ENTRI

Civil Engineering

4

RCC

CONTENTS

Introduction 1

Plain concrete: It is a mixture of sand, gravel, cement and water resulting into solid mass usually used in mass concreting (*i.e.*, dams).

Note: Tensile strength is one-tenth of compressive strength.

Reinforced concrete: Concrete with reinforcement embedded in it. Bond between steel and surrounding concrete ensures **strain compatibility**. Reinforcing steel imparts ductility to concrete.

Grade of concrete: Compressive strength measured by standard test on concrete cube (150 mm size) tested 28 days after casting and continuous curing. Strength of cube is the average of three specimens of a sample, where **individual variation** should not be more than **± 15% of average.** If variation is more, test result of the **sample** is declared invalid.

Acceptance criteria of compressive strength

Specified Grade **(1)**	**Mean of the group of 4 Non overlapping Consecutive test results in N/mm²** **(2)**	**Individual test Result in N/ mm²** **(3)**
M 15	$\geq f_{ck}$ + 0.825 × established standard deviation (rounded off to nearest 0.5 N/mm²) or f_{ck} + 3 N/mm², whichever is greater	f_{ck}^{-3} N/mm²
M 20 or above	$> f_{ck}$ + 0.825 × established standard deviation (rounded of to nearest 0.5 N/mm²) or f_{ck} + 4 N/mm², whichever is greater	f_{ck}^{-4} N/mm²

Flexural strength: Both the below condition should be satisfied.

(*i*) The mean strength determined from any group of four consecutive test results exceeds the specified characteristic strength by atleast 0.3 N/mm^2.

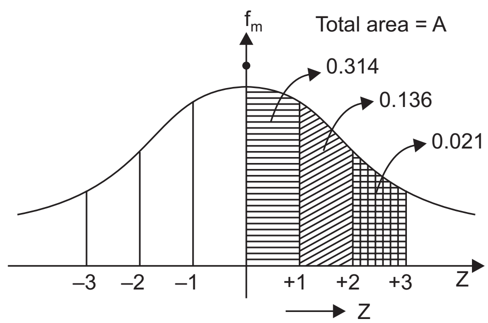
(*ii*) The strength determined from any test result is not less than the specified characteristic strength less 0.3 N/mm^2.

Variation in strength: Strength of cube varies as **Normal probability distribution curve**.

(*i*) Mean strength $f_m = \dfrac{\Sigma(\text{Strength of sample i.e., } f)}{\text{No. of samples i.e., } m}$

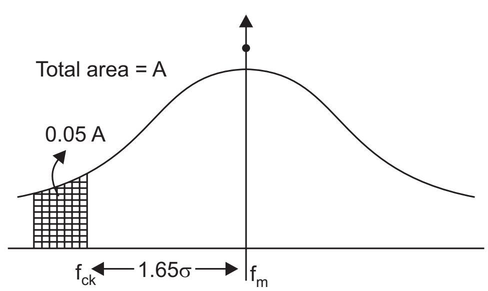
(*ii*) Standard deviation $\sigma = \sqrt{\dfrac{\Sigma(f - f_m)^2}{m}}$

(*iii*) Frequency density $y = \dfrac{1}{\sqrt{2\pi}} e^{-z^2/2}, \; z = \dfrac{f - f_m}{\sigma}$



Characteristic strength (f_{CK}): Strength below which not more than 5% of test results are expected to fall.

$$f_{CK} = f_m - 1.65\sigma$$



- Minimum curing period for OPC is 7 days.
- Grade's of concrete

  (*i*) M_{10}, M_{15}, M_{20} → ordinary

  (*ii*) M_{25}– M_{55} → standard

  (*iii*) M_{60}– M_{90} → High strength

  where 'M' stands for 'mix'
- Minimum grade for RCC → M_{20}

Concrete Mix Design: It's economical selection of relative proportions of various ingredients of concrete such that it remains workable in fresh state and impermeable and durable in hardened state.

Nominal mix (upto M20 grade only) | Design mix (IS 10262–1982)

Nominal mix
(upto M_{20} grade only)

- Specified in terms of **total mass** of aggregate, and volume of water to be used per 50 kg of cement.

1 bag of cement = 34.5 litres

	cement	: FA	: CA
M 5	1	: 5	: 10
M 7.5	1	: 4	: 8
M 10 =	1 +	: 3 +	: 6
M 15	1	: 2	: 4
M 20	1	: 1.5	: 3

Design mix
(IS 10262 – 1982)

Steps involved are

(*i*) Target mean strength.

(*ii*) Water cement ratio from charts.

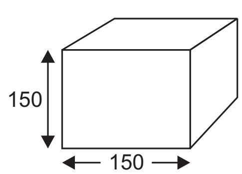
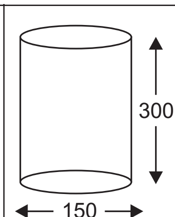
(*iii*) Water content based on workability and ratio of fine and coarse aggregate, by mass based on type and grading of aggregate.

(*iv*) Find cement content from (*ii*) and (*iii*) above

(*v*) Mass of FA and CA from absolute volume principle.

(*vi*) Weight of ingredent per batch based on capacity mixer.

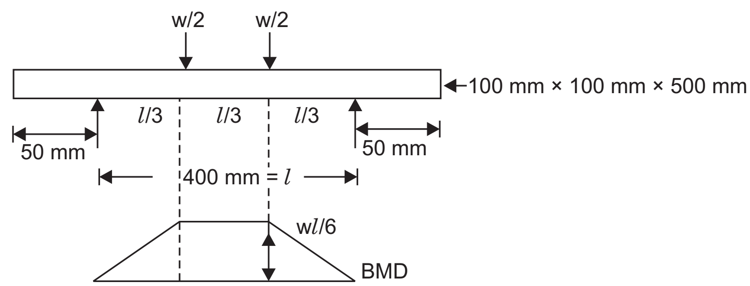
Compressive strength of concrete in structures:

		For size **beyond 450 mm** there is no decrease in compressive strength Hence taken as
Compressive strength = f_{CK}	$0.8\ f_{CK}$	$0.67\ f_{CK}$

Modulus of Rupture of concrete

It is used to determine the onset of cracking or the load at which cracking starts.

$$f_{Cr} = 0.7\sqrt{f_{CK}}$$

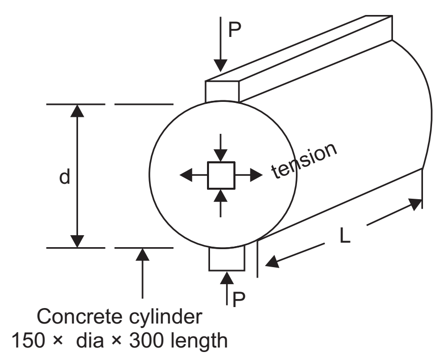


Tensile strength of concrete

Splitting tensile strength

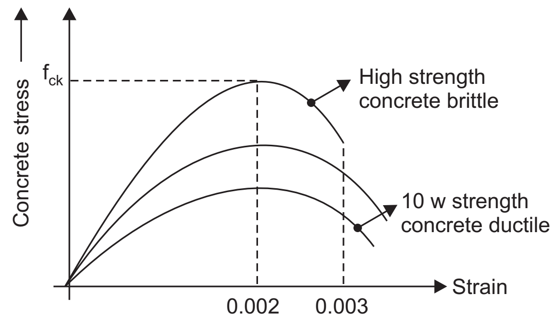
$$f_{Ct} = \frac{2P}{\pi d L} = 0.66\, f_{Cr}$$

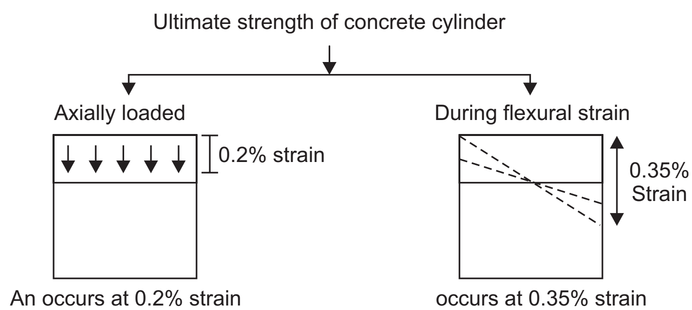
Direct tensile strength = $(0.5 - 0.625)\, f_{cr}$



Stress-stain curve of concrete

- Descending part of high strength concrete is steeper.
- Point where curve ends is crushing strain.
- Curves are linear upto 0.6 times the peak stress.
- Secant modulus of elasticity of concrete is taken at a stress of around $0.33\, f_{CK}$.



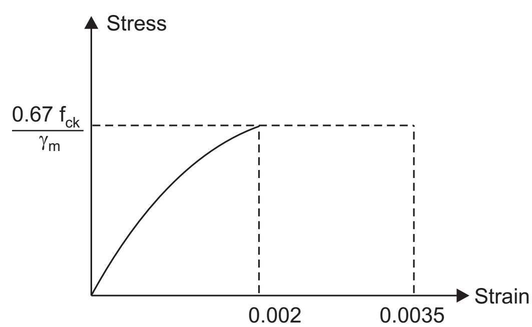


Design stress-Strain curve for strength of concrete

Partial safety factor for material strength (γ_m)

(*i*) Collapse = 1.5

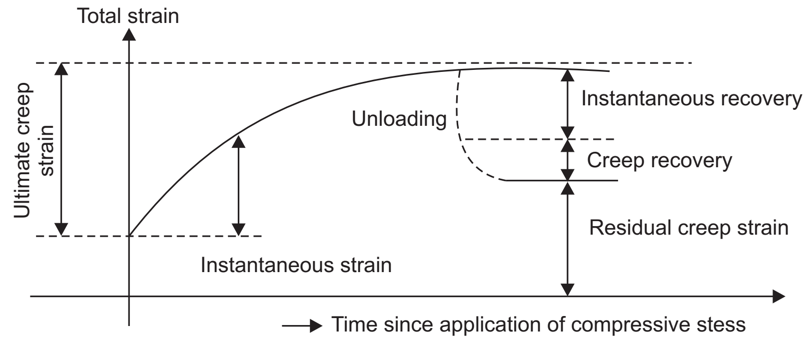
(*ii*) Serviceability = 1.0



Creep in Concrete

Time dependent component of total strain is creep.

- Occurs due to internal movement of adsorbed water, loss of moisture, growth of microcracks, sliding between gel particles.



- Advantages of creep
  - (*a*) Reduction in cracking stress developed due to restrained shrinkage.
  - (*b*) Reduction in stress due to deferential settlement in indeterminate structures.
- Disadvantages of creep
  - (*a*) Increased deflection of beams, slabs and columns (which can even buckle).
  - (*b*) Gradual transfer of load from concrete to reinforcing steel in compression members.
  - (*c*) Loss of prestress.

Creep increases when following are

Low/small
(*i*) Aggregate content
(*ii*) Relative humidity
(*iii*) Size/thickness ratio

High/large
(*i*) Cement content
(*ii*) Water/cement ratio
(*iii*) Temperature
(*iv*) Loading at early stage
(*v*) Loading over long period

Note:
High strength concrete, Adding reinforcement, delaying application of partition wall and finishes, steam curing under pressure reduce the effect of creep.

Shrinkage: Shortening in length of a member or contraction of concrete per unit length due to drying when concrete sets.

Plastic Shrinkage

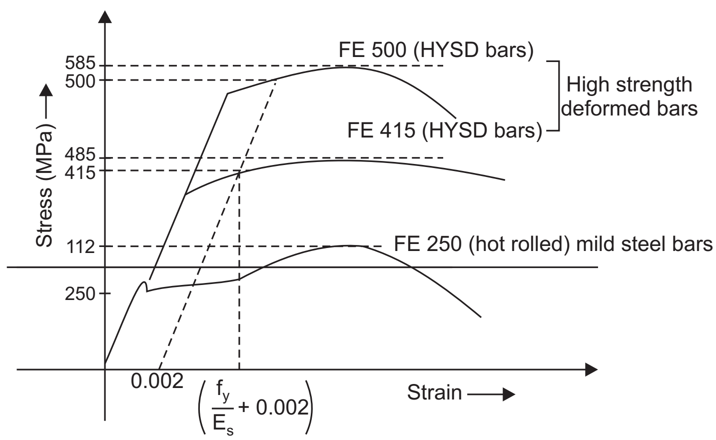
- Shrinkage when concrete is in its plastic stage itself due to loss of water by evaporation or by absorption by aggregate.
- Presented by using **expanding cement** or **Aluminium Powder**.

Drying Shrinkage

- Everlasting process by loss of water hold in gel pores when concrete is kept in drying condition.
- Presented by using harder and bigger size of aggregate, Bigger size of member and increasing humidity.

Reinforcement

In Fe 250, Fe 415, Fe 500 :- 250 MPa, 415 MPa and 500 MPa is the guaranted minimum strength and can be treated as characteristic strength of steel respectively.



Note:
By cold working (stretching and twisting) yield strength increases but ductility falls (i.e., HYSD bars have high yield strength than mild steel).

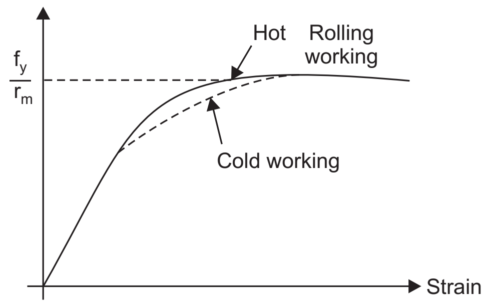
- HYSD bars fails at less elongation as compared to mild steel bars.

Design stress strain curve for strength of steel

Partial Factor of safety for material strength (γ_m)

(*i*) Collapse = 1.5

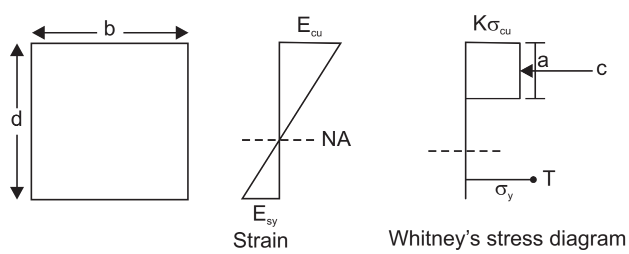
(*ii*) Serviceability = 1.0



Design methods

1. Working stress method.
2. Unit state method.
3. Ultimate load method.

Ultimate load Method: (Whitney's theory)



a = Depth of rectangular stress block

= 0.537 d in accordance to whitney.

σ_{cu} = Ultimate compressive strength of concrete cubes (28 days)

$K\sigma_{cu}$ = average stress

= 0.85 σ_{cu} in accordance with whitney.

This theory is based on the assumption that ultimate strain in concrete is 0.3% and compressive stress at the extreme fiber of the section corresponds to this strain. Whitney replaced the actual parabolic stress diagram by a rectangular stress diagram.

2 Limit State Method

The acceptable limit for the safety and serviceability requirement of a structure or structural element before occuring of failure

Limit state of serviceability

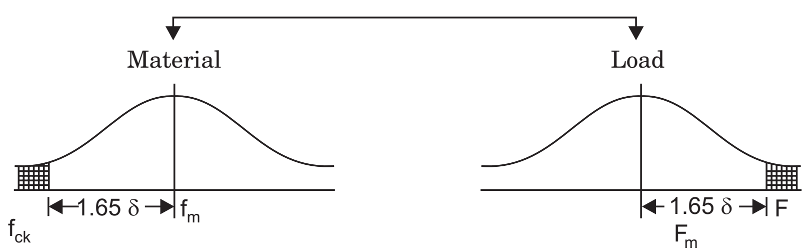
- Satisfactory performance under service load
- Deflections, cracking, vibration, leakage, loss of durability etc.

Limit state of collapse

- Adequate margin of safety for normal over loads.
- Flexure, compression, shear, torsion, over turning, sliding, buckling, fatigue.

> **Note:**
> Structure will return to its original state it has reached only till its limit state of serviceability. However, after reaching limit state of collapse it will not region its shape.

Characteristic Shape



$f_{ck} = f_m - 1.65\,\delta$ $\qquad$ $F = F_m + 1.65\,\delta$

- To account for uncertainties **reliability based analysis** was performed and **partial factor of safety** were established for material as well as load.
- (f_m – 1.65f) and (F_m + 1.65F) are limits with in which "probability of lying test result" is maximum and it is called **confidence limit**.

(*a*) Partial factor of safety in material property

$$f_d = \frac{f_{ck}}{\gamma_{ms}}$$

Concrete

$$f_d = \frac{0.67\, f_{ck}}{1.5} = 0.45\, f_{ck}$$

Steel

$$f_{ck} = \frac{f_y}{1.15} = 0.87\, f_y$$

(*b*) Partial factor of safety under various load combinations

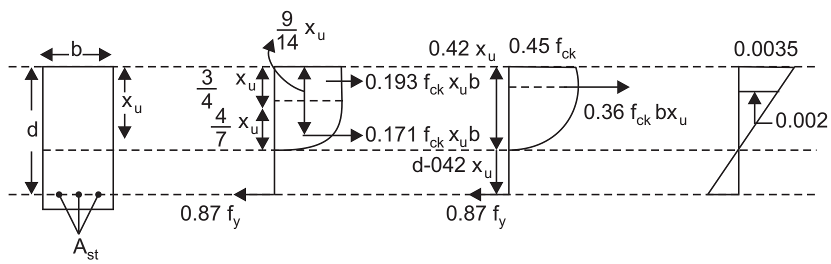
$$F_d = F\gamma_f$$

S. No.	Discription	Collapse	Servicability
1.	D.L + L.L	1.5	1
2.	D.L + (W.L) or (E.L) combination		
	(*i*) for normal case		
	D.L + W.L (or E.L) D.L + W.L (or E.L) (*ii*) For checking stability against overturning/stress reversal	1.5	1
	D.L + W.L (or E.L)	0.9	1
3.	(D.L.) + L.L + W.L (or E.L) combination		
	D.L	1.2	0.8
	W.L (or E.L)	1.2	0.8

Assumptions of limit state of collapse: Flexure

1. Plane section before bending remains plane after bending (**Strain compatibility**)
2. Maximum strain in concrete at the outermost compression fibre is taken as 0.0035 in bending (Regardless of grade of concrete by **Max. Principal strain theory**)
3. Relationship between compressive stress distribution in concrete and strain in concrete may be assumed to be rectangular, Trapezoidal, parabolic or any other shape which results in prediction of strength in substantial agreement with the result of test.

4. Tensile strength of concrete is ignored.



5. For design purpose partial factor of safety for steel γ_{ms} = 1.15 and the stress in steel is derived from stress-strain curve.
6. **Maximum** strain in tension reinforcement in the section at failure shall not be less than

$$\in_{st} = \frac{f_y}{1.15E_s} + 0.002 = \frac{0.87f_y}{E_s} + 0.002$$

This assumption restricts depth of neutral axis.

Limiting Depth of Neutral Axis

	$x_{u\,lim/d}$	$R_u = \frac{M_{u\,lim}}{bd^2}$	p_t, lim
Fe 250	0.53	0.148 f_{ck}	0.088 f_{ck}
Fe 415	0.48	0.138 f_{ck}	0.0479 f_{ck}
Fe 500	0.46	0.133 f_{ck}	0.038 f_{ck}

$$\left(\frac{x_u}{d}\right)_{lim} = \frac{0.0035}{0.0055 + \frac{0.87f_y}{E_s}}$$

Note:
Failure always occurs due to crushing of concentrate on compression face in both the cases.

3 Beams

Singly Reinforced Beam

(*i*) To determine M.O.R (M_u) when beam cross-section is given

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{CK} b}$$

$$x_4 = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$x_u = x_u$ lim	$x_u < x_u$ lim	$x_u > x_u$ limit
Balanced	Under reinforced	Over reinforced

(*a*) **Balanced section** $M_u = M_{u\,lim}$

$M_{u\,lim} = R_u bd^2$ or $M_{u\,lim} = 0.87 f_y A_{st} (d - 0.42 x_{u\,lim})$

(*b*) **Under reinforced section**

$M_u = 0.36 f_{ck} x_u (d - 0.42 x_u)$ or $M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$

(*c*) **Over reinforced section:** x_u is limited to $x_{u\,lim}$ and M_u is calculated as in balanced section.

(*ii*) To determine area of steel, when concrete cross-section and applied moments are known.

$$M_u \text{ applied} = 0.87 f_y A_{st}\left(d - \frac{A_{st} f_y}{f_{CK} b}\right)$$

(*iii*) To determine the *x*-section for given bending moment (M_u)

(*a*) $M_u = R_u bd^2$

- Take $d = 2b$ and get '*b*' then *d*
- Round off '*d*' to the nearest **upper** 50 like 500, 550, 600, 650, etc.
- $D = d + 50$

(*b*) $M_u = 0.87 f_y A_{st}\left(d - 0.42 \times \frac{0.87 f_y A_{st}}{0.36 f_{CK} b}\right)$

Get A_{st}

(*c*) Apply checks

- $\frac{A_{st\,min}}{bd} > \frac{0.85}{f_y}$, $A_{st\,max} = 0.04\, bd$, $x_u < x_{u\,lim}$

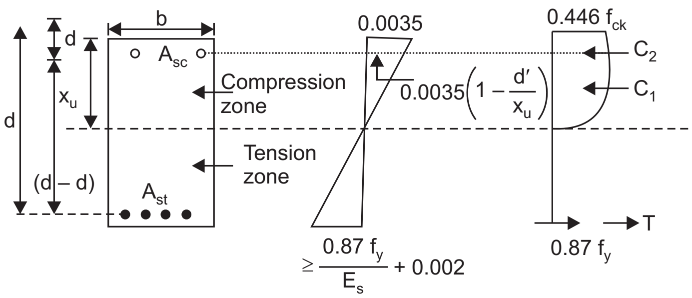
Doubly Reinforced Beam: If $M_u > M_{u\ lim}$, then either section dimensions need to be modified or higher grade of steel/concrete to be used. It section dimensions are restrained then compression reinforcement is provided such that neutral axis does not shifts downward by providing tension steel greater than $A_{st\ lim}$.

Advantage

(*a*) Prevents Beam in Reversal of moments.

(*b*) Reduction in long term deflection due to Shrinkage and creep.

(*i*) To determine Area of steel for given bending moment ($M_{u\ applied} > M_{u\ lim}$) and retrained dimension's (*b* and *d*).



(*a*) $M_{u\ lim} = R_u bd^2$

(*b*) $M_{u\ lim} = 0.87 f_y A_{st\ 1} (d - 0.42\, x_{u\ lim})$, Get $A_{st\ 1}$

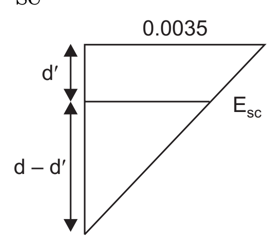
(*c*) $M_{u\ applied} - M_{u\ lim} = 0.87 f_y A_{st\ 2} (d - d')$, Get $A_{st\ 2}$

where d' is selected such that

$$0.05 < \frac{d'}{d} < 0.2$$

(*d*) Get E_{SC} from strain diagram

$$\frac{0.0035}{E_{SC}} = \frac{d}{d - d'}$$



(*e*) Get f_{SC} from E_{SC} or from d/d' values. (given in question)

Stress (f_{SC})	Strain ($\in_{SC}$)
$0.8\, f_{yd}$	0.00144
$0.85\, f_{yd}$	0.00163
$0.90\, f_{yd}$	0.00192
$0.95\, f_{yd}$	0.00241
$0.975\, f_{yd}$	0.00276
$1.00\, f_{yd}$	0.00380

For Fe 415 d'/d values

d'/d	0.05	0.10	0.15	0.20
f_{st}(N/mm^2)	355	353	342	329

(*f*) Get A_{SC} from $0.87\, f_y\, A_{st2} = A_{SC} \cdot f_{SC}$.

(*ii*) To determine M.O.R (M_u) when beam cross-section is given

(*a*) Get $x_{u\,\lim}$ from d (i.e., $0.53\, d$, $0.48\, d$ or $0.46\, d$)

(*b*) Get x_u from

$$0.36\, f_{CK}\, b\, x_u + (f_{SC} - f_{CC})\, A_{SC} = 0.87\, f_y\, A_{st}$$

where $f_{CC} = 0.45\, f_{CK}$

	f_{SC}
Fe 250	217
Fe 415	350
Fe 500	400

if $\quad 0.05 < \dfrac{d'}{d} < 0.2$

otherwise trial and error.

(*c*) As $x_u < x_{u\,\lim}$

$$M_u = 0.36\, f_{ck}\, b\, x_u\, (d - 0.42\, x_u) + f_{sc}\, A_{sc}\, (d - d')$$

Flanged beam: In monolithic construction, slab and beams are cast together. If slab in such cases is in compression zone they become effective in adding significantly to the area of concrete in compression in beam.

Effective flange width for T and L beams

Monolithic L-Beam

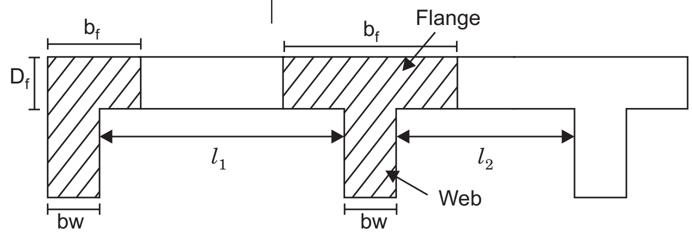
(*i*) $b_f = b_w + 3D_f + \dfrac{l_0}{12}$

$b \nless b_w + \dfrac{l_1}{2}$

Monolitic T-Beam

(*ii*) $b_f = b_w + 6D_f + \dfrac{l_0}{6}$

$b_f \nless b_w + \dfrac{l_1 + l_2}{2}$

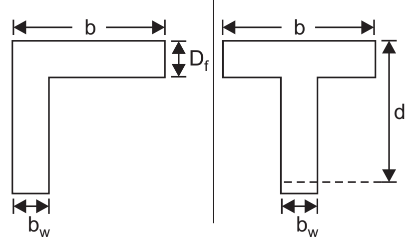


Isolated L-Beam

(*i*) $b_f = b_w + \dfrac{0.5\, l_0}{\dfrac{l_0}{b} + 4} \le b$

Isolated T-Beam

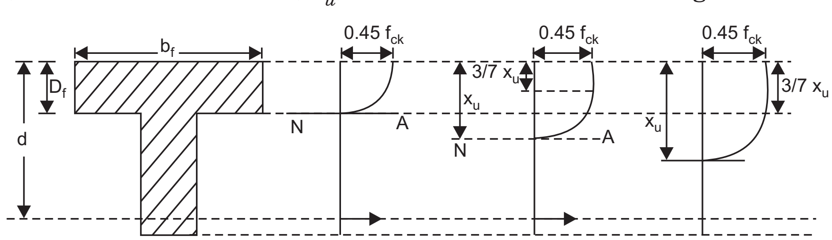
(*ii*) $b_f = b_w + \dfrac{l_0}{\dfrac{l_0}{b} + 4} \le b$



b_f = effective flange width (the width of flange with constant compressive stress equal to the peak actual flexural compressive stress which leads to the same longitudinal compressive force as due the original stress distribution).

l_0 = distance between point of zero moment in the beam.

(*i*) To determine M.O.R (M_u) when beam cross-section is given



(*a*) When Neutral axis lies in the flange ($x_u < D_f$)

$$x_u = \frac{0.87\, f_y\, A_y}{0.36\, f_{CK}\, b_f} \qquad M_u = 0.36\, f_{CK}\, b_f\, x_u\, (d - 0.42\, x_u)$$

$$M_u = 0.87\, f_y\, A_{st}\, (d - 0.42\, x_u)$$

(*b*) When Neutral axis lies in the web $\left(x_u < \frac{7}{3} D_f\right)$

$$y_f = 0.65\, D_f + 0.15\, x_u$$

Get x_u from 0.36

$f_{CK}\, b_w\, x_u + 0.45\, f_{CK}\, (b_f - b_w)\, y_f = 0.87\, f_y\, (A_{st1} + A_{st2})$

If $x_u < x_{u\,lim}$

$$M_u = 0.36\, f_{CK}\, b_w\, x_u\, (d - 0.42\, x_u) + 0.45\, f_{CK}\, (b_f - b_w)\, y_f \left(d - \frac{y_f}{2}\right)$$

OR If $x_u > x_{u\,lim}$, $M_{u\,lim} = 0.36\, f_{CK}\, b_w\, x_{u\,lim}\, (d - 0.42\, x_{u\,lim})$

$$+ 0.45\, f_{CK}\, (b_f - b_w)$$

$$(0.65\, D_f + 0.15\, x_{u\,lim}) \left(d - \frac{0.65\, D_f + 0.15\, x_{u\,lim}}{2}\right)$$

(*c*) When Neutral axis lies in the web $\left(x_u > \frac{7}{3} D_f\right)$

Get x_u from $0.45\, f_{CK}\, (b_f - b_w)\, D_f + 0.36\, f_{CK}\, b_w\, x_u = 0.87\, f_y\, A_{st}$

If $x_u < x_{u\,lim}$

$$M_u = 0.45\, f_{CK}\, (b_f - b_w)\, D_f \left(d - \frac{D_f}{2}\right) + 0.36\, f_{CK}\, b_w x_u (d - 0.42\, x_u)$$

OR If $x_u > x_{u\,lim}$

$M_u = 0.36\, f_{CK}\, b_w\, x_{u\,lim}\, (d - 0.42\, x_{u\,lim})$

$$+ 0.45\, f_{CK}\, (b_f - b_w)\, D_f \left(d - \frac{D_f}{2}\right)$$

Note:

Inverted beams are designed as rectangular beams, because slab in tension zone does to resists any compression. These beams are recommended for architectural requirement only.

4 Shear

Types

Flexure
Shear associated with change in bending moment along the span

Punching
Shear associated with possibility of punching a thin member by concentrated load

Torsion
Shear associated with a member subjected to torsion.

Note:
Flat slab is a **beam less** floor slab supported directly by columns.

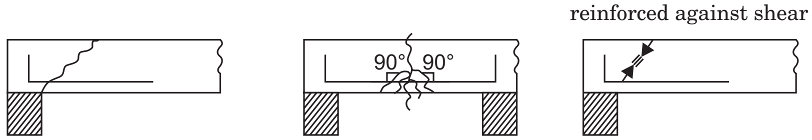
Modes of Failure Due to Shear

Modes of failure due to shear

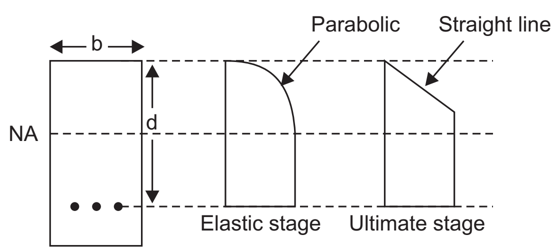
Diagonal torsion failure
- Large shear force and less bending moment.

Flexure shear failure
- Less shear force and large bending moment.

Diagonal compression failure
- Crushing of concrete when beams are heavily reinforced against shear



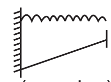
Shear stress distribution



$$\text{Nominal shear strength} = \tau_u = \frac{V_u}{b_d}$$

For beam of varying depth

$$\tau_u = \frac{Vu \pm \dfrac{M_u \tan\beta}{d}}{bd}$$



(–ve sign)

Design Shear Strength of Concrete in Beams

(*i*) Without shear reinforcement

$$\tau_c = \frac{0.85\sqrt{0.8 f_{ck}}\,(\sqrt{1+5\beta}-1)}{6\beta}$$

where $\beta = \dfrac{0.8\,f_{ck}}{6.89\,p_t} > 1$

$$p_t = \frac{A_{st}}{bd}\times 100$$

A_{st} = Area of longitudinal reinforcement which continues at least one effective depth beyond the section being considered except at support where the full area of tension reinforcement may be used provided the detailing is as per code.

P	WSM		LSM	
	M20	M25	M20	M25
$0 \le 0.15$	0.18	0.19	0.28	0.29
0.25	0.22	0.23	0.36	0.36
0.50	0.30	0.31	0.48	0.49
0.75	0.35	0.36	0.56	0.57
1.00	0.39	0.40	0.62	0.64

Note:

τ_c depends on both f_c and A_{st}.

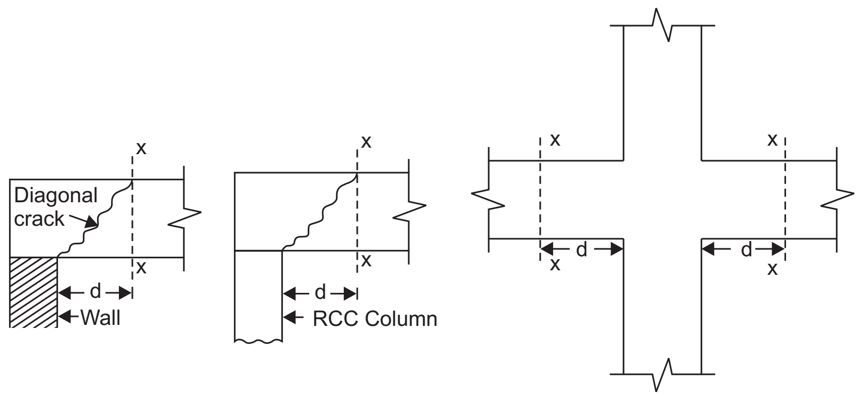
Maximum shear stress with shear reinforcement ($\tau_{c\,max}$)

M	15	20	25	30	35	40 and above
LSM	2.5	2.8	3.1	3.5	3.7	4.0
WSM	1.6	1.8	1.9	2.2	2.3	2.5

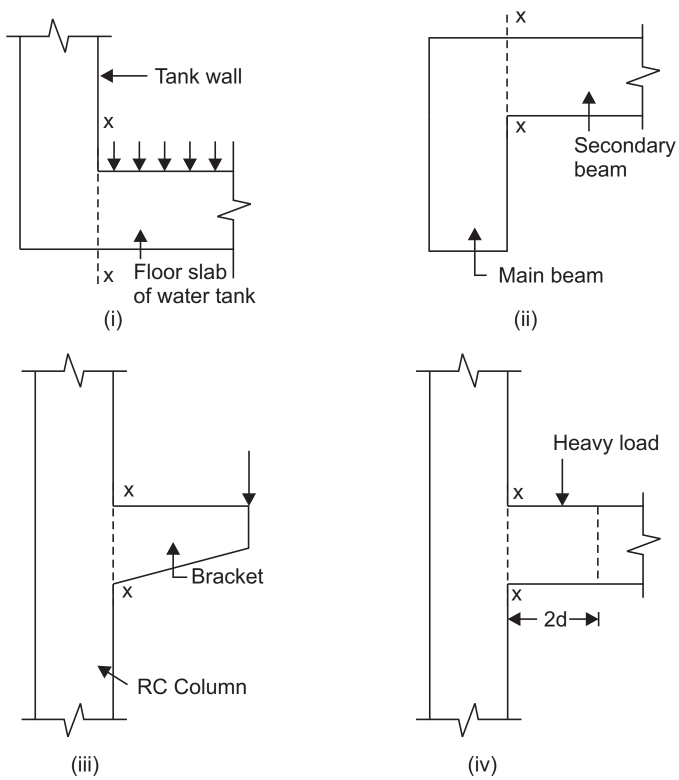
for LSM $\tau_{c\,max} \approx 0.63\sqrt{f_{ck}}, \tau_u \not> \tau_{c\,max}$

Note: $\tau_{c\,max}$ depends only on concrete (*i.e*, f_{ck})

Critical Section for Shear



(*a*) Critical section X-X at *d* from the face of the support

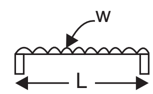


(*b*) Critical section X-X at the face of the support

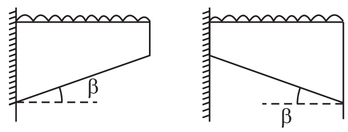
Design for shear

(*i*) Get factored shear force V_u

$$V_u = 1.5\ V \quad \text{or} \quad Vu = 1.5\left(\frac{wL}{2}\right)$$



(*ii*) $\tau_u = \dfrac{V_u}{bd}$ or $\tau_u = \dfrac{V_u \pm \dfrac{M_u \tan\beta}{d}}{bd}$ if



(–ve sign) (+ve sign)

(*iii*) Get $\tau_{c\,max}$ from code or $\tau_{c\,max} \approx 0.63\sqrt{f_{ck}}$

(*iv*) It $\tau_{c\,max} < \tau_u$ Redesign the section by changing b or d.

(*v*) It $\tau_u < \tau_{c\,max}$ then get τ_c from I.S. code (which depends on f_{ck} and A_{st})

(*vi*) Now if $\tau_u - \tau_c < 0.4$ N/mm^2, provide reinforcement for minimum shear

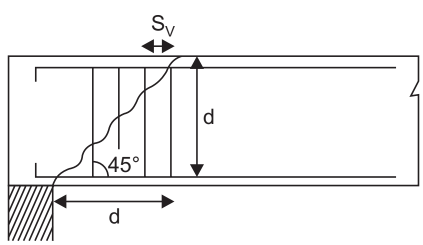
$$\frac{d}{S_V}(0.87\,f_y)\,A_{SV} = 0.4\,bd$$

Get S_V

$\dfrac{d}{S_V}$ = No. of stirrups till critical section for shear *i.e.* till distance d from face of support (as S_V = spacing between stirrups)

A_{SV} = Total cross-sectional area of stirrups leg effective in shear.

f_y = yield strength of stirrup's subjected to maximum of 415 N/mm^2



(*vii*) If $\tau_u - \tau_c > 0.4 N/mm^2$ then get spacing S_V by

$$\frac{d}{S_V}(0.875 f_y) A_{SV} = (\tau_u - \tau_c)bd$$

(*viii*) Check for maximum spacing of stirrups as per code

Max. S_V

$$= \text{Minimum of} \begin{cases} 0.75\, d(\text{Vertical stirrups}),\ d\ (\text{Inclined stirrups}) \\ S_V\ (\text{as obtained from } (vi) \text{ or } (vii)) \\ 300 \text{ mm} \end{cases}$$

Note:

For inclined stirrups or a series of bent up bars

$$\left(\frac{d}{S_V}\right)(0.87 f_y)(\sin\alpha + \cos\alpha) A_{SV} = (\tau_u - \tau_c)bd$$

α = Angle of inclination of stirrups with the horizontal

Minimum shear reinforcement: It is provided

(*i*) to prevent bursting of concrete cover.

(*ii*) to prevent tension failure due to shrinkage, thermal stresses and internal cracks.

(*iii*) to avoid brittle shear failure.

(*iv*) to hold reinforcement in place while concreting.

(*v*) to make the section effective with the tie effect.

5 Bond and Anchorage

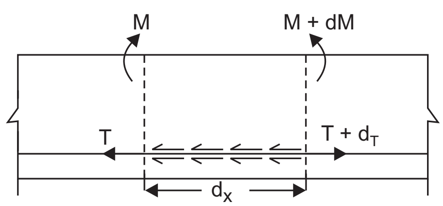
Bond is the means by which relative movement between concrete and steel is prevented and the intensity of adhesive force is called stress. Bond transfers the axial force by providing '**strain compatibility**' and '**composite action**', of concrete and steel.

Bond is due to combined effect of adhesive resistance, frictional resistance, and mechanical resistance (for deformed bars)

Bond stress

- Flexural bond stress
  (Due to change in bonding moment)
  • Development length (L_d) is provided
- Anchorage bond stress
  (Due to tension or compression)
  • Anchorage length is provided

Flexural Bond Stress



Note: Bond stress can be reduced by providing an increased number of bars of small diameter rather than small number of large diameter bars.

$$\tau_{bd} = \frac{V}{jd(\sum O)}$$

V = Shear force at any section = $\frac{dm}{dx}$

d = effective depth of the section

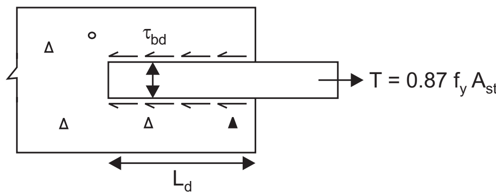
ΣO = Summation of all perimeter of reinforcement

$\Sigma O = n(\pi\ \phi)$

n = number of bars

ϕ = diameter of reinforcement

Anchorage Bond Stress: (Due to tension)



$$L_d = \frac{0.87\, f_y \phi}{4\tau_{bd}} \qquad A_{st} = \frac{\pi}{4}\phi^2$$

L_d = development length

τ_{bd} = Average bond stress

ϕ = Nominal diameter

Permissible Bond Stress in Tension

Grade of concrete	M20	M25	M30	M35	M40 and above
τ_{bd} (N/mm²)	1.2	1.4	1.5	1.7	1.9

For deformed bars above value is increased by **60%**.

For bars in compression above value is increased by **25%**.

For ready Reference

	M20		M25	
L_d	T	C	T	C
Fe 250	460ϕ	37ϕ	39ϕ	32ϕ
Fe 415	47ϕ	38ϕ	41ϕ	33ϕ

T = Tension

C = Compression

Development Length Due to Flexure

$$L_d \le \frac{M_1}{V} + L_O$$

L_O = max (d, 12 ϕ)

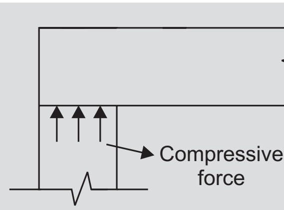
M_1 = MOR of the section to be assuming all reinforcement at the section to be stressed to 0.87 f_y

V = Shear force at the section due to design load.

Note:

When the ends of the reinforcement are confined by compressive stresses then M_1 is increased by 30%

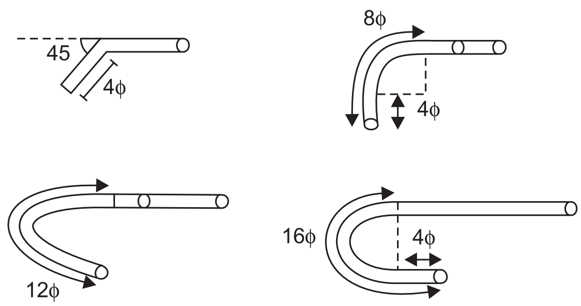
i.e., $$L_d \le \frac{1.3M_1}{V} + L_O$$



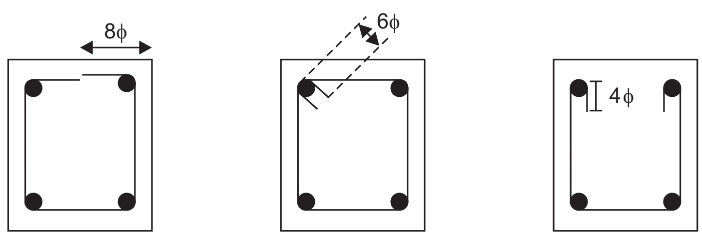
Bundled bars: The development length of each bundled bars shall be increased by 10% when two bars are bundled, by 20% when three bars are bundled and 33% when four bars are bundled.

Bends and Hooks: (IS : 2502)

The anchorage value of bend shall be taken as 4 times the diameter of bar for each 45° bend subject to a maximum of 16 times the diameter of bars.



For stirrups



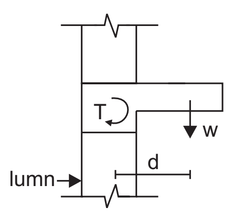
6 Torsion

Equilibrium Torsion/Primary Torsion

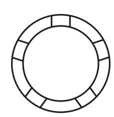
- Torsion which can be determined only by using static equilibrium conditions.
- Torsion induced in beams curved in plan and subjected to gravity loads.
- Also occurs in beams where the transverse loads are eccentric with respect to the shear centre of the *x*-section.

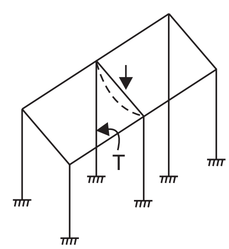
Compatibility Torsion or Secondary Torsion

- It is induced in a structural member by rotation applied at one or more point along the length of member through inter connected members, instead of by directly applied load.



Cantilever slab with 'L' beam


Circular beam



Effect of Torsional moment

(*i*) Beam fails in diagonal tension.

(*ii*) Longitudinal reinforcement is provided in the form of bars placed **closed to the periphery** where as transverse reinforcement is in the form of closed rectangular stirrups placed perpendicular to the beam axis.

(*iii*) Longitudinal reinforcement helps in reducing the crack width through **dowel action.**

(*iv*) Stirrups resist shear due to vertical loads and torsion.

Note:
As per IS code, clause 41, For design of torsion section located at a distance less than '*d*' from the face of the support may be designed for the same torsion as computed at '*d*' where '*d*' is the effective depth.

Design for Torsion:

Given V_u = Ultimate Vertical Shear at the Section

T_u = Ultimate Torsional moment

M_u = Factored Bending moment at the cross-section

b, d, D, f_{CK} and f_y

(*i*)
$$V_e = V_u + 1.6\,\frac{T_u}{b}$$

V_e = Equivalent shear at the section

$$\tau_e = \frac{V_e}{bd}$$

$$\tau_{c\,max} \approx 0.63\sqrt{f_{CK}}$$

Get τ_c for minimum % of tensile reinforcement. If $\tau_c < \tau_c < \tau_{c\ max}$ then Both Longitudinal and transverse reinforcement is required.

(*ii*) Longitudinal reinforcement

$$M_e = M_u + M_t$$

M_e = Equivalent Bonding moment at the section

$$M_t = \frac{T_u}{1.7}\left(1 + \frac{D}{b}\right)$$

Case (*a*): If $M_t < M_u$ = No compression reinforcement is required. Still provided two no's of hanger bar's (in compression side) of 10mm ϕ

Case (*b*): If $M_t > M_u$ = Compression reinforcement required (i.e., A_{SC})

$M_t - M_u = 0.87\,f_y\,A_{SC}\,(d - d')$, get A_{SC}

For A_{st}, $M_e = 0.87\,f_y\,A_{st}\left(d - \dfrac{0.42 \times 0.87\,f_y\,A_{st}}{0.36\,f_{CK}\,b}\right)$, Get A_{st}

(*iii*) Transverse reinforcement

$$\left(\frac{d_1}{S_V}\right)(0.87\,f_y)\,A_{SV} = \frac{T_u}{b_1} + \frac{V_u}{2.5}$$

Get S_V

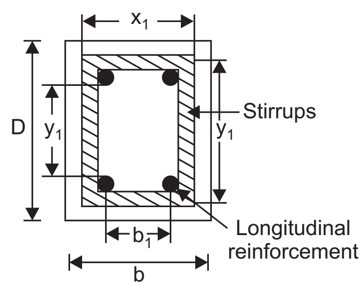
where,

b_1 = *c/c* distance between corner bars in the direction of the width.

d_1 = *c/c* distance between corner bars in the direction of the depth.

b_1 = *b*-clear cover – 2 diameter of stirrups

$$- 2\left(\frac{\text{diameter of longitudinal bar}}{2}\right)$$



A_{SV} = Area of cross-section of two legged closed loops enclosing the corner longitudinal bar (For *e.g.* $2 \times \frac{\pi}{4}(8)^2$ if dia is 8 mm)

Now get x_1 from b_1 and y_1 from d_1

x_1 =Short dimension of stirrup

$$= b_1 + 2\left(\frac{\text{diameter of longitudinal bar}}{2}\right) + 2\left(\frac{\text{diameter of stirrups}}{2}\right)$$

y_1 = Long dimension of stirrup

$$= d_1 + 2\left(\frac{\text{diameter of longitudinal bar}}{2}\right) + 2\left(\frac{\text{diameter of stirups}}{2}\right)$$

Note:
Better to calculate b_1, d_1, x_1 and y_1 from diagram only.

(*iv*) S_V will be minimum of $\begin{cases} S_V \text{ obtained in } (iii) \\ x_1 \\ \dfrac{x_1 + y_1}{4} \\ 300 \text{ mm} \end{cases}$

(*v*) Additional Longitudinal reinforcement shall be provided along the two faces when the cross-sectional dimensions either b or D of the member exceeds 450 mm.

Hence, side face reinforcement to be provided = $\dfrac{0.1}{100} bD$

on one side = $\dfrac{1}{2}\left(\dfrac{0.1bD}{100}\right)$

Note:
Generally 10 mm ϕ bars provided, so number of bars

$$= \frac{\frac{1}{2}\left(\frac{0.1bd}{100}\right)}{\frac{\pi}{4}(10)^2} \approx 2$$

So provided 2 no's of 10 mm ϕ bar's with spacing < 300 mm.

(*vi*) $\dfrac{A_{st\,min}}{bd} = \dfrac{0.85}{f_y}$ and $A_{st\,max} = 0.04\,bD$

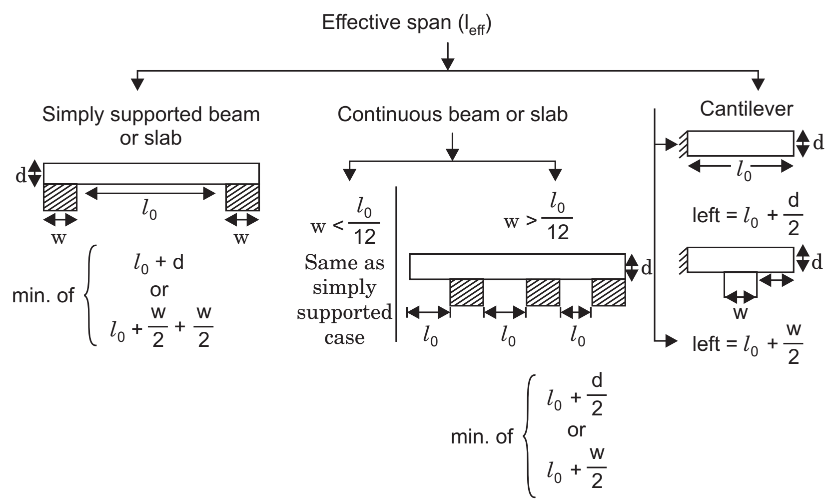
(*vii*) Minimum spacing between longitudinal tension bars

(*a*) In horizontal direction, min $\begin{cases} \text{max bar dia} \\ \text{coarse aggregate} + 5 \text{ mm} \end{cases}$

(*a*) In vertical direction, min $\begin{cases} \text{max bar dia} \\ \dfrac{2}{3} \text{ coarse aggregate} \\ 15 \text{ mm} \end{cases}$

7 Codal Provisions for Beam and Slab

EFFECTIVE SPAN (L_{eff})



Note:
For frames effective span is its centre to centre distance between members.

- In case of spans with roller or rocket bearings, the effective span shall always be the distance between the centres of bearings.

Check for Deflection

1. Final deflection due to all loads including the effect of temperature, creep and shrinkage should not exceed $\frac{\text{span}}{250}$.
2. Deflection including effect of creep, temperature and shrinkage occurring after creation of partition and application of finishes should not exceed $\frac{\text{span}}{350}$ or 20 m which ever is less.

Control of Deflection

$$\frac{\text{span}}{\text{depth}} < \left(\frac{l}{d}\right)_{\text{basic}} \times \frac{10}{\text{span}} \times k_t \times k_c \times k_f$$

(*i*) If span ≤ 10 m then $\left(\frac{l}{d}\right)_{\text{basic}}$ values are

Cantilever beam	7
Simply supported beam	20
Continuous beam	26

Type of slab	Type of reinforcement	
	Mild steel	**Fe 415**
Simply supported	35	28 = 35 × 0.8
Continuous	40	32 = 40 × 0.8

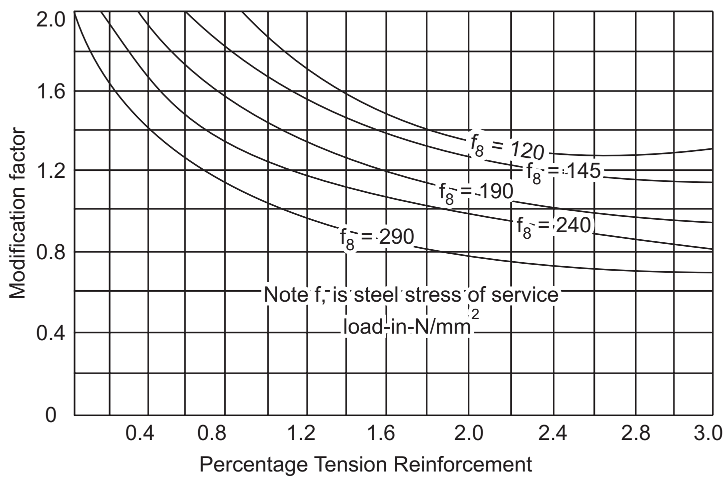
Where as $\left(\frac{10}{\text{span}}\right)$ factor is neglected.

Note:
For slab's its short span to overall depth ratio.
- For Fe 415 reinforcement values in case of slab, multiply mild steel value's by **0.8**.

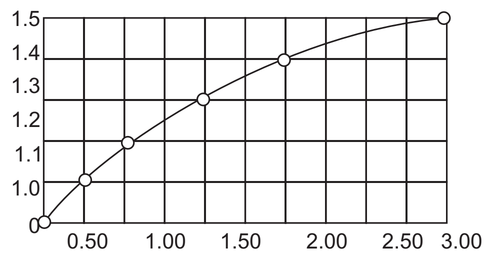
(*ii*) If span > 10 m, then $\left(\frac{10}{\text{span}}\right)$ factor is also multiplied (But if the case is of cantilever beam > 10 m then actual deflection calculations should be made)

(*iii*) K_t = modification factor for tension reinforcement depending upon area and stress of steel for tension reinforcement.

Note:
This factor allows the designer to make shallow members by increasing area of tension reinforcement.

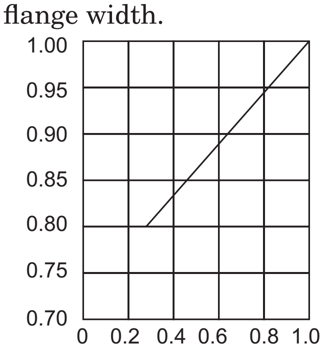


(*iv*) K_c = modification factor for compression reinforcement depending on area of compression reinforcement



Note:
It's always greater than 1, as compression reinforcement reduces shrinkage and increases the stiffness of the beam.

(*v*) K_f = modification factor for flanged beams depends on ratio of web width to flange width.



Note:
Here K_t and K_c should be calculated based on area $b_f d$.

Slenderness Limits to Ensure Lateral Stability

Slenderness limits to ensure lateral stability

- Simply supported beam or continuous beam

  $$\text{Clear span} < \text{min.}\left[\begin{array}{l} 60\,b \\ \dfrac{250\,b^2}{d} \end{array}\right.$$

- Cantilever beam

  $$\text{Clear span} < \text{min.}\left[\begin{array}{l} 25\,b \\ \dfrac{100\,b^2}{d} \end{array}\right.$$

Steel Reinforcement

Steel reinforcement

- Tension reinforcement
  - $\dfrac{A_{st}\,\text{min}}{bd} = \dfrac{0.85}{f_y}$
  - $A_{st}\,\text{max} = 0.04\,BD$
- Compression reinforcement
  - $A_{sc}\,\text{min} = 0$
  - $A_{sc}\,\text{max} = 0.04\,BD$

Side face reinforcement: Provided when **D > 750 mm**.

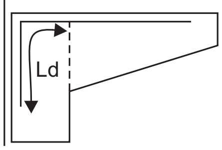
The total area of such reinforcement shall not be less than 0.1 percent of web area and shall be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness whichever is less.

Cover: To protect steel against corrosion

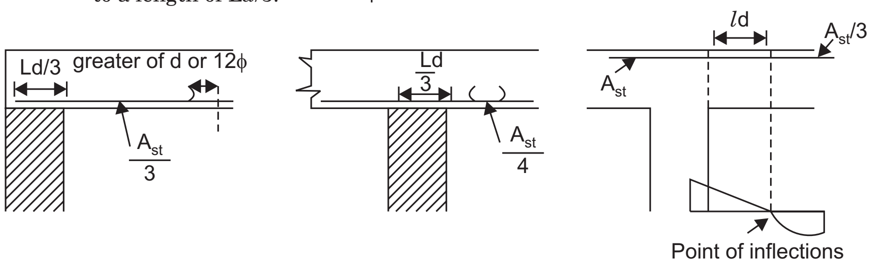
Environmental condition	Minimum grade of conc.	Nominal cover
Mild	M20	20 mm
Moderate	M25	30 mm
Severe	M30	45 mm
Very severe	M35	50 mm
Extreme severe	M40	75 mm

Extent of Reinforcement

- Positive movement tension reinforcement
  - Simply supported: Atleast 1/3rd of tensile reinforcement
  - Continuous: Atleast 1/4th of tensile reinforcement
  - Along the same face into the support to a length of Ld/3.
  - Cantilever



- Negative moment tension reinforcement
  - Atleast 1/3rd of total reinforcement provided at the support shall not extent beyond the point of inflection for a distance of no less than "d or 12ϕ or 1/16 of clear span" which ever is greater



Arrangement of loads in Continuous beams

(*i*) **For maximum moment:** Dead load on all spans while live loads on **two adjacent spans**.

(*ii*) **For maximum span moment:** Dead load on all spans while live loads on **alternate spans**.

Types of load	Span moments		Support moments	
	Near middle of end span	At middle of interior span	At support next to the end support	At other interior supports
(1)	(2)	(3)	(4)	(5)
Dead load and imposed load (fixed)	$+\frac{1}{12}$	$+\frac{1}{16}$	$-\frac{1}{10}$	$-\frac{1}{12}$
Imposed load (not fixed)	$+\frac{1}{10}$	$+\frac{1}{12}$	$-\frac{1}{9}$	$-\frac{1}{9}$

Moment Coefficients

Types of load	At end support	At support next to the end support		At all other interior supports
		Outer side	Inner side	
(1)	(2)	(3)	(4)	(5)
Dead load and imposed load (fixed)	0.4	0.6	0.55	0.5
Imposed load (not fixed)	0.45	0.6	0.6	0.6

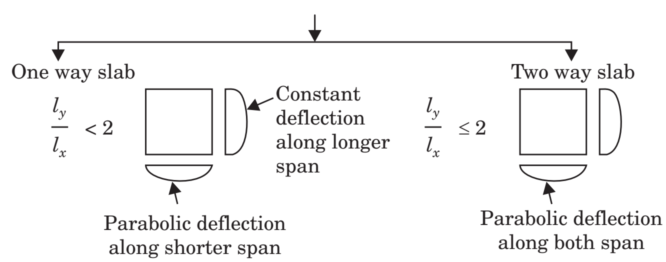
Shear Coefficients

Note:
If the span of two sides of support are different or the loading are different then support moment will be calculated from both sides and average will be taken.

- Dia of reinforcement bars generally available are 6, 8, 10, 12, 14, 16, 18, 20, 22, 25, 28, 30, 32, 36, 40 mm.
- Mild steel (Fe 250) is more ductile hence preferred for earthquake zones or where there are possibilities of vibration, impact, blast etc.

Slabs: Plate elements having depth much smaller than its other two dimensions.

They carry distributed load primarily by bending



Note:
Even if $\frac{l_y}{l_x} \leq 2$ but the slab is supported only on two parallel edges then it will be treated as one way slab.

Minimum reinforcement in slabs (in either direction)

(*i*) Fe 250 → 0.15% of total cross-sectional area.

(*ii*) Fe 415/Fe 500 → 0.12% of total cross-sectional area.

Maximum diameter of reinforcing bar $= \frac{1}{8} \times$ Depth of the slab(D)

Maximum Distance between Bars

Maximum distance between bars

- Main bar (bottom bar): S = min ($3d$, 300 mm)
- Secondary/distribution bar: S = min ($5d$, 450 mm)

Designs of One Way Slab

Given dimensions of room ($l_y \times l_x$), superimposed/dead load, f_y, f_{ck}

(*i*) Get 'd' from deflection criteria (Let it be d_{provided}).

(*ii*) Assume clear cover (20 mm) and dia of main bar (10 mm), get D.

(*iii*) $L_{\text{eff } x}$ and $L_{\text{eff } y}$ from end conditions.

(*iv*) Check for one way or two way slab. If one way slab then

(*v*) Get factored Bending moment, $M_u = 1.5 \frac{W l_{\text{eff}}^2}{8}$

where w = (dead load + super imposed load) per metre of **width**.

(*vi*) Check for d_{required}

$M_u \leq 0.36 f_{ck} x_{u\,lim} b (d - 0.42 x_{u\,lim})$ where b = 1000 mm

If $d_{\text{required}} < d_{\text{provided}}$ Adjust values of d_{provided} such that slab remains under reinforced

(*vii*) Get A_{st} from, $M_u = 0.87 f_y A_{st} (d_{\text{provided}} - 0.42 x_u)$

where $x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck}(1000)}$

(*viii*) Check for A_{st} min *i.e.* $A_{st} > A_{st}$ min

(*ix*) Spacing between bars $= \frac{1000}{\left(\frac{A_{st}}{\frac{\pi}{4}\phi^2}\right)}$

(*x*) Get number of bar's needed

(*xi*) Provide distribution bar similarly and check its spacing as per criteria.

(*xii*) Design for shear.

(*xiii*) Check for development length.

(*xiv*) Final diagram showing cross-section of slab and detail of bars and spacing in slab.

Design of Two way Slab

Design of two-way slab

- Corners not held down (simply supported case) Grassoff Rankine Method
  - (*i*) $M_x = \dfrac{W_x l_x^{\,2}}{8}$ $\quad My = \dfrac{W_y ly^2}{8}$
  - (*ii*) $V_x = \dfrac{1}{3} W_{lx}$ $\quad V_y = \left(\dfrac{r}{2+r}\right) W_{lx}$
- Corners held down
  - Pigeruos method
    - $M_x = r_x{}' \dfrac{Wl_x^2}{8}$
    - $M_y = r_y{}' \dfrac{wl_y^2}{8}$
  - I.S. code method
    - $M_x = \alpha_x W l_y^{\,2}$
    - $M_y = \alpha_y W l_x^{\,2}$

8 Column

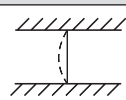
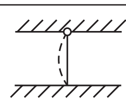
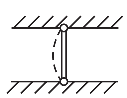
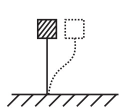
It is a compression member who's slenderness ratio is greater than 3. If slenderness ratio is less than 3 it is termed as pedestral.

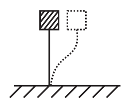
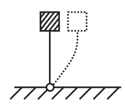
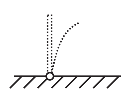
$$\text{Slenderness } (\lambda) \text{ ratio} = \frac{\text{Effective length}}{\text{Least lateral dimension}}$$

λ **< 12** Short column, fails under ultimate loads by crushing.

λ **> 12** Long column, fails due to large lateral deflection under relatively low compressive loads.

Effective length of column: Length between the points of contra-flexures of a buckled column.

Degree of end restraint of compression member (1)	**Symbol** (2)	**Theoretical value of effective length** (3)	**Recommended value of effective length** (4)
Effectively held in position and restrained		0.50 l	0.65 l
Effectively held in position at both ends, rotation at one end		0.70 l	0.80 l
Effectively held in position at both ends, but not restrained against rotation.		1.00 l	1.00 l
Effectively held in position and restrained against rotation at one end, and at other restrained against rotation but not held in position.		—	1.2 l

Effectively held in position and restrained against rotation in one end, and at the other partially restrained against rotation but not held in position		1.5 l	1.5 l
Effectively held in position at one end but not restrained against rotation, and at the other end restrained against rotation but not held in position.		2.00 l	2.00 l
Effectively held in position and restrained against rotation at one end but not held in position nor restrained against rotation at the other end.			

Minimum Eccentricity

Rectangular section

$$e_{x,\min} = \max \begin{cases} \dfrac{l_{ex}}{500} + \dfrac{D_x}{30} \\ 20\text{ mm} \end{cases}$$

$$e_{y,\min} = \max \begin{cases} \dfrac{l_{ey}}{500} + \dfrac{D_y}{30} \\ 20\text{ mm} \end{cases}$$

Non rectangular and non circular section (generally square)

$$e_{\min} = \max \begin{cases} \dfrac{l_e}{300} \\ 20\text{ mm} \end{cases}$$

Minimum longitudinal reinforcement

(*i*) **Column:** 0.8% of gross cross sectional area

(*ii*) **Pedestral with plain concrete columns:** 0.15% of gross sectional area

(*iii*) **Concrete walls:**

(*a*) In general, 0.15% of gross cross sectional area

(*b*) If welded wire fabric or deformed bars (Fe 415/Fe 500) then 0.12% of gross cross-sectional area ($\phi < 16$ mm)

(*c*) If wall thickness > 200 mm, two layers of vertical reinforcement needed.

(*d*) Spacing of bars = min (3*d*, 450 mm)

Maximum Longitudinal Reinforcement

- Its 6% of gross cross-sectional area of the column.
- Can even be reduced to 4% at lapped splice locations for better placement and compaction.

Other specifications

(*i*) Minimum diameter of longitudinal bar = 12 mm (but diameter should not exceed 12 mm for small sized column's *i.e.*, $D \leq 200$ mm)

(*ii*) Max centre to centre spacing of reinforcement = 300 mm

(*iii*) Number of bar's

(*a*) For rectangular columns = 4

(*b*) For circular columns = 6

(*iv*) Longitudinal bars should be placed close to periphery for better flexural resistance.

(*v*) Cover to reinforcement

(*a*) Minimum = 40 mm

(*b*) Can be reduced to 25 mm for small sized column

(*c*) Even in aggressive environment maximum cover is limited to 75 mm.

Lateral ties: Diameter of lateral ties is governed by criteria of stiffness not by strength. Hence, it is independent of grade of steel.

$$\text{Tie diameter } \phi_t \geq \begin{cases} \dfrac{\phi \text{ longitudinal max}}{4} \\ 6 \text{ mm} \end{cases}$$

$$\text{Tie spacing } S_t \leq \begin{cases} D \\ 16\,\phi \text{ longitudinal min} \\ 300 \text{ mm} \end{cases}$$

Design of short column

(*i*) Check for short column $\lambda < 12$

(*ii*) If $e_{min} \leq 0.05$ D then its short axially loaded column

(*iii*) For A_{SC}

(*a*) For short axially loaded column

$$P_u = 0.4 f_{CK} (A_g - A_{SC}) + 0.67 f_y A_{SC}$$

(*b*) For truly axially loaded column $e = 0$

$$P_u = 0.45 f_{CK} (A_g - A_{SC}) + 0.75 f_y A_{SC}$$

Note:
It can also be used when member is subjected to combined axial load and biaxial bending and also used when $e > 0.05$ D

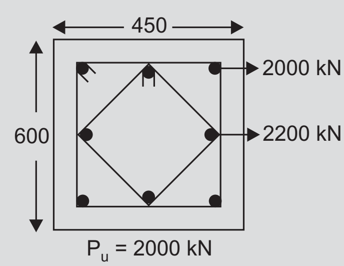
(*iv*) Provide A_{SC}, check for maximum and minimum reinforcement criteria, check for minimum diameter of longitudinal bar.

(*v*) Provide lateral ties check for diameter and spacing.

Note:
For general Idea

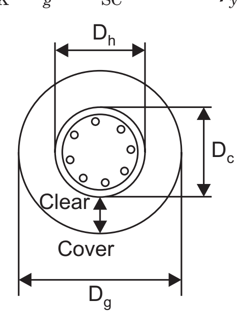
P_u	2000 kN	2200 kN
B × D	450 × 600 mm	500 × 600 mm
Bars	4 # 25 + 4 # 20 ϕ	8 # 25 ϕ

Lateral ties are placed in BARFI cut



Short axially loaded column with helical reinforcement

(*i*) $P_u = 1.05 (0.4 f_{CK} (A_g - A_{SC}) + 0.67 f_y A_{SC})$



(*ii*) Diameter of helical reinforcement should be selected such that

$$0.36\frac{f_{CK}}{f_y}\left(\frac{A_g}{A_C}-1\right) \le \frac{V_h}{V_C}$$

where,

A_g = gross cross-sectional area = $\frac{\pi}{4}(D_g)^2$

$D_c = D_g - 2d_c$

d_c = clear cover to tie distance

$$V_n = \frac{1000}{p} \times \pi D_n \times \frac{\pi}{4}\phi_h^2$$

D_n = diameter of helix

ϕ_h = diameter of helical reinforcement

p = pitch of helix

V_C = Volume of core in unit length of column

$V_C = 1000 \times A_C$

(*iii*) pitch < $\begin{cases} \frac{\text{core diameter}}{6} \\ 75\text{ mm} \end{cases}$ pitch > $\begin{cases} 3(\text{diameter of tie}) \\ 25\text{mm} \end{cases}$

Other Specifications on Column's Slenderness Limit

(*a*) Unsupported length between end restrains ≯ 60 times least lateral dimension

(*b*) If in any given plane one end of column is unrestrained then its unsupported length ≯ $\frac{100B^2}{D}$

Design of column by WSM

(*i*) For short column ($\lambda \le 12$)

$P_u = \sigma_{SC} A_{SC} + \sigma_{CC} A_C$

σ_{SC} = Stress in compression steel

$A_C = A_g - A_{SC}$

σ_{CC} = Stress in concrete

(*ii*) **For long column**

$Pu = C_r(\sigma_{SC} A_{SC} + \sigma_{CC} A_C)$

C_r = reduction factor

$\lambda > 12$ $\quad C_r = 1.25 - \frac{l_{eff}}{48b}$

l_{eff} = effective length

b = least lateral dimension

$\lambda > 50$ $\quad C_r = 1.25 - \dfrac{l_{eff}}{160 i_{min}}$

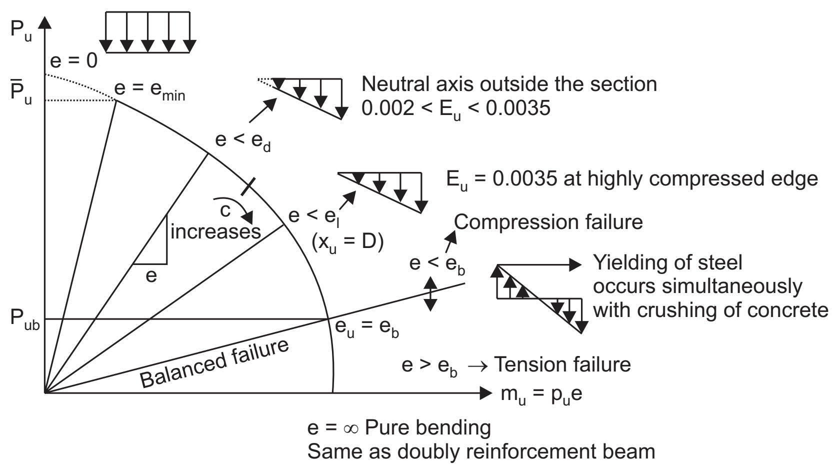
i_{min} = least radius of gyration.

$$i_{min} = \sqrt{\frac{I}{A}}$$

I = moment of inertia

A = cross-sectional area

Column Subjected to Axial Compression and Uniaxial Bending



9 Footing

Requirements of the Design of Foundation

(*i*) Foundation should sustain without exceeding safe bearing capacity of the soil.

(*ii*) Avoid differential settlement.

Depth of Foundation: Minimum 50 cm (IS 1080 – 1962)

As per Rankine's Formula

$$d = \frac{q_c}{\gamma}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2$$

d = minimum depth of foundation

q_c = gross bearing capacity of soil

γ = density of soil

ϕ = Angle of response of soil

Design Considerations

(*i*) Minimum Nominal Cover = 50 mm

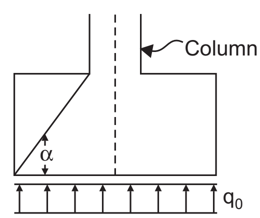
(*ii*) Minimum Thickness at the edge of Footing

(*a*) If footing rests on soil = 150 mm

(*b*) If footing rests on top of piles = 300 mm

(*c*) For plain Concrete pedestral footing without any longitudinal tension steel.

$$\tan\alpha \nleq 0.9\sqrt{\frac{100\,q_o}{f_{ck}}+1}$$



q_o = Calculated maximum bearing pressure at the base of pedestral.

f_{ck} = Characteristic strength of concrete at 28 days in N/mm^2

(*iii*) Critical section of maximum bending moment.

(*a*) At the face of the column, pedestral or wall for footing supporting a concrete column, pedestral or reinforced concrete wall.

(*b*) Halfway between the centre-line and edge of the wall for footing under masonary wall.

(*iv*) Critical section for one way shear

Footing slab rests on	distance from face of column, pedestral or wall
Soil	d
Piles	$d/2$

(*v*) Critical section for two way shear → at a distance $d/2$ around the column on a perimeter.

Permissible shear stress when reinforcement is not provided should be less than $K_S\ \tau_C$ where

$$K_S = (0.5 + \beta_C) < 1,$$

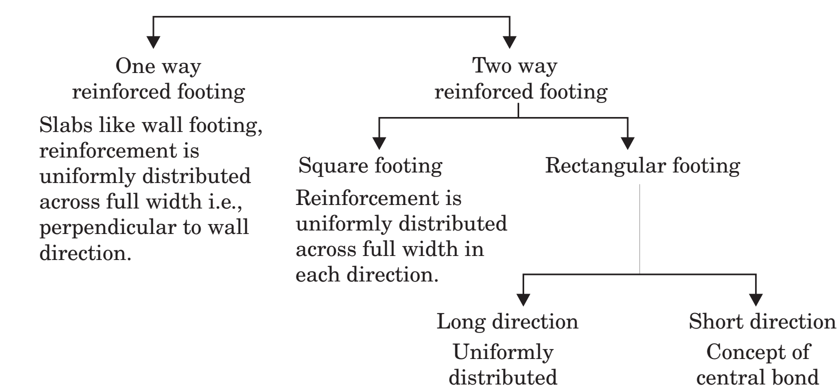
$$\tau_C = 0.25\sqrt{f_{CK}}$$

$$\beta_C = \frac{\text{Short side of column}}{\text{Long side of column}}$$

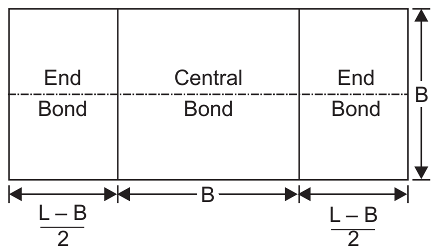
Note:
Generally, thickness of slab is governed by shear, for WSM its $K_S \times 0.16\ \sqrt{f_{CK}}$

(*vi*) Critical section for checking development length → in a footing slab be the same planes as those of bending moments.

(*vii*) Tensile reinforcement



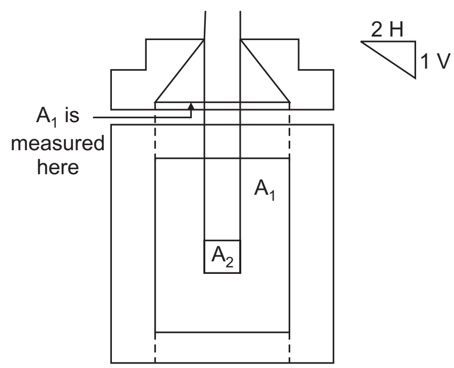
Concept of Central band



Reinforcement in central band = $\left(\dfrac{2}{\beta+1}\right)$ (Total reinforcement in short direction)

where $$\beta = \frac{\text{Longer dimension of footing}}{\text{Shorter dimension of footing}}$$

(*viii*) **Transfer of load at the base of column:** Compression forces are transferred through direct bearing while tension forces are transferred through developed reinforcement.



Permissible bearing stresses on full area of concrete

(*a*) WSM $\sigma_{br} = 0.25\, f_{CK}$

(*b*) LSM $\sigma_{br} = 0.45\, f_{CK}$

$$\sigma_{br} = 0.45\, f_{CK}\sqrt{\frac{A_1}{A_2}} \quad \text{where} \quad \sqrt{\frac{A_1}{A_2}} \le 2$$

A_1 = maximum supporting area of footing for bearing which is geometrically similar to and concentric with loaded area A_2

A_2 = loaded area at the base of the column.

(*xi*) Dowel bars

(*a*) Minimum number needed = 4

(*b*) diameter of dowel < diameter of column bar + 3 mm

(*c*) minimum area of dowel = 0.5% of gross cross-sectional area of supported column.

(*d*) column bars of diameter larger than 36 mm, in compression only can be dowelled at the footing with bars of smaller size of the necessary area.

Design of Footing

Given: load (p_u), Bearing capacity of soil (q_u), column size, f_{CK}, f_y

(*i*) Size of foundation

Column load = P, Foundation load (P_F) = 0.1 P

Total load P_T = P + 0.1 P = 1.1 P (even for LSM)

$$\text{Area required} = \text{Area} = \frac{P_T}{q_u}$$

Choose L and B such that $A_{provided} > A_{required}$

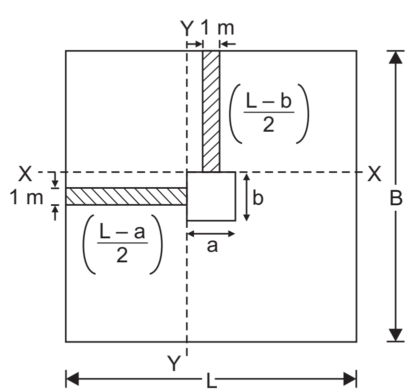
$$\text{Net soil pressure, } w = \frac{P}{A} = \frac{\text{Column load}}{\text{Area provided}} \quad \text{(WSM)}$$

$$w = \frac{1.5\,P}{A} \quad \text{(LSM)}$$

(*ii*) Bending moment

$$M_x = \frac{w(B-b)^2}{8}$$

$$M_y = \frac{w(L-a)^2}{8}$$



Note:
For LSM its 1.5 w

(*iii*) Get required depth '*d*' (put b = 1000 mm)

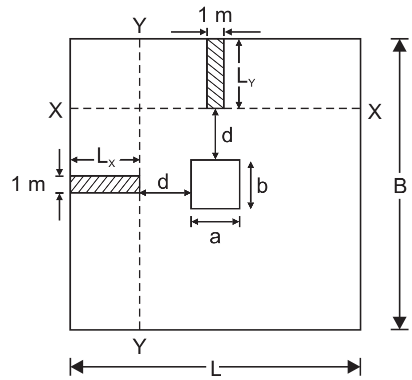
$$M_{max} = Q\,bd^2 \quad \text{(WSM)}$$
$$M_{u\,max} = Q\,bd^2 \quad \text{(WSM)}$$

(*iv*) One way shear check

$$V_x = W.1 \times L_y = W\left[\left(\frac{B-b}{2}\right) - d\right]$$

$$V_y = W.1 \times L_x = W\left[\left(\frac{L-a}{2}\right) - d\right]$$

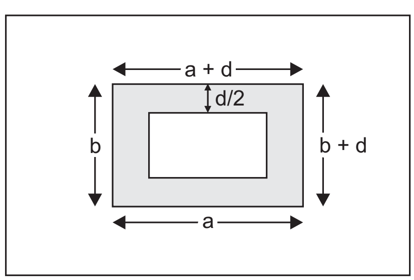
$$V_{max} = \max(V_x, V_y)$$



Nominal shear stress $\tau_V = \dfrac{V_{max}}{Bd}$ $\qquad$ τ_C is taken from IS 456

(*v*) Two way shear check

$$\text{Punching stress developed} = \frac{\text{Net punching force}}{\text{Cross-sectional resisting area}}$$



Net punching force, $P_{net} = P - W(a + d)(b + d)$

Cross-sectional resisting area = $2((a + d) + (b + d)) \times d$

Punching stress developed < Permissible punching stress

(*vi*) Area of steel for longer span

$$M_y = A_{st}\,(\sigma_{st}\,jd) \text{ WSM}$$

$$M_{uy} = 0.87\,f_y\,A_{st}\,(d - 0.42\,x_u) \text{ LSM}$$

This reinforcement is equally distributed over entire width B. Here area of steel is calculated for 1 m width. Calculate for width B, then distribute uniformly

(*vii*) Area of steel for shorter span

$$M_x = \sigma_{st}\,A_{st}\,jd, \text{ WSM}$$

$$M_{ux} = 0.87\,f_y\,A_{st}\,(d - 0.42\,x_u), \text{ LSM}$$

This A_{st} is provided in the central band width B.

Then find total reinforcement in short direction **By central band concept formulae**.

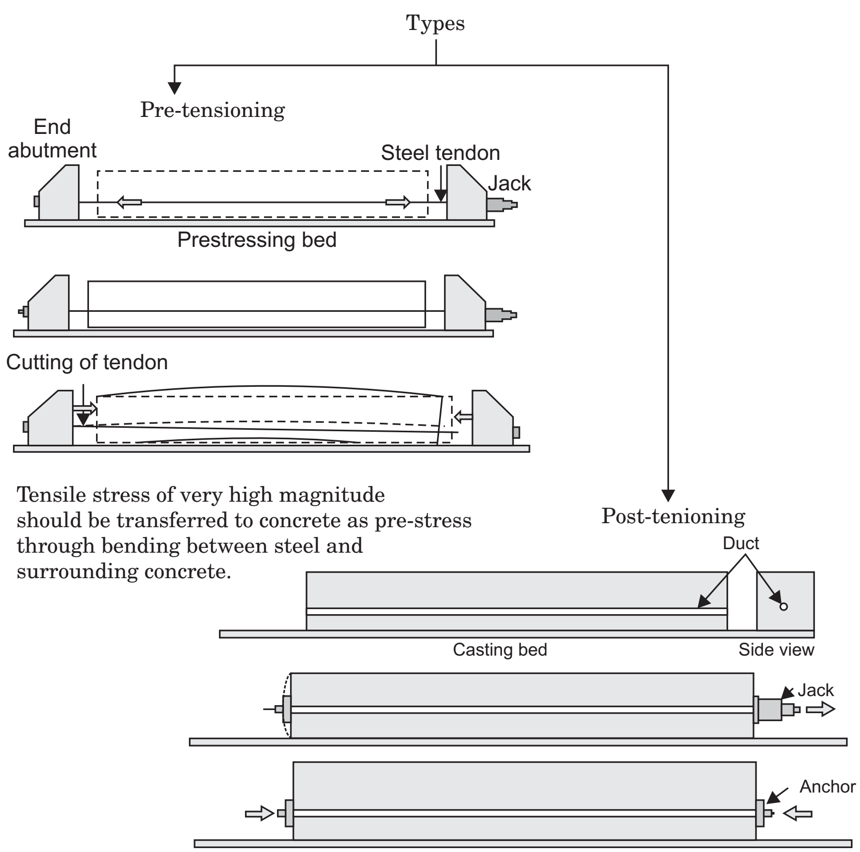
Reinforcement in each end bands

$$= \frac{\text{Total reinforcement} - A_{st} \text{ in central band}}{2}$$

10 Pre-Stressed Concrete

A concrete in which internal stress of suitable magnitude and distribution are introduced so that stresses resulting from external load are counteracted to a desired degree.

Note:
In pipes or liquid storage tanks the hoop tensile stresses can be effectively counteracted by circular prestressing.



Transfer of stress is through bearing at end sector.

- By using high strength concrete, loss of prestress can be reduced.
- High strength concrete results in reduction of cross-sectional dimensions Hence, ultimately reduced dead weight.

Note:
Normally, due to creep and shrinkage loss in strain is approximately 0.0008 Hence, stress loss is $0.0008 \times 2 \times 10^5 = 160$ N/mm^2. Hence, High strength steel is used

- **Advantages:** Full section is utilised as section remains uncraked at service load, shear resistance capacity is increased, high span/depth ratio is possible, increases speed of construction.

  Limitations: Skilled technology, high cost, better quality control

Types if pre-stressing

- Source of force
  - Hydraulic
  - Mechanical
  - Electrical
  - Chemical
- Place of force application
  - External (bridges)
  - Internal (Sleepers)
- Time of force application
  - Pre tensioning
  - Post tensioning
- Profile of cable tendon
  - Linear (Beams, Poles)
  - Circular (Tanks, silo's)
- Direction of cable tendons
  - Uniaxial
  - Biaxial (sides)
  - Multi axial (dome)

As per IS: 1343 -1980

- **Type 1 Full-prestressing**
  - No tensile stress in concrete under service loads.
- **Type 2 Limited Prestressing**
  - Tensile stress under service loads is within the cracking stress of concrete.
- **Type 3 Partial Prestressing**
  - Tensile stress due to service loads, the crack width remains within allowable limit.

	Pre-tensioning	Post-tensioning
Stages	Anchoring of tendons, placing jacks, applying tension, casting concrete then cutting of tendons.	Casting of concrete, placing tendons, placement of anchorage block and jack, applying tension to tendons, seating wedges then cutting of tendons

Devices Used	Prestressing bed, End Abutments, Shuttering/mould, Jack, Anchoring device, Harping device	Casting bed, mould/ Shuttering, Ducts, Anchoring devices, Jacks.
Advantages	• Suitable for Bulk production • Large Anchorge device not needed	• Heavy cast in place members can be easily post- tensioned • Less waiting period in casting bed. • Transfer of prestress is independent of length.
Disadvan -tages	• Prestressing bed required • Good bond necessary between transmission length.	• Requirement of anchorage device and grouting equipment.

Post-tensioning system

System (Country)	Type of tendon	Range of force	Arrange-ment of tendons in duct	Type of Anchorage
Freyssinet (**F**rance)	Wires and stands	Medium Large	Annular, spaced by helical wire core.	Concrete wedge
Gifford-up all-ccl (**G**reat Britain)	Wires	Small medium	Evenly spaced by perforated spacers	Split Conical wedge
Lee-mc-**call** (Great Britain)	Bar threaded at ends	Small medium large	Single bars	High strength nut
Magnel-**Blaton** (**B**elgium)	wires	Small medium large	Horizontal rows of four wires spaced by metal griller	Flat steel wedge in sandwich plates

Note :- Great Britain has 'all'.

11 Analysis of Pre-stress and Bending Stresses

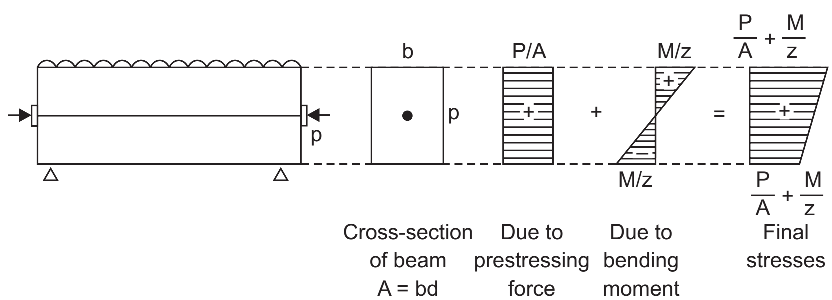
Assumptions:

1. Concrete is homogenous elastic material.
2. Withing working stresses, Hooke's law is valid.
3. Plane section before bending remains plane after bending.
4. Stress variation in steel due to external load is negligible.
5. Stress in reinforcement does not change along the length of member.

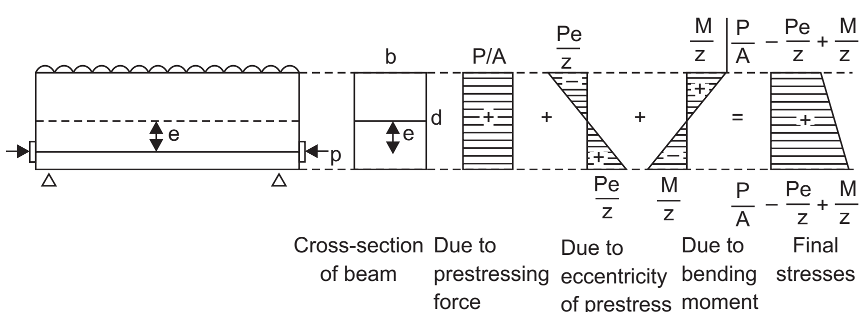
Analysis of Members Under Flexure

1. **Stress concept**

(*a*) Concentric tendon



(*b*) Eccentric tendon



Note:
The bending moment at which visible cracks develop in prestressed concrete members is called cracking moment. The tensile stresses developed when crack become visible at the soffit of beams depend upon the type and distribution of steel and quality of concrete in the beam.

Load factor against cracking

$$= \frac{\text{Live load required for cracking}}{\text{Live load actually acting}}$$

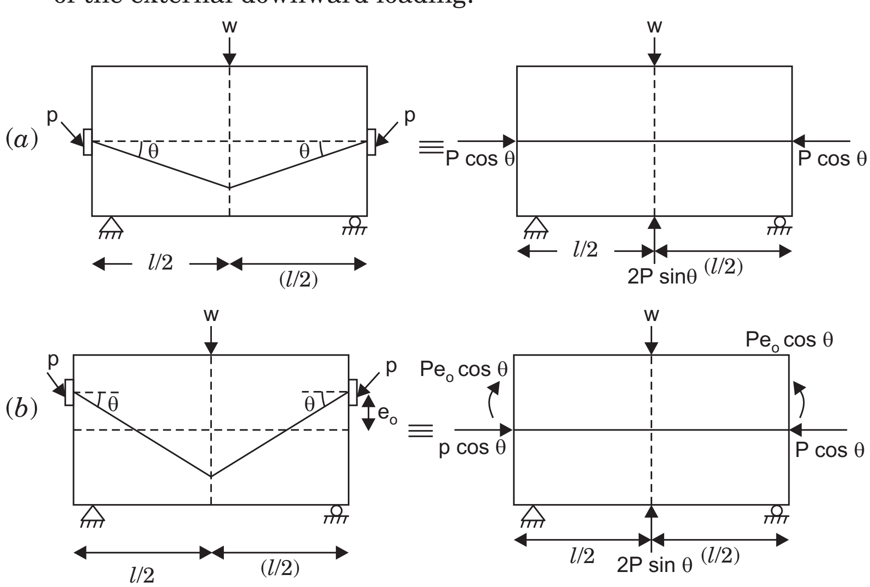
$$= \frac{\text{Live load moment required for cracking}}{\text{Actual live load moment}}$$

Load factor against cracking with respect to total load

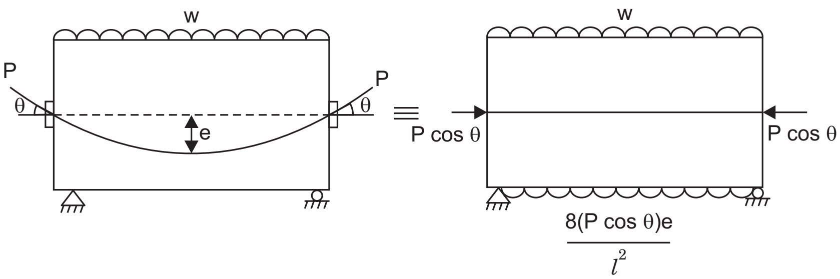
$$= \frac{\text{Total load causing cracking}}{\text{Total load actually acting}}$$

$$= \frac{\text{Total Bending moment required to cause cracking}}{\text{Total Bending moment actually acting}}$$

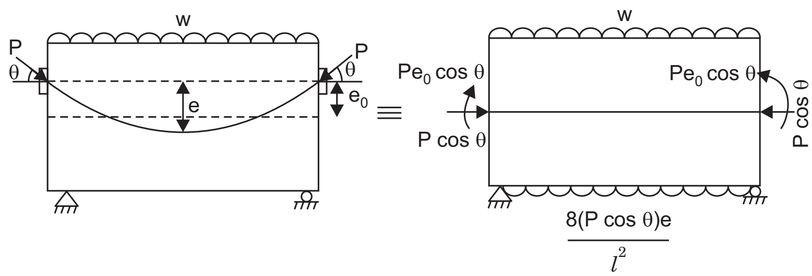
2. **Load Balancing Concept:** Bent tendon exerts an upward pressure on the concrete beam and will therefore counteract a part or whole of the external downward loading.



(c)



(d)



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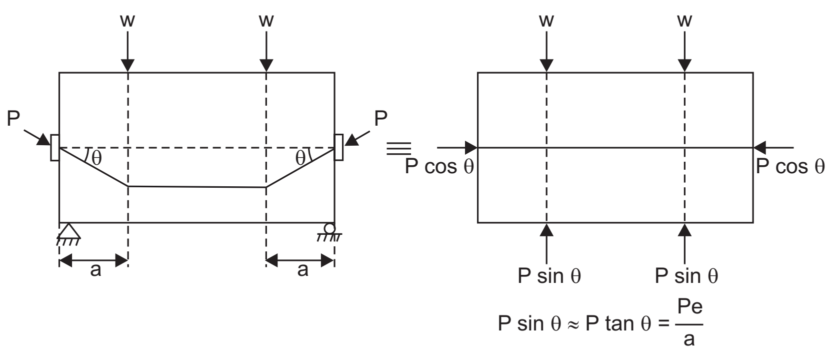
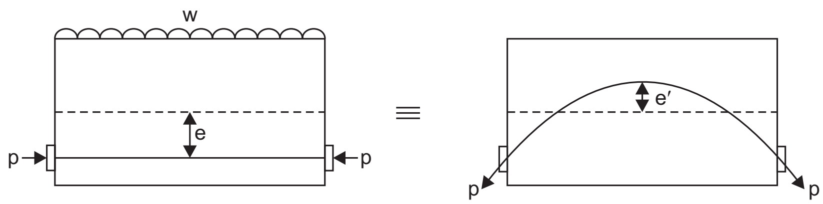


Note:

Equation of parabolic profile is $y = \dfrac{4ex(l - x)}{l^2}$

Type of load	Bending moment diagram	Cable profile
Point load	Triangular	Triangular
udl	Parabolic	Parabolic
Two point load	Trapezoidal	Trapezoidal

3. **Thrust line or pressure line concept:** The combined effect of prestressing force (P/A + P*e*/Z) and externally applied load (M/Z) will result in a distribution of concrete stresses that can be revolved into a single force and the locus of point of application of resultant force is called pressure or thrust line.



$$\sigma_{b/t} = \frac{P}{A} \pm \frac{Pe}{Z} \mp \frac{M}{Z} \qquad \sigma_{b/t} = \frac{P}{A} \mp \frac{Pe'}{Z}$$

$$\downarrow \qquad\qquad \uparrow$$

$$\sigma_{b/t} = \frac{P}{A} \pm \frac{Pe}{z} \mp \frac{P}{Z}\left(\frac{M}{P}\right) \rightarrow \sigma_{b/t} = \frac{P}{A} \mp \frac{P}{Z}\left(-e + \frac{M}{P}\right)$$

So, $e' = -e + \frac{M}{P} \Rightarrow e + e' = \frac{M}{P}$

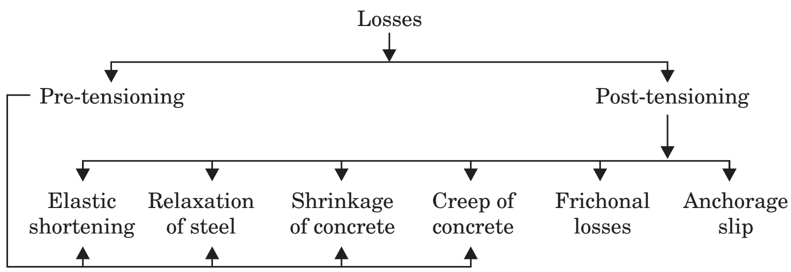
Note:
As P is a constant force, hence if

e	**M**	**then *e′***
Constant	Parabolic	Parabolic
Constant	Linear	Linear
Parabolic	Parabolic	Parabolic if $\frac{M}{P} \neq e$ zero if $\frac{M}{P} = e$

- In prestressed section load carrying mechanism consist of constant force (F) and changing lever arm while in reinforced sections it consists of changing force with a constant lever arm.

12 Losses in Pre-stress

Various reductions of prestressing force are termed as losses in prestress.



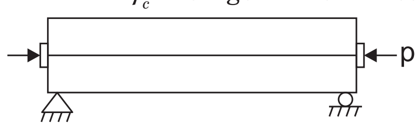
- Elastic shortening, frictional losses, Anchorage slip are short term losses where as other losses are time dependent.
- In Post-tensioning there is no loss of elastic shortening when all bars are simultaneously tensioned.
- There may be losses due to sudden change in temperature, especially in steam curing of pretensioned units.
- Total loss in Pretensioning is more than post tensioning.

1. Elastic Shortening Loss

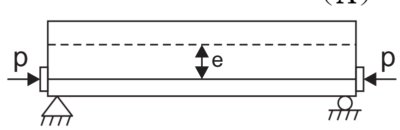
(*a*) Pre-tensioned members, Pre-stressing loss = $m \times f_{c\ \text{average}}$

where m = modular ratio

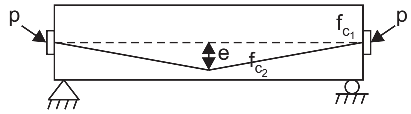
f_c average = stress in concrete at the level of tendon

(*i*) 

$$\Delta f_s = m\left(\frac{P}{A}\right) \text{ as } e = 0$$

(*ii*) 

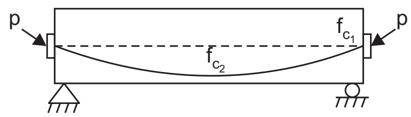
$$\Delta f_s = m\left[\frac{P}{A} + \frac{Pe.e}{I} - \frac{M_d e}{I}\right]$$

(*iii*) 

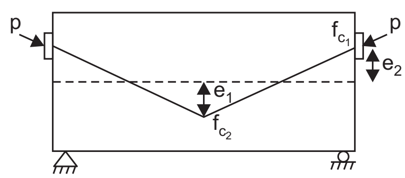
$$\Delta f_s = m\left(\frac{f_{c_1} + f_{c_2}}{2}\right)$$

$$f_{c_1} = \frac{P}{A}$$

$$f_{c_2} = \frac{P}{A} + \frac{Pe.e}{I} - \frac{M_d e}{I}$$

(*iv*) 

$$\Delta f_s = m\left[f_{c_1} + \frac{2}{3}(f_{c_2} - f_{c_1})\right]$$

(*v*) 

$$\Delta f_s = m\left[\frac{f_{c_1} + f_{c_2}}{2}\right]$$

$$f_{c_1} = \frac{P}{A} - \frac{P_{e_o}.e_o}{I}$$

$$f_{c_2} = \frac{P}{A} + \frac{P_{e_1}\cdot e_1}{I} - \frac{M_d e_1}{I}$$

(*b*) **Post-tensioned members:** In case of single tendon, there is no loss as the applied prestress is recorded after the elastic shortening of the member. However, when bars are successively tensioned and anchored then losses will occur as

(*i*) when first bar tensioned, no loss

(*ii*) when second bar tensioned, no loss in 2nd but loss in 1st bar

(*iii*) when third bar tensioned, no loss in 3rd but loss in 1st and 2nd bar,

The same concept of averaging will be used when bar's are not stretched simultaneously.

2. **Relaxation of steel:** Decrease in stress with time under **constant strain**. It depends on type of steel, temperature and initial prestress, normally, it is taken as **2 to 5%** of initial stress in steel.

3. **Shrinkage of concrete**

$$\text{Loss of stress} = \varepsilon_{SS} \times E_S$$

$$\varepsilon_{SS} = \text{Total residual shrinkage strain}$$

$$= 3 \times 10^{-4} \text{ for pretensioning,}$$

$$= \frac{2 \times 10^{-4}}{\log_{10}(t+2)} \text{ for post tensioning,}$$

t = age of concrete at transfer in **days.**

E_S = Modulus of elasticity of steel.

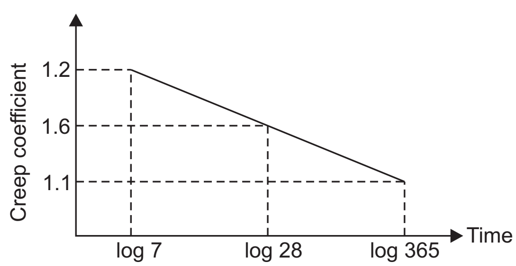
4. **Creep of concrete:**

$$\text{Loss of stress} = m\,\phi\,f_c$$

$$\phi = \text{creep-coefficient} = \frac{\text{ultimate creep strain}}{\text{elastic strain}}$$

$$m = \frac{E_S}{E_C}$$

Creep losses are generally 2–3% of initial prestressing force.

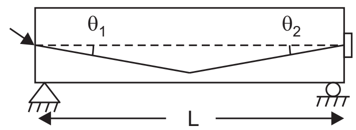
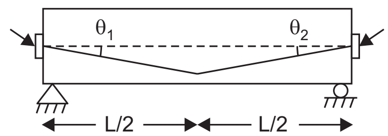


5. **Frictional losses:**

Loss of stress = Initial stress ($\mu\alpha + Kx$)

μ = Coefficient of friction in curve (Generally between 0.25 to 0.55)

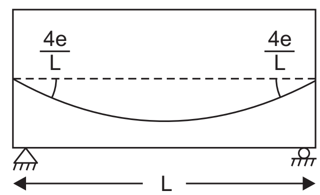
K = Wooble correction factor (0.0015 to 0.0050 per metre length of tendon)

		x	α
Jacking from one end		L	$\theta_1 + \theta_2$
Jacking from both ends		$\frac{L}{2}$	$\max(\theta_1, \theta_2)$

For parabolic profile,

Jacking at one end, $\alpha = 2\theta = \frac{8e}{L}$

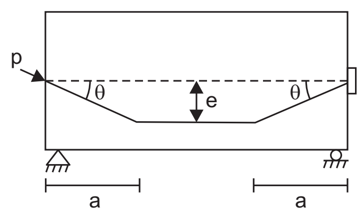
Jacking from both ends $\alpha = \theta = \frac{4e}{L}$



- For trapezoidal profile

Jacking at one end $\alpha = 2\theta = \frac{2e}{a}$

Jacking from both ends $\alpha = \theta = \frac{e}{a}$



Note:
In Pre tensioned members, as there is no concrete during stretching of tendons hence this loss doesn't occur.
In post tensioned members, frictional loss is generated due to **curvature of tendon** and **vertical component of prestressing force**.

6. Anchorage slip:

$$\text{Loss of stress} = \left(\frac{\Delta}{L}\right)E_S$$

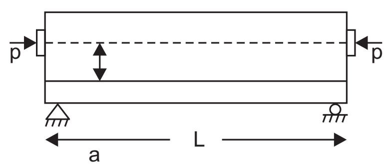
E_S = young modulus of steel in N/mm^2
Δ = Anchorage slip in mm
L = Length of cable in mm

Note:
The percentage loss due to anchorage slip is higher for shorter members as compared to longer members.

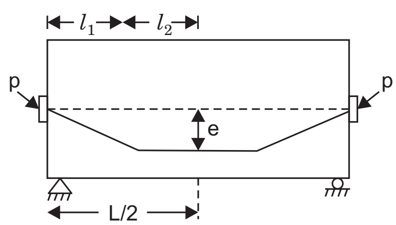
13 Deflection of Pre-stressed Beam

Short term deflection under **uncracked condition** can be computed using elastic theory by using **area moment method** (Mohr's method). Concrete beam deflects **upwards** on the application or transfer of prestress.

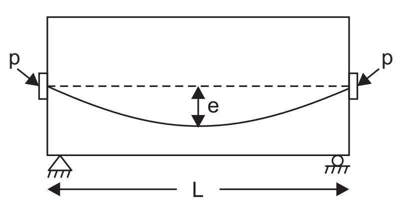
Bending moment at any section is the product of prestressing force and eccentricity at that section.



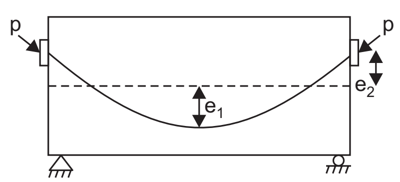
$$\Delta = \frac{PeL^2}{8EI}$$



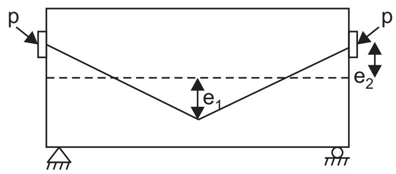
$$\Delta = \frac{-Pe}{6EI}\left(2l_1^2 + 6l_1l_2 + 3l_2^2\right)$$



$$\Delta = \frac{-5}{48}\frac{PeL^2}{EI}$$



$$\Delta = \left[\frac{-5}{48}\frac{PL^2}{EI}(e_1 + e_2)\right] + \left[\frac{Pe_2L^2}{8EI}\right]$$



$$\Delta = \left[\frac{-PL^2}{12EI}(e_1 + e_2)\right] + \left[\frac{Pe_2L^2}{8EI}\right]$$

Note:

Downward deflection due to self wt or imposed load is $\frac{5w}{384}\frac{L^4}{EI}$

14 IS Code Recommendations for Pre-stressed Concrete

(*i*) Grade of concrete
M 40 for pre tensioned members
M 30 for post tensioned members

(*ii*) **Design mix:** Only 'design mix concrete' can be used with cement content preferably less than 530 kg/m^3

(*iii*) Flexure tensile strength $F_{cr} = 0.7\sqrt{f_{ck}}$

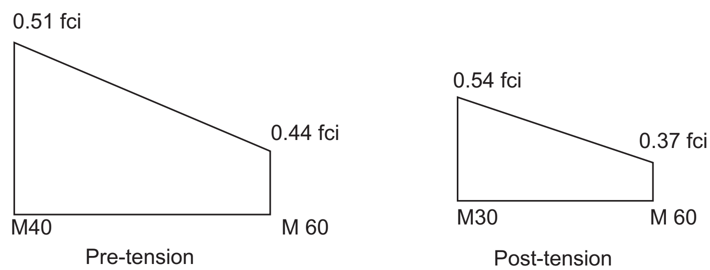
(*iv*) Short term modulus of elasticity $E_c = 5000\sqrt{f_{ck}}$

(*v*) Modulus of elasticity of steel

Type of steel	E_s (kN/mm^2)
Plain cold drawn wires	210
High tension steel bars rolled or heat treated	200
Strands	195

(*vi*) Allowable stresses in concrete

(*a*) Allowable compressive stresses under flexure



where fci = cube strength at transfer

(*b*) **Allowable compressive stresses under direct compression:** 80 % of the compressive stress under flexure (*c*) Allowable tensile stresses under flexure

Type 1	Number tensile Stress
Type 2	3N/mm^2 can be increased to 4.5 N/mm2 for temporary loads
Type 3	Table 8 of IS 1373 : 1980 provides hypothetical values

(*d*) **Maximum Initial Prestress:** Should be such that, maximum tensile stress immediately behind the anchorage should not exceed 80% of the ultimate tensile strength of wire

(*vii*) **Minimum cover:**

Pre tensioned work	200 mm
Post tensioned work	max (30 mm, size of cable)

Note:
For pre tensioned work in aggressive environment cover shall be increased by 10 mm.

(*viii*) Spacing of tendons

(*a*) For single wires = max $\left(3d, \frac{4}{3} \text{(max aggregate size)}\right)$
where d = dia of wire

(*b*) For cable or large bars = max (40 mm, max size of cable, 5 mm + max aggregate size)

(*c*) For grouped cables

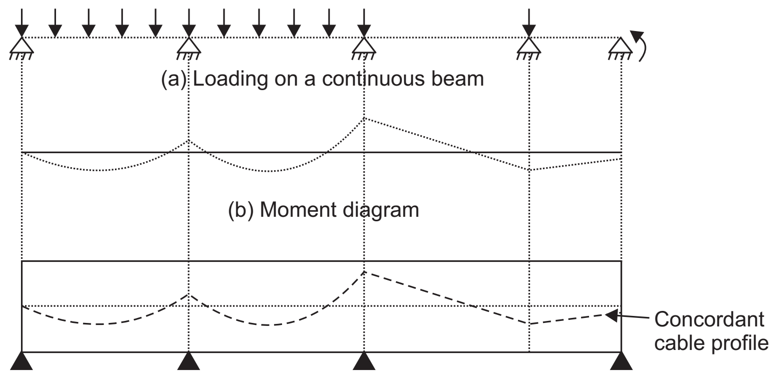
(*i*) Minimum horizontal spacing = max (40 mm, 5 mm + max aggregate size)

(*ii*) Minimum vertical spacing = 50 mm.

(*ix*) maximum deflection allowable

(*a*) Final deflection due to loads including effect of temp, shrinkage, creep	$\frac{\text{span}}{250}$
(*b*) Deflection including effect of temp. shrinkage, creep accuring after partition erection and application of finishes	max $\left(20 \text{ mm}, \frac{\text{span}}{250}\right)$

(*x*) **Concordant cable profile:** For continuous beams prestressing generates reactions at the support. These reactions cause additional moments along the length of beam (secondary moments). If the profile of cable is properly selected such that it does not produces reactions at the support or secondary moments in the span then its called concordant cable profile.



(c) Concordant profile