt{x}$	$b_x \alpha x$
	$\frac{3\omega x^2}{b_x d_x^2}$	$d_x \alpha x$	$b_x \alpha x$
	$\frac{3\omega x}{b_x d_x^2}$	$d_x \alpha \sqrt{x}$	$d_x \alpha x$

Equation of Neutral axis in Unsymmetrical bending (No Twisting case)

$$y = \left(\frac{I_x}{I_y}\tan\theta\right)x$$

Slope of N.A = $\tan\phi = \frac{I_x}{I_y}\tan\theta$



NOTE:- Neutral axis is always located between the couple vector and the minor principal axis.

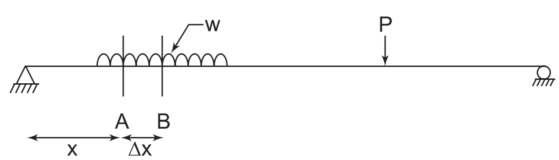
8

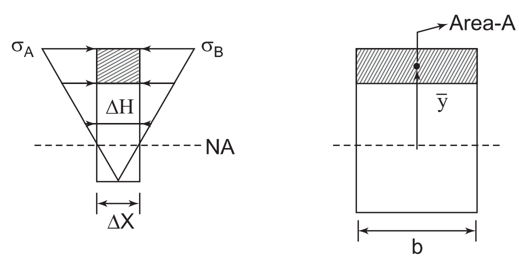
Shear Stress in Beams

Normal stress is produced by bending and shear stress is produced by shear force. The Vertical shearing stress is accompanied by a horizontal shearing stress of equal magnitude known as **complimentary shear stress**.

$$\Delta H = \int_A (\sigma_A - \sigma_B)\, dA$$

$$\boxed{\Delta H = \frac{VA}{I} \bar{y}\, \Delta x}$$





Where,

ΔH = Shear force in length Δx of beam

V = S.F at the section where shear stress is to be found.

A = Moment of area of section above the level at which shear stress is to be found out.

I = Moment of inertia of complete section about N.A.

Shear **force** per unit length = $\frac{\Delta H}{\Delta x} = \frac{VA\bar{y}}{I}$

Shear **Stress** at the level y from N.A = $\boxed{q = \frac{VA\bar{y}}{Ib}}$

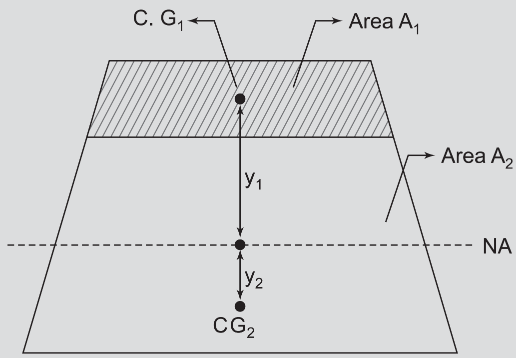
b = width of section at the level where shear stress is to be found.

Note:-

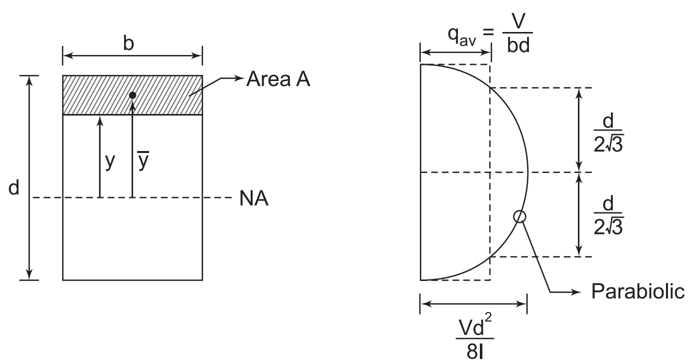
$$\boxed{A_1 y_1 = A_2 y_2}$$

A_1 = Shaded area

A_2 = unshaded area



Shear stress in rectangular section

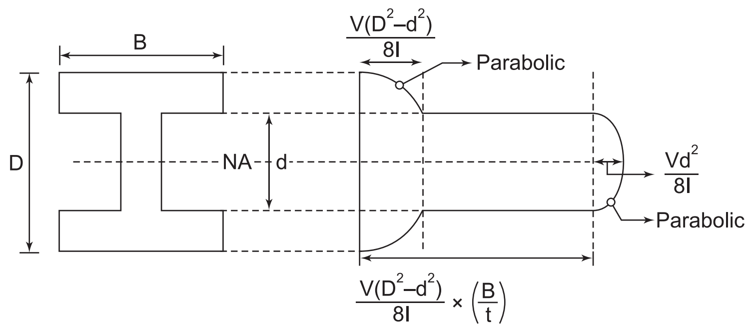


$$\text{Shear stress} = \frac{v}{2I}\left(\frac{d^2}{4} - y^2\right)$$

$$\tau_{max} = \frac{3}{2}\tau_{avg}$$

$$\tau_{avg} = \frac{V}{bd}$$

Shear stress in I – section

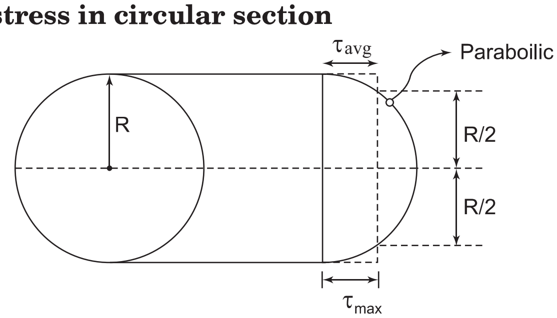


Shear stress in flange $\Rightarrow \frac{v}{2I}\left(\frac{D^2}{4} - y^2\right)$

Shear stress in flange $\Rightarrow \frac{V\left(D^2 - d^2\right)}{8I} \times \frac{B}{t} + \frac{v}{2I}\left(\frac{d^2}{4} - y^2\right)$

Note:- In I-section's nearly 80-85% shear is resisted by web.

Shear stress in circular section



Shear stress $= \tau = \frac{4}{3}\left(\frac{V}{A}\right)\left(1 - \frac{y^2}{R^2}\right)$

$$\tau_{max} = \frac{4}{3}\tau_{avg}$$

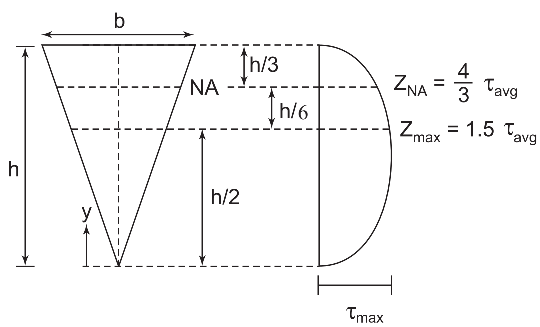
$$\tau_{avg} = \frac{V}{A}$$

Shear stress in triangular section

$$\text{Shear stress} = \frac{Vy(h-y)}{3I}$$

$$\tau_{max} = \frac{3}{2}\tau_{avg}$$

$$\tau_{NA} = \frac{4}{3}\tau_{avg}$$

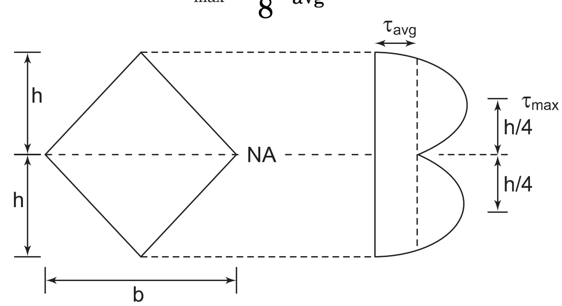


Distance b/w N.A and τ_{max} location $= \frac{h}{6}$

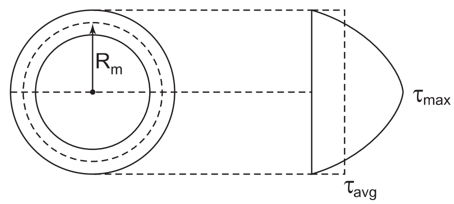
Shear stress in Quadrilateral section about diagonal

$$\text{Shear stress} = \tau = \frac{V}{bh^3}(h-y)(2y+h)$$

$$\tau_{max} = \frac{9}{8}\tau_{avg}$$



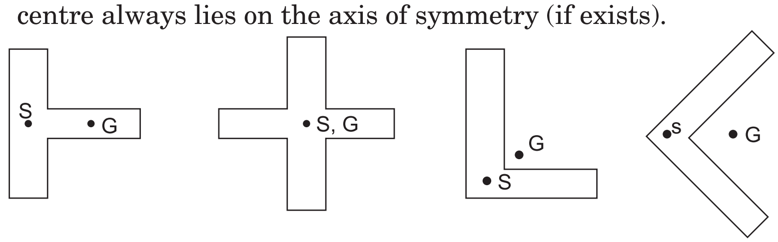
Shear stress of thin walled section



$$\tau_{max} = 2\,\tau_{avg}$$

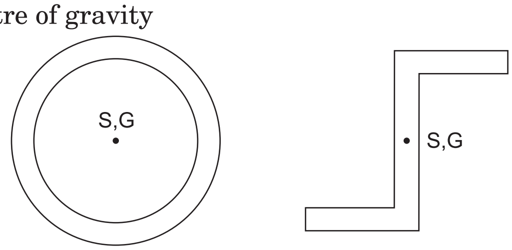
$$\tau_{avg} = \frac{V}{A}$$

Shear Centre:- It is the point through which if transverse bending load passes, the beam will have no twisting. It is the point through which resultant of shearing force on the section passes. Shear centre always lies on the axis of symmetry (if exists).



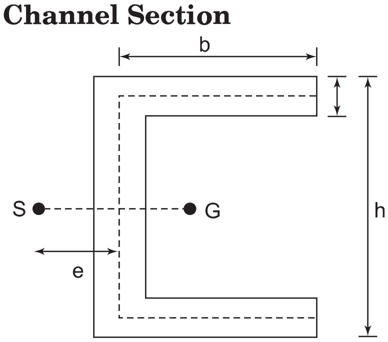
S = Shear Centre

G = Centre of gravity



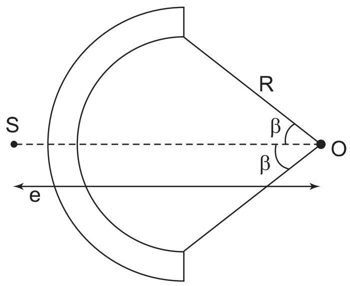
Distance of Shear Centre of Important sections:-

1. Channel Section



$$\beta = \frac{b^2h^2t}{4I}$$

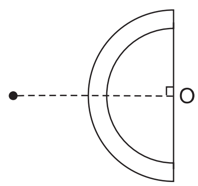
2. Circular arc



$$\boxed{\left(\frac{2R}{\beta - \sin\beta\cos\beta}\right)(\sin\beta - \beta\cos\beta)}$$

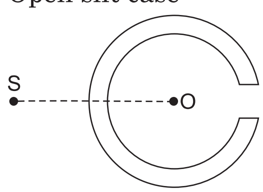
Special cases

(a) Semi-circular :



$\beta = 90°$ $\boxed{e = \frac{2R}{\pi/2}}$

(b) Open slit case

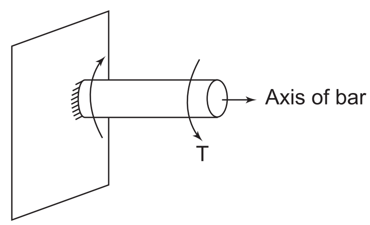


$\boxed{e = 2R}$

Torsion of Circular Shaft

9

Torsion means twisting of a member when it is loaded by torques that tend to produce rotation about the longitudinal axis of the bar



Pure torsion:- When its cross-sections are subjected to only torsional moments and not accompanied by axial forces or bending moment or transverse shear. For pure torsion, bar should be prismatic.

Assumptions in Torsion formula

1. Circular section remains circular
2. Plane section remains plane and do not warp. (warping occurs in Non-circular sections)
3. Stress do not exceed proportional limits.
4. Shaft is loaded by twisting couples in planes that are perpendicular to the axis of shaft.

Torsion formula

$$\boxed{\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}}$$

T = Torque

J = Polar moment of Inertia

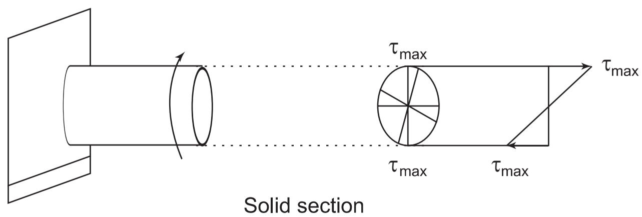
τ = Shear stress

r = distance from centre of shaft

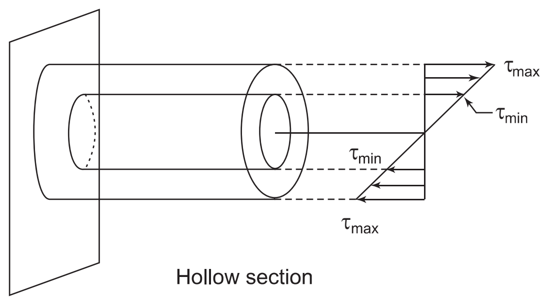
G = shear modulas

θ = Angle of twist

L = Length of shaft



Solid section



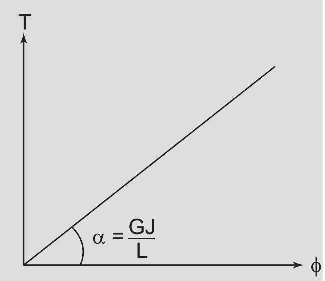
Hollow section

Note:-

T – φ plot is used to determine

The value of G using Torsion text

$$T = \left(\frac{GJ}{L}\right)\phi$$

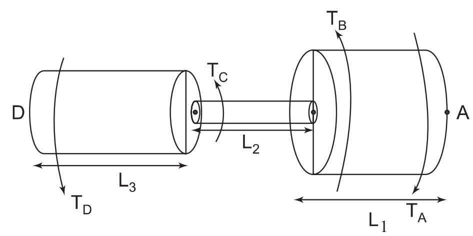


Get α from the graph and then by knowing J and L we can easily get the value of G.

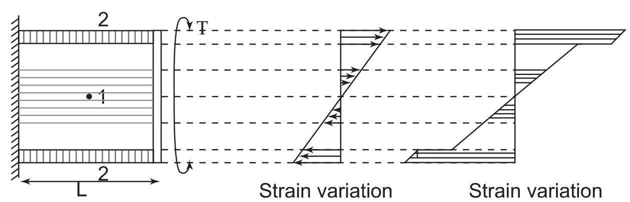
Compound shaft

1. Series connection

$$\phi_{AD} \Rightarrow \phi_A - \phi_D$$
$$\Rightarrow (\phi_A - \phi_B) + (\phi_B - \phi_C) + (\phi_C - \phi_D)$$
$$\boxed{\phi_{AD} \Rightarrow \phi_{AB} + \phi_{BC} + \phi_{CD}}$$



2. Parallel Connection



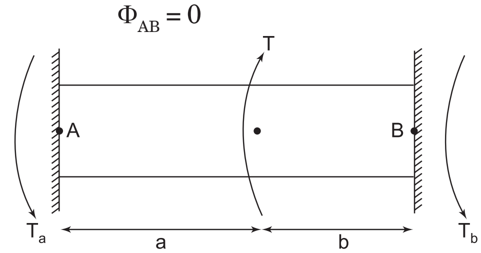
$$\boxed{\theta_1 = \theta_2} \Rightarrow \frac{T_1}{G_1}\frac{L}{J_1} = \frac{T_2}{G_2}\frac{L}{J_2}$$

$$\phi = \frac{TL}{G_1J_1+G_2J_2}$$
$$T_1 = \frac{G_1J_1}{G_1J_1+G_2J_2} \quad T_2 = \frac{G_2J_2}{G_1J_1+G_2J_2}$$

Torsion in fixed beam (statically Indeterminate)

$$\boxed{T_a = \frac{Tb}{a+b}} \quad \boxed{T_b = \frac{Ta}{a+b}}$$

$$\Phi_{AB} = 0$$



Torsional strain energy

$$U = \frac{1}{2}T.\theta = \frac{1}{2}T.\frac{TL}{GJ} = \frac{\tau_{max}^2}{4G} \times \text{Volume of shaft}$$

Strain energy density = $\dfrac{\text{Strain energy}}{\text{Volume of shaft}} = \dfrac{1}{2} \times \tau \times \gamma$

Power transmitted by shaft

$$\boxed{P = T \times \omega}$$

T = Torque

ω = Speed of rotation

N = Rotation per minute

Where $\omega = 2\pi f$

or $\omega = \dfrac{2\pi N}{60}$

Thin walled Hollow shaft

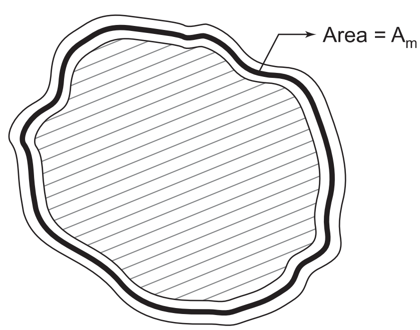
$$\boxed{\tau \times t = \frac{T}{2A_m}}$$

Where, τ = Shear stress

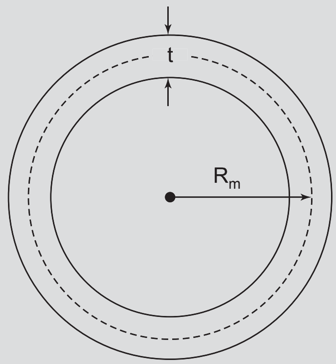
T = thickness of the section

T = Torque applied

A_m = Area under mean circle.

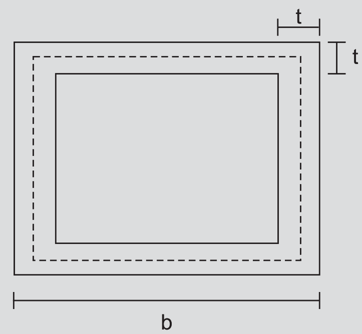


Note:-



$$\boxed{A_m = \pi R_m^2}$$

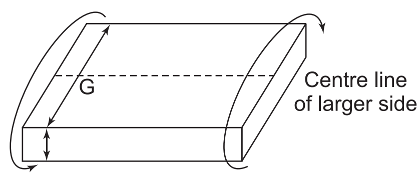
Hollow Thin circular shaft



$$\boxed{A_m = b^2}$$

Hollow thin circular shaft

Torsion of Non-circular members



$$\tau_{max} = \frac{T}{C_1 ab^2} \quad \text{Where } b < a$$

$$\phi = \frac{TL}{C_2 ab^3 G}$$

Shear stress at corners will be zero, Where as it will be maximum along the centre line of larger side.

10 Columns

Vertical members carrying vertical loading and moments is called column.

Failure of column's

- Long column — Buckling
- Intermediate column — Buckling and Crushing
- short column — Crushing

Assumptions of Euler's theory

1. Applicable to long columns only.(i.e Buckling failure only)
2. Material is isotropic, homogenous and linear elastic.
3. Purely axial loading.
4. Perfectly straight axis of column after unloading.

Euler's Formula

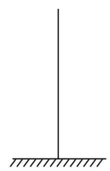
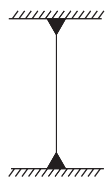
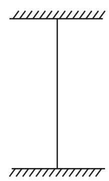
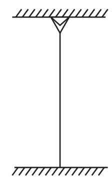
$$\boxed{P_e = \frac{\pi^2 EI_{min}}{L_{eff}^2}}$$

P_e = Buckling load

I_{min} = Minimum MOI about centroidal axis.

L_{eff} = Effective length of column.

Effective length of column based on end conditions

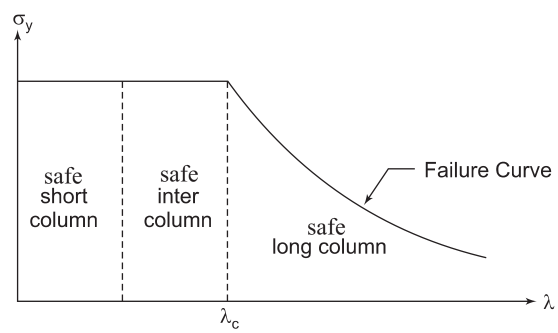
End condition	One end fixed one end free	Both end Hinged	Both end Fixed	One end fixed one end Hinged
L_{eff} (Theoretical)	2L 	L 	L/2 	$\frac{L}{\sqrt{2}}$ 
L_{eff} (As per IS code.	2L	L	0.65L	0.8L

Validating of Euler's theory

$$P_e = \frac{\pi^2 E A r_{min}^2}{L_{eff}^2}$$

$$P_{critical} = \frac{P_e}{A} = \frac{\pi^2 E}{\left(L_{eff}/r_{min}\right)^2}$$

$$\boxed{P_{cr} = \frac{\pi^2 E}{\lambda^2}}$$



Where, λ = Slenderness ratio, ie ratio of effective length to least radius of gyration.

For mild steel $P_{cr} = 250$ $E = 2 \times 10^5$ N/mm²

Hence $\boxed{\lambda_c = 88.89}$

ENTRI

Rankine's Formula:- For Both **Short** and **Long** column

$$\frac{1}{P_R} = \frac{1}{P_c} + \frac{1}{P_e}$$

P_R = Rankine load

P_c = Crushing load = $\sigma_c \times A$

P_e = Euler's load

$$P_R = \frac{P_c}{1 + \frac{P_c}{P_e}} = \frac{\sigma_c A}{1 + \frac{\sigma_c A}{\left(\frac{\pi^2 E}{\lambda^2}\right)}} = \boxed{\frac{\sigma_c A}{1 + \alpha\lambda^2} = P_R}$$

Where, A = Area of column

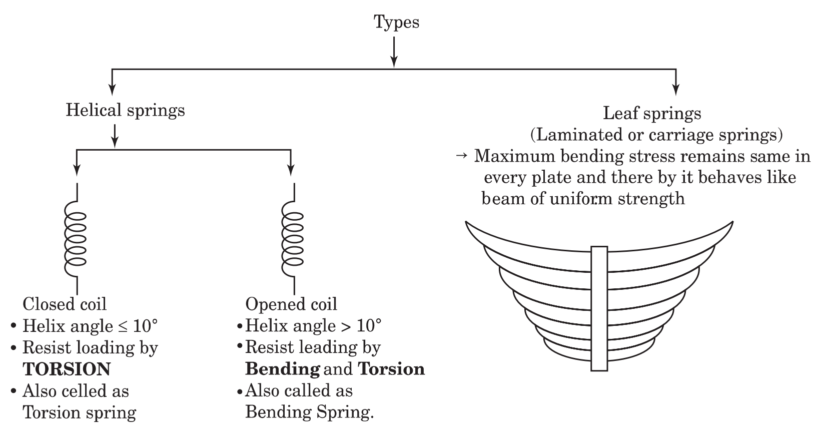
α = Rankines constant = $\frac{\sigma_c}{\pi^2 E}$

Note: For **long column** in which **eccentric loading** is applied, **secant formula** is used.

ENTRI

11 Springs

When load is applied on the spring it either gets deflected or distorted. and it recovers its original shape when load is released. During deflection or distortion it **absorbs** energy and **releases** the same as unloaded.



Proof load:- Greatest load that the spring can carry without getting permanently distorted.

Proof Stress:- Max stress in the spring when proof load is applied.

Proof Resilience:- Strain energy stored when proof load is applied.

Spring Constant:- It's the stiffness of the spring measured in load per unit deflection.

Closed coil helical spring

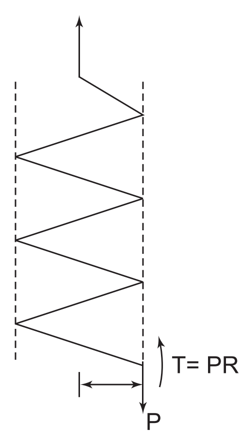
$$\text{Strain energy } U = \frac{1}{2}\frac{T^2L}{GJ}$$

$$T=PR \qquad L=2\eth Rn \qquad J=\frac{\eth d^4}{32}$$

Axial deflection (δ) : $\delta = \frac{\partial U}{\partial P} = \frac{64PR^3n}{Gd^4}$

Stiffness of Spring (K) $K = \frac{P}{\delta} = \frac{Gd^4}{64R^3n}$

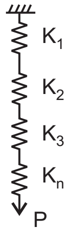
Proof load $P_{max} = \frac{\pi d^3}{16R} \times \sigma_{max}$



Note:- If the spring is cut into two halves, then the stiffness of each half will be doubled to that of original spring.

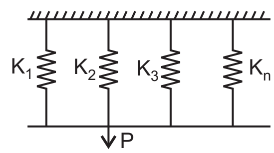
Equivalent Spring Constant

1. Series connection



$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + ... + \frac{1}{K_n}$$

2. Parallel connection



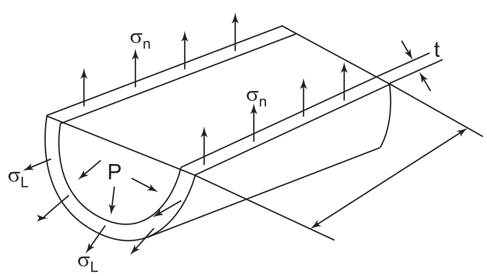
$K_{eqa} = K_1 + K_2 + K_3 + ... K_n$

12

Pressure Vessels

Thin shells are the shells in which thickness of the wall is less than $\frac{1}{10}$th or $\frac{1}{15}$th of its internal diameter. In thin shells normal stress is uniformly distributed throughout the thickness of the wall. While if thickness of wall is greater than $\frac{1}{10}$th or $\frac{1}{15}$th of internal diameter it's called Thick Shell.

Thin Cylinder subjected to internal pressure



1. Hoop Stress = $\sigma_h = \frac{pd}{2t}$
2. Longitudinal Stress = $\acute{o}_L = \frac{pd}{4t}$
3. Radial pressure = Inside = p
   outside = o
4. Hoop Strain $\varepsilon_h = \frac{pd}{4tE}(2 - \grave{\imath})$
5. Longitudinal Strain $\varepsilon_h = \frac{pd}{4tE}(1 - 2\mu)$

6. Volumetric Strain $\varepsilon_h = \varepsilon_1 + 2\varepsilon_h$

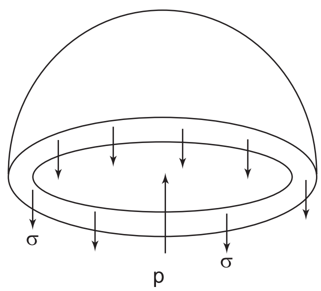
$$\varepsilon_v = \frac{pd}{4tE}(5 - 4\mu)$$

7. Maximum shear stress in plane of σ_h and σ_l

$$\tau_{max\,(in\,plane)} = \frac{pd}{8t}$$

8. Absolute maximum shear stress $\tau_{abs\,max} = \frac{pd}{4t} + \frac{p}{2}$

Thin sphere subjected to internal pressure



1. Hoop stress = Longitudinal stress

$$\sigma_h = \sigma_1 \; \frac{pd}{4t}$$

2. Hoop strain = Longitudinal strain

$$\sigma_h = \sigma_l \; \frac{pd}{4t\,E}(1 - \mu)$$

3. Volumetric strain $\varepsilon_v = 3\varepsilon_h = 3\varepsilon_l$

$$\varepsilon_v = \frac{3pd}{4t\,E}(1 - \mu)$$

4. Maximum shear stress in plane = 0
5. Absolute maximum shear stress $\tau_{abs\,max} = \frac{pd}{8t}$

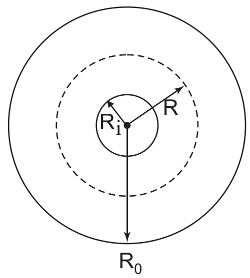
Lame's theorem:- Analysis of **thick shells**

Assumptions:-

1. Material is homogenous, isotropic and linear elastic.
2. Plane section of cylinder, perpendicular to longitudinal axis remains plane under pressure.

Lame's equation's for thick cylinder

1. Hoop stress → Tensile $\sigma_h = \dfrac{B}{R^2} + A$



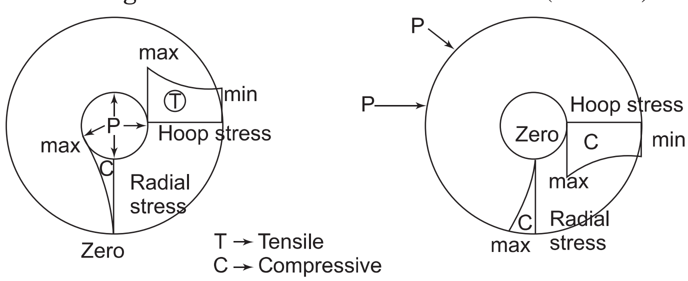
2. Longitudinal stress $\sigma_l = \dfrac{pR_i^2}{R_o^2 - R_i^2}$

3.Radial stress → compressive $\sigma_R = \dfrac{B}{R^2} - A$

Variation of stress in thick cylinder

Note:- 1. Radial and Hoop compression vary hyperbolically.

2. Longitudinal stress remains constant (Tensile)



Due to internal pressure Due to external pressure

Lame's equation for thick sphere

1. Hoop stress = Longitudinal stress

$$\sigma_n = \sigma_l = \frac{B}{R^3} + A \text{ (Tensile)}$$

2. Radial stress

$$\sigma_R = \frac{2B}{R^3} - A \quad \text{(Compressive)}$$

Note:- Both lame's constant A and B are positive for internal pressure and both are negative for external pressure.