FORCE METHOD OF ANALYSIS

Statically indeterminate structure are always choose over statically determinate structures.

Internal stresses are nduced in indeterminate structures due to

(a) Temperature variation

(b) Differential settlement of support

(c) Change in length due to fabrication error.

In statically indeterminate structures, following conditions are to be satisfied: (a) Force displacement relationship

(b) Equilibrium equation (c) Compatibility equation

	Force Method	Displacement Method
(i)	Also known as compatibility method, method of consistent deformation, flexibility method	Also known as Equilibrium method/Stiffness method
(ii)	Forces (BM, SF) are taken as unknown	Diplacement (Δ,θ) are taken as unknown
(iii)	Compatibility equations are used to find redundants	Equilibrium equations are use to find redundants.
(iv)	BM, SF are found using equilibrium equations	Δ, θ are found using load displacement equation
(v)	Used when $D_S < D_K$	Used when $D_S > D_K$
(vi)	Other force methods are: Virtual work method, Strain energy method, Castigliano's theorem, claperon's three moment theorem (continuous beams) Column analogy method (rigid frame with fixed support), flexibility matrix method.	Other methods are moment distribution, Kani's method, slope deflection, stiffness matrix method.

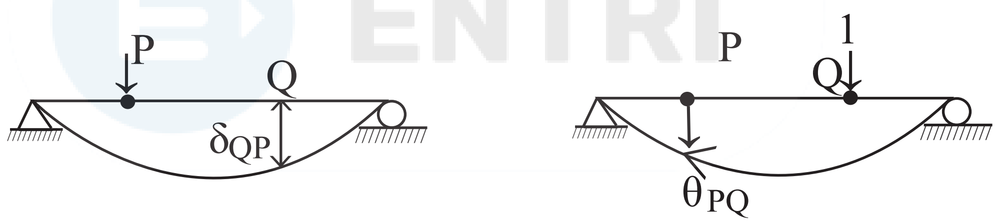
Principal of Superposition:

1. Hooke's law should be valid. 2. Small deformations.

The total displacement or internal loadng at a point in a structure subjected to various external loading can be determined by adding together the displacements or internal loading caused by each of the external loads acting seperately for application of superposition principal.

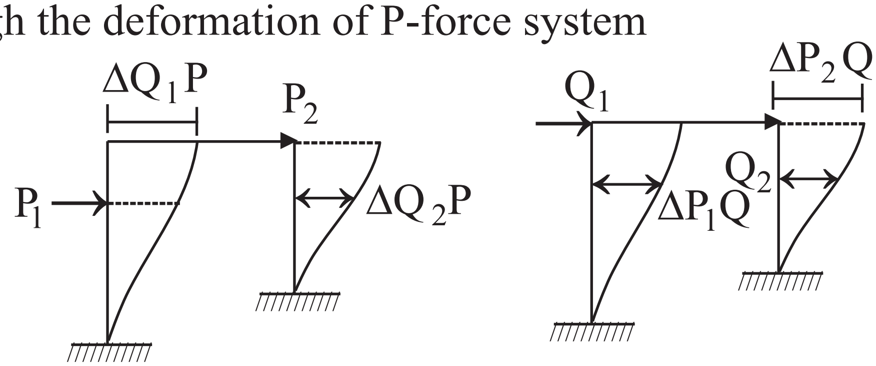
Various Force Methods

1. **Method of Consistent Deformation:** Beam is assumed to be composed of:
   (a) Primary structure which is obtained by removing the redundants & loading the resulting beam with external loading only.
   (b) Secondary structure which contains loading of redundant reaction only.
   (c) Redundant's are found out. Then writing the compatibility conditions.
2. **Maxwell's reciprocal theorem:** It is a special cases of Bettis law. If only two force P & Q are acting & magnitude of P and Q are unity, then $\boxed{\Delta_{PQ} = \Delta_{QP}}$ where, Δ_{QP} = Deflection at Q due to unit load at P, Δ_{PQ} = Deflection at P due to unit load at Q



$$\boxed{\delta_{QO} = \theta_{PQ}}$$

3. **Betti's Theorem:** In it, the virtual work done by a P-force system in going through deformation of Q - Force system is equal to the virtual work done by the Q-force system in going system in going through the deformation of P-force system



P-system of forces　　Q-system of forces

$$\boxed{P_1 \Delta P_1 Q + P_2 \Delta P_2 Q = Q_1 \Delta Q_1 P + Q_2 \Delta Q_2 P}$$

4. Castigliano's Theorems:

Castigliano's 1st theorem	Castigliano's 2nd thoerem
(a) The first partial derivative of total internal energy (strain energy) in a structure with respect to any particular deflection component at a point is equal to the force applied at that point & in the direction corresponding to the deflection component. $\frac{\partial U}{\partial \delta} = P$ or $\frac{\partial U}{\partial \theta} = M$	(a) The first partial derivative of total internal energy in a structure with respect to the force applied at any point is equal to the deflection at the point of application of that force in the direction of its line of action $\frac{\partial U}{\partial P} = \delta$ or $\frac{\partial U}{\partial M} = \theta$
(b) Castigliano's 1st theorem is applicable to linearly or non-linearly elastic structures in which the temperature is constant & the supports are unyielding.	(b) Castigliano's 2nd theorem is applicable to linearly elastic (Hookean material structures with constant temperature & unyielding supports

Self-straining is caused due to settlement of support of redundant structure (λ) or by any initial misfit of a member by an amount λ too short or too long.

$$\frac{\partial U}{\partial R} = \lambda,$$

λ = Small displacement in the direction of Redundant force R.

5. Theorem of least work: For any statically indeterminate structure, the redundant should be such as to make the total energy within the structure a minimum. This theorem is a special case of castigliano's 2nd theorem.

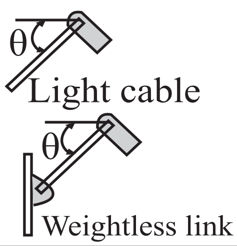
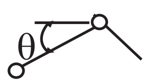
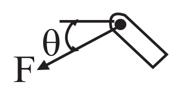
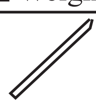
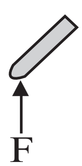
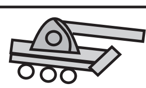
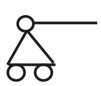
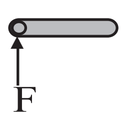
DETERMINACY, INDETERMINACY & STABILITY OF STRUCTURE

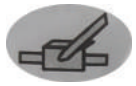
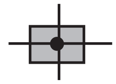
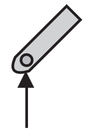
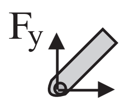
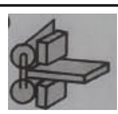
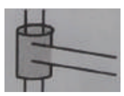
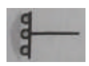
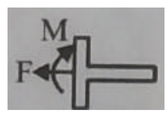
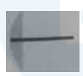
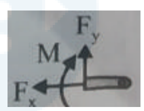
* The three types of joint most commonly are the pin connection, the roller support & the fixed support.

* Pin connected joint or hinge gives two reactions, one against vertical movement & another against horizontal movement (R_x & R_y) but offers no resistance to the angular rotation of the beam at the hinge.

* A pin connected joint & a roller support allow some freedom for slight rotation, but fixed joint allows no relative rotation between the connected members & is consequently more expensive to fabrication.

* Roller support gives only one reaction acts perpendicular to the surface of the point of contact & offers no resistance to the angular rotation of the beam at the roller support, also no resistance to in-plane lateral movement.

Type of Connection	Idealized Symbol	Reaction of Constraints	Number of unknowns/ constraints.
(1) 			**One Unknown.** The reaction is a force that acts in the direction of the cable or link.
(2)  smooth contacting surface			**One unKnown.** The reaction is a force that acts perpendicular to the surface at the point of contact.
(3)  Rollers  Rocker	 		**One Unknown.** The reaction is a force that acts perpendicular to the surface at the point of contact.

No.	Support		Reaction	Unknowns
(4)	 Smooth pin-connected collar			**One Unknown.** The reaction is a force that acts perpendicular to the surface at the point of contact.
(5)	 Smooth pin or hinge			**Two Unknowns.** The reaction are two force components.
(6)	 Slider  Fixed connected collar			**Two Unknowns.** The reaction are a force & a moment.
(7)				**Three Unknowns.** The reaction are the moment & the two force.

Determinacy and Indeterminacy

Stability :

Classification
- External → Related to support condition
- Internal → Related to geometry of the structure

Conditions for external stability in 2D structure :

In 2D structure 3 reactions are available at support.

These reactions on the support should be.

(a) Non-Parallel, (b) Non-Trivial, (c) Non-Concurrent

External Stability: If a body is sufficient constraint by external reaction such that rigid body movement of structure does not occur, then the structure is stable externally.

Necessary Conditions for External Stability

1. There should be three reactions that are neither concurrent nor parallel (in plane structure).
2. Reactions should be non-concurrent, non-parallel & non-coplaner for space structure.

Internal Stability: When part of the structure moves appreciably with respect to the other part, the structure is to be unstable internally.

In a plane structure, to ensure external stability.

In 3D structure 6 reactions are available at fixed supports & all these 6 reactions should follow the same condition mentioned above.

Internal Stability:

In rigid structures internal instability may occur due to formation of mechanism. (3 hinges colliness)

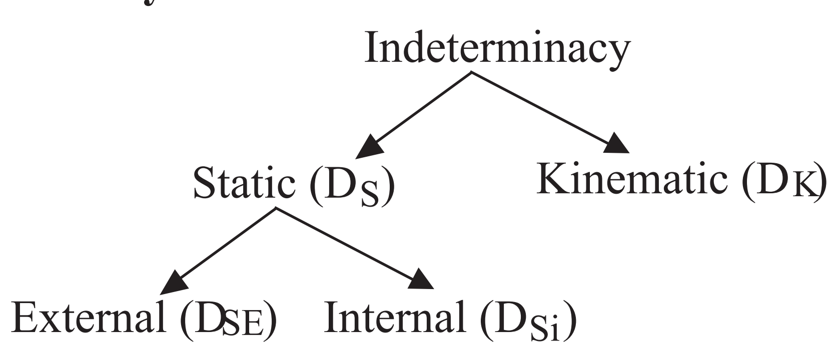
In trusses internal unstability occurs due to deficiency of member & their arrangement.

In trusses for internal stability of 2D truss min no. of members required.

$m \geq 2j - 3$ $\qquad$ j = No. of Joints

* If above condition is satisfied then triangle also formed at each part of truss to ensure internal stability.
* If any phologonal block is open in a structure then it is internally unstable.
* Overall Stability: For overall stability of a structure external stability in must.

Static Interminancy:



So, $\boxed{D_S = D_{SE} + D_{Si}}$

No. of additional reactions required to analyse a structure is called static indeterminancy.

* Total static determinacy: $D_s = D_{se} + D_{si}$
* D_s for 2D truss: $m + r_e - 2j$
* D_s for 3D truss: $m + r_e - 3j$
* D_s for 2D rigid frames: $D_s = 3C - r'$

  Where, C - no. of cuts required to produce open stable tree like structure

  r' - no. of restraints added to make structure perfectly rigid.
* D_s for 3D rigid frames: $D_s = 6C - r'$

Special Point:

D_s for beam, $D_s = D_{se} + D_{si}$, $D_s = r - s$ (because beam have $D_{si} = 0$)

Kinematic Indeterminacy:

Refers to degree of freedom at all joint.

* For 2D rigid frames: $D_k = 3j - r_e + r_r - n_r$
* For 3D rigid frames: $D_k = 6j - r_e + r_r - n_r$

Where, $j \rightarrow$ no. of joints, $r_e \rightarrow$ reactions released,

$r_r \rightarrow$ reactions available at supports,

$n_r \rightarrow$ no. of members axially rigid.

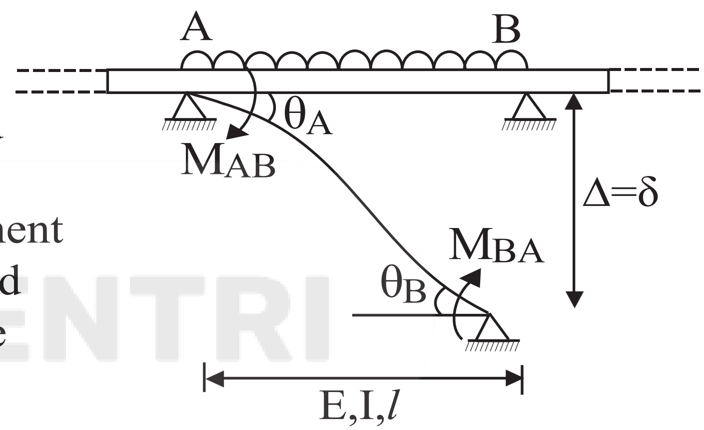
Slope-Deflection Method

Slopes & deflections are combinedly called displacements.
In this method, we establish a relationship between degrees of freedom (θ, Δ) & member end moments. This relationship is called slope deflection relationship.

* Method of super position (also known as G.A. maney method.) is used to find out slope deflection relationship.

Slope deflection equation:

1. Continuous Beam



M_{AB} = Internal member end moment at 'A'

M_{FAB} = Member end moment due to external load when all joints are considered fixed,

θ_B = Rotation at B,

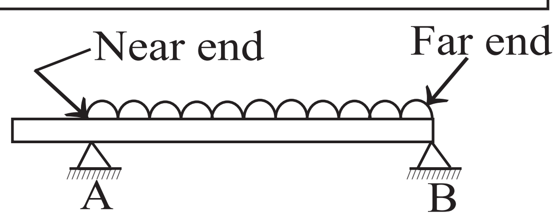
θ_A = Rotation at A, θ_δ = deflection of joint B wrt A

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right)$$

2. When one end is pin supported

$$M_{AB} = M_{FAB} - \frac{M_{FBA}}{2} + \frac{3EI}{L}\left(\theta_A - \frac{\delta}{L}\right)$$

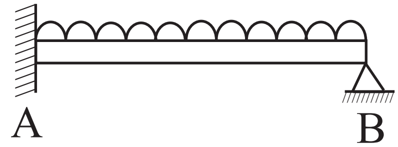


M_{FAB} = member & moment due to external load considering the continuous end (near end) to be fixed.

δ = Settlement of B with respect to A, θ_A = Rotation at A

Calculation of Fixed End Moments in case of one end being hinged, from the fixed end moments corresponding to the case of both end fixed.

$$\text{Fixed End Moment at A} = M_{FAB} - \frac{M_{FBA}}{2}$$



where, M_{FAB} & M_{FBA} are fixed end moments at A & B repectively

Equilibrium Equation:

No. of equilibrium equations required = Degree of freedom of structure

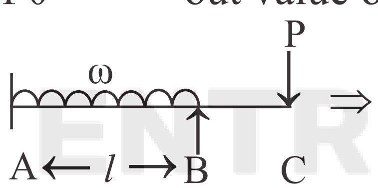
Types of equilibrium equation

↓

- Joint Equilibrium equation used to find out value of θ
- Shear equations used to find out value of δ

$M_{BA} + M_{BC} = 0$ ⇐

Equation at joint B



⇒ $R_A + R_B = wl + P$

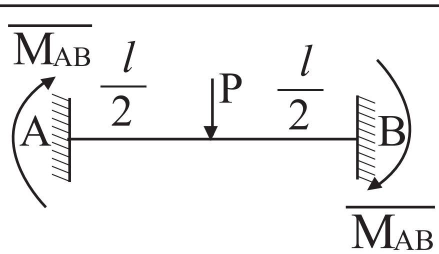
If δ is horizontal $\sum F_H = 0$

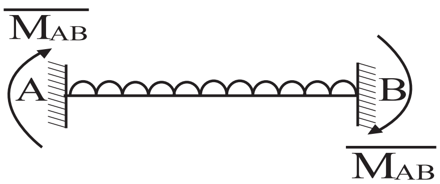
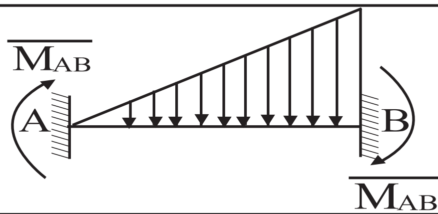
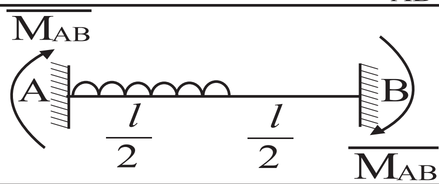
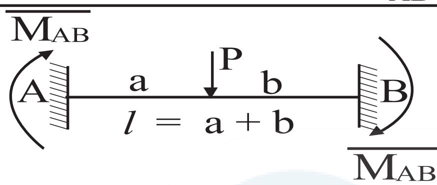
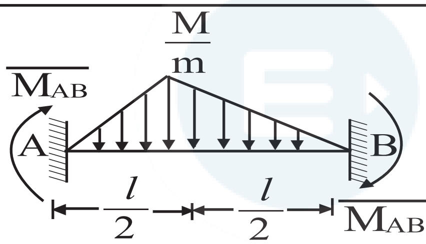
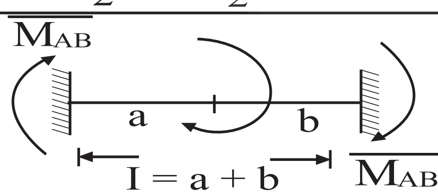
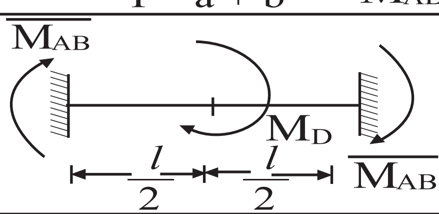
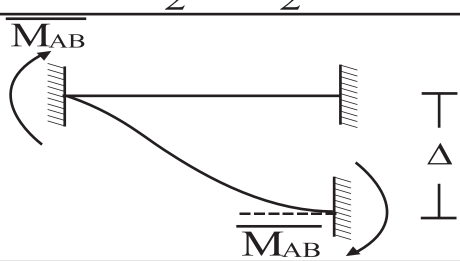
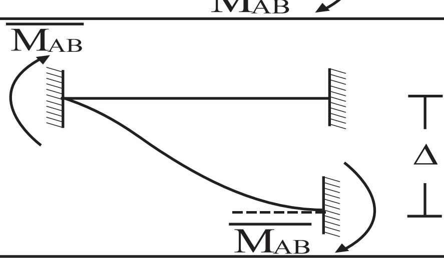
If δ is vertical $\sum F_V = 0$

Stepd for Analysis in Slope Deflection Method:

(a) Calculation of fixed end moments
(b) Relate member end moments to joint displacement
(c) Formulate equilibrium equations
(d) Solve the equations
(e) Back-Substitution of known displacements in equations of (b)
(d) Draw sketch of SFD & BMD

Fixed End Moments

Loading Diagram	M_{AB}	M_{BA}
	$\frac{-Pl}{8}$	$\frac{Pl}{8}$

	$\frac{-Wl^2}{12}$	$\frac{Wl^2}{12}$
	$\frac{-Wl^2}{30}$	$\frac{Wl^2}{20}$
	$\frac{-11}{192} Wl^2$	$\frac{5}{192} Wl^2$
	$\frac{-Pab^2}{l^2}$	$\frac{Pa^2b}{l^2}$
	$\frac{-5}{96} Wl^2$	$\frac{5}{96} Wl^2$
	$\frac{M_o b(2a - b)}{L^2}$	$\frac{M_o a(2b - a)}{L^2}$
	$\frac{M_0}{4}$	$\frac{M_0}{4}$
	$\frac{-6EI\Delta}{l^2}$	$\frac{-6EI\Delta}{l^2}$
	0	$\frac{-3EI\Delta}{l^2}$

ARCHES

Arches

3-Hinged arches ($D_s = 0$) | 2-Hinged arches ($D_s = 1$)

- * **An arch is subjected to** thrust, shear force & bending moment.
- * A three hinged arch is subjected to normal **thrust & radial shear and bending moment.**
- * A linear arch is subjected to **normal thrust only.**
- * In general, bending moment in arch is significantly reduced & axial force act as compressive force (thrust), so most appropriate answer is shear force & thrust.

Special Points : The lintels are preferred to arches because

(a) Arches required more headroom to span the opening like doors, windows etc.

(b) Arches are difficult in construction.

(c) Arch requires strong abutments to withstand arch thrust (because arch is subjectected mostly by axial thrust)

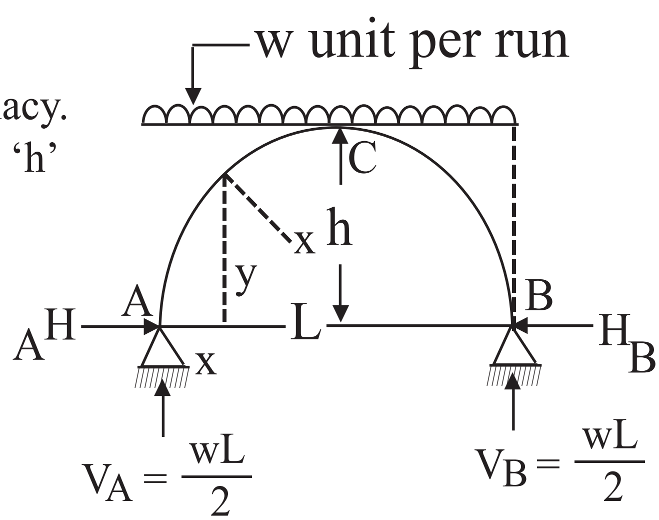
Three Hinged Arches

For three hinge parabolic arch, when subjected to UDL, then moment at all point in 0.

Determines with $D_s = 0$

D_s = Degree of static indeterminacy.

Parabolic Arch of Span ‘L’ & rise ‘h’ carrying a U.D.L. of w over the whole span.



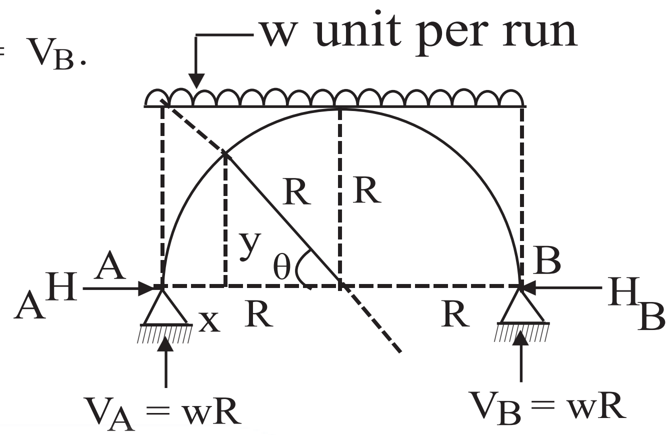
Profile, $y = \frac{4h}{l^2} x(l - y)$ $\qquad H_A = H_B = \frac{wl^2}{8h}$; $V_A = V_B = \frac{wL}{2}$

Moment at only section × from A - $M_x = V_A x - \frac{Wx^2}{2} - Hy$

$M_x = 0$ $\qquad$ Where, H_A = Horizontal thrust = H_B

V_A = Vertical reaction at A $= \frac{wl}{2} = V_B$.

Semicircular Arch having Radius 'R' carrying a U.D.L. 'w' over the whole span.



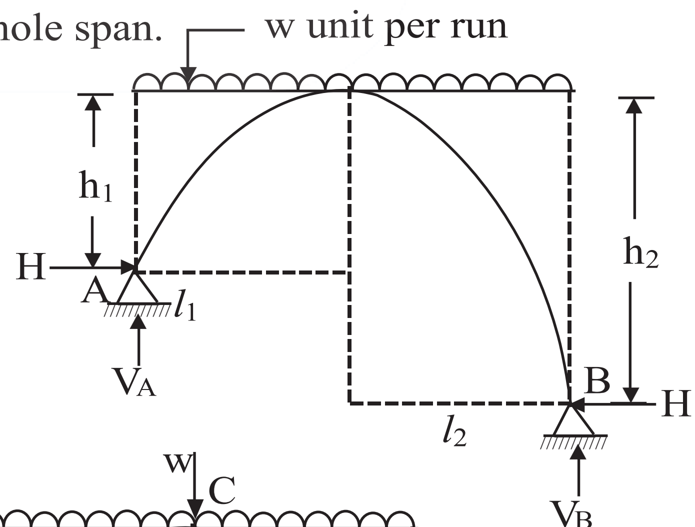
$$H_A = H_B = \frac{wR}{2}$$

$$M_x = \frac{wR^2}{2}(\sin^2\theta - \sin\theta)$$

$$M_{max} = \frac{-wR^2}{8} \text{ (at } \theta = 30^\circ\text{)}, \quad BM_C = 0 \text{ (at } \theta = 90^\circ\text{)}$$

Parabolic Arch Having Abutment is at Different Levels

When it is subjected to UDL over whole span.

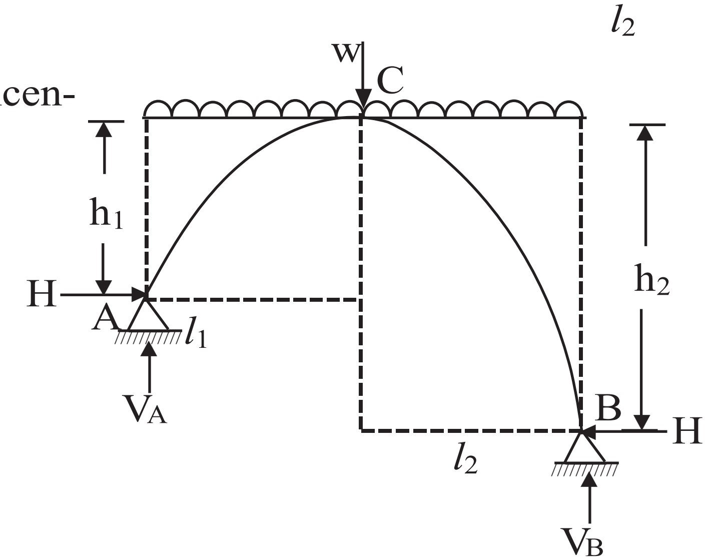


$$l_1 = \frac{1\sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}} ;$$

$$l_2 = \frac{1\sqrt{h_2}}{\sqrt{h_1} + \sqrt{h_2}}$$

$$H_A = H_B = \frac{wl^2}{2(\sqrt{h_1} + \sqrt{h_2})^2}$$

When it is subjected to concentrated load w at crown.



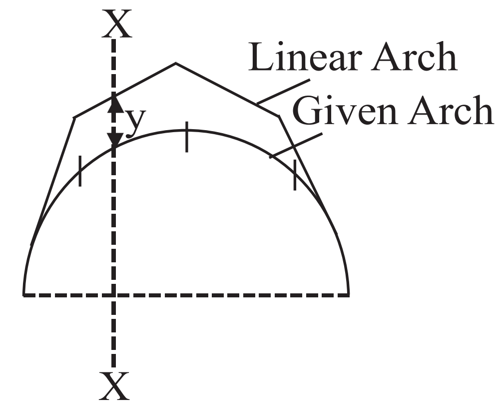
$$H = \frac{wl}{(\sqrt{h_1} + \sqrt{h_2})^2}$$

Eddy's Theorem :

As per eddy's theorem $M_x \propto y$

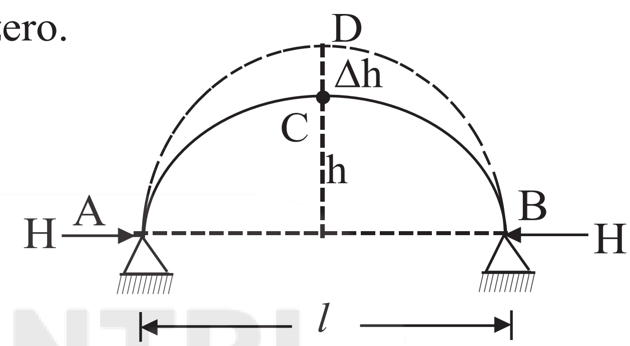
M_x = B.M. at any section

y = distance between given arch & linear arch.



Temperature Effect on Three Hinged Arches

Since free rise is occur at the crown. Therefore no Horizontal Reaction will develope since internal stresses are zero.



Free expansion

$$\Delta h = \left(\frac{l^2 + 4h^2}{4h}\right)\alpha t$$

where,

Δh = free rise in crown height.

l = length of arch   h = rise of arch

t = rise in temperature in ºC.

α = coefficient of thermal expansion.

$H \alpha \frac{1}{h}$ where, H = Horizontal thrust & h = rise of arch.

$$\%\text{ Decrease in horizontal thrust} = \frac{\delta h}{h} \times 100$$

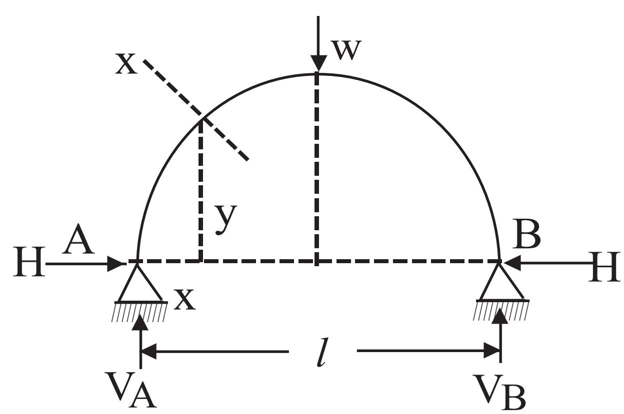
∴ If arch is initially loaded & temperature increases. Then, Horizontal Reaction is decreases & vertical Reaction Remains unaffected.

Two Hinged Arches

$D_s = 1$

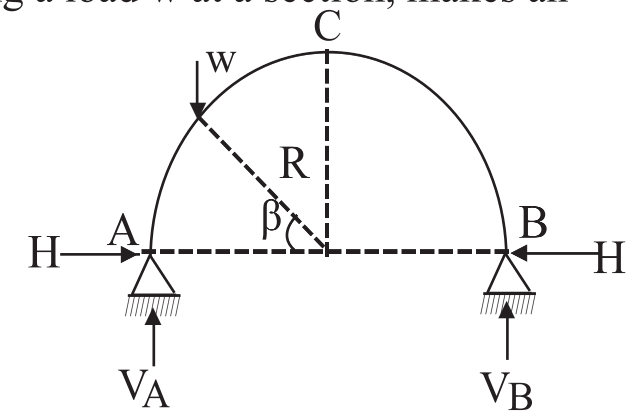
Use minimum potential energy theorem & final Horizontal Reaction H will be given as,

$$H = \frac{\int \frac{My}{EI} ds}{\int \frac{y^2}{EI} ds}$$



where, M = Beam moment caused by vertical forces.

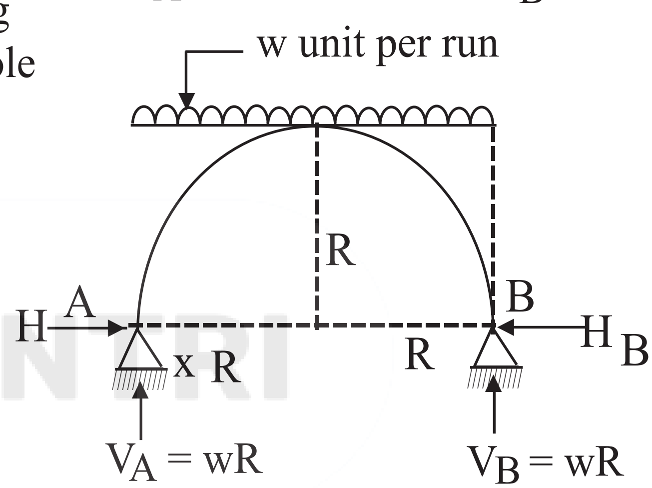
Semicircular arch of radius ‘R’ carrying a load w at a section, makes an angle with horizontal.



$$H_A = H_B = \frac{W}{\pi}\sin^2\beta$$

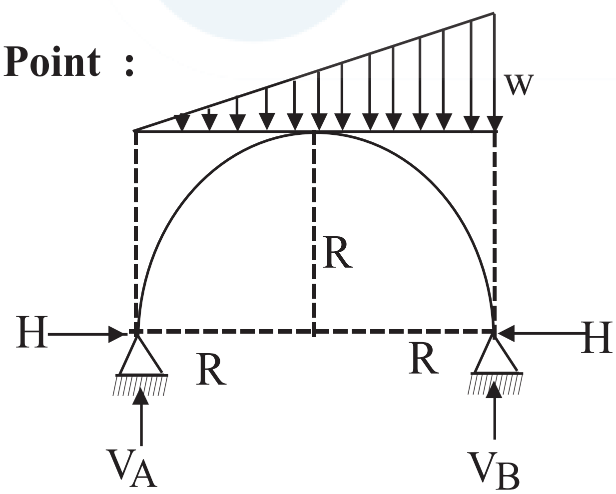
$$\left(\text{At crown } \alpha = 90^\circ,\ H = \frac{W}{\pi}\right)$$

Semicircular arch of radius R carrying a UDL w per unit length over the whole span.



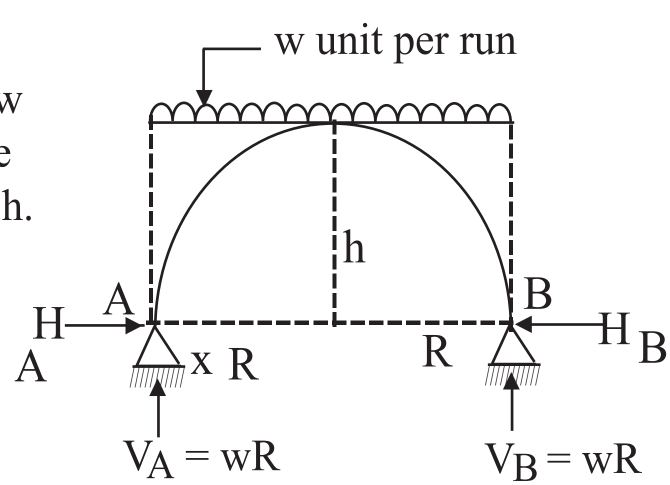
$$H_A = H_B = \frac{4}{3}\cdot\frac{wR}{\pi}$$

Special Point :



$$H_A = H_B = \frac{4}{3}\cdot\frac{wR}{\pi}$$

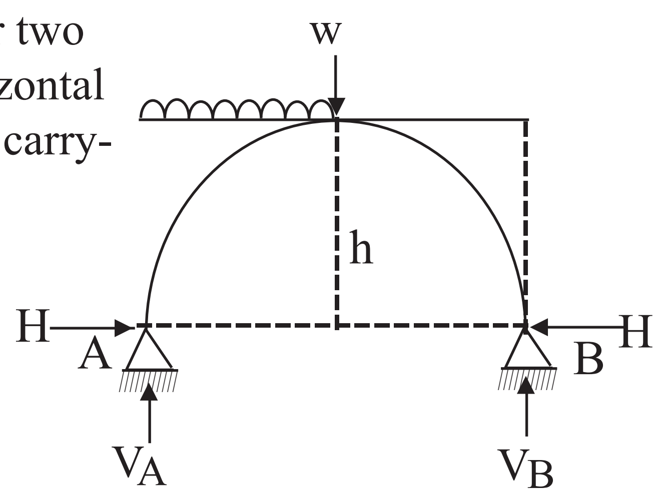
* Parabolic arch carries a UDL of w per unit run on entire span. If the span of the arch is L & its rise is h.



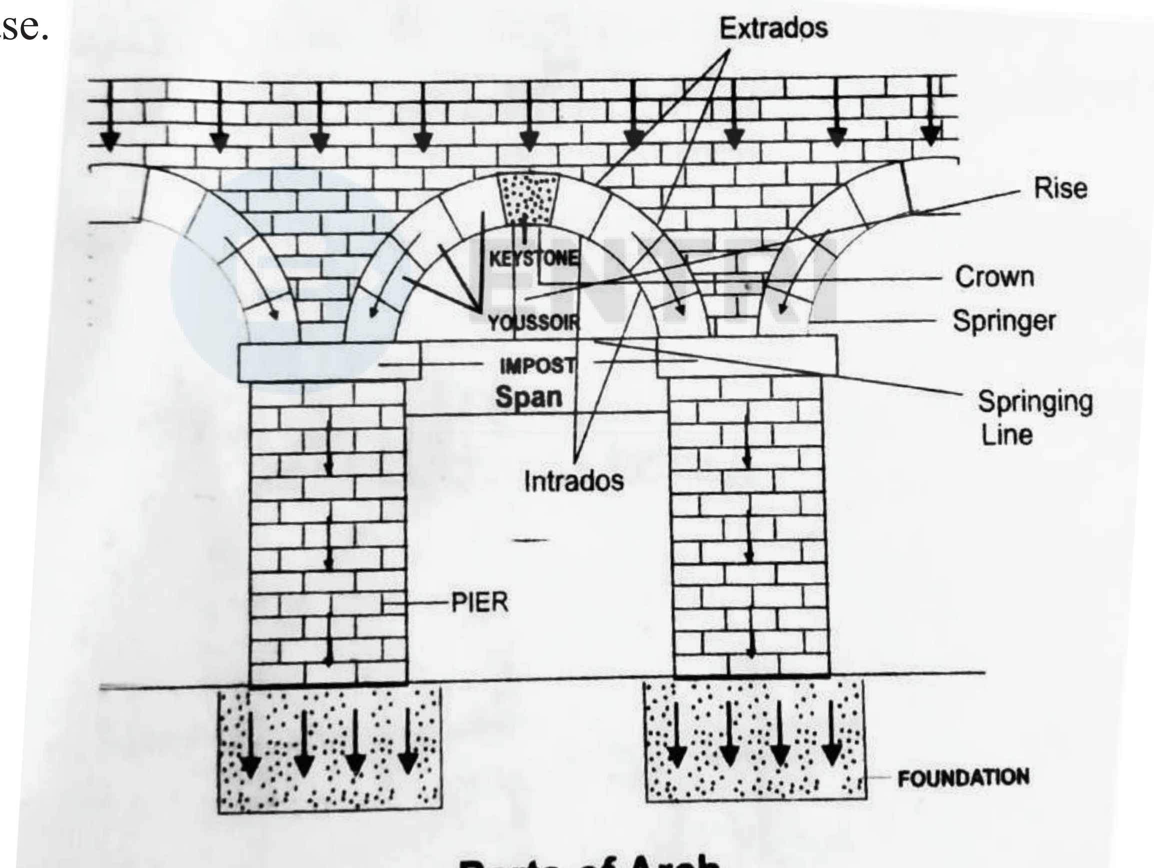
$$H_A = H_B = \frac{wl^2}{8h}$$

Special Point : Horizontal thrust for two hinged parabolic arch is equal to horizontal thrust for three hinged parabolic arch carrying UDL over entire span.

$$H = \frac{wl^2}{16h}$$



In 2 - Hinge parabolic arch, temperature increase then horizontal thrust increase.

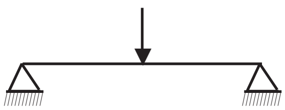


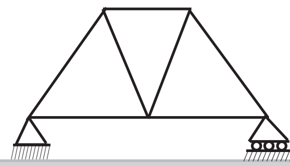
Parts of Arch

TRUSSES

A truss is a structure composed of slender members joined together at end points by bolting/riverting or welding. Ends of the members are joined to a common plate called gusset plate.

* Truss transmits load in axial direction as tension of compression.

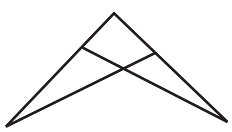
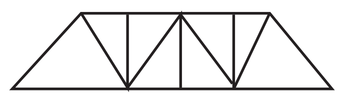
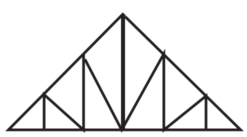
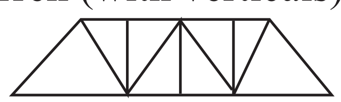
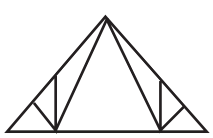
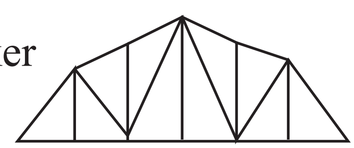
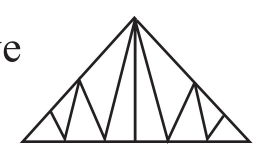
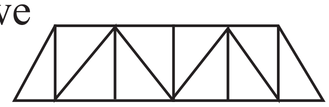
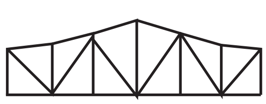
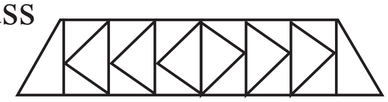




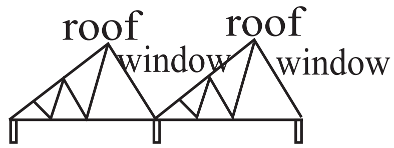
Special Points :

* Beams resist bending moment & shear force.
* A beam is a structural member of sufficient length compared to lateral dimension.
* Beams may be concrete, steel even composite beam, having any type, sections such as angles, channels, I section rectangular square etc.

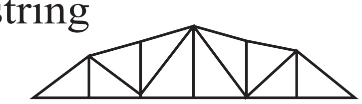
Types of Trusses

Roof Trusses	Bridge Trusses
(a) Scissors 	(a) Pratt 
(b) Pratt 	(b) Warren (with verticals) 
(c) Fan 	(c) Parker 
(d) Howe 	(d) Howe 
(e) Warren 	(e) K-truss 

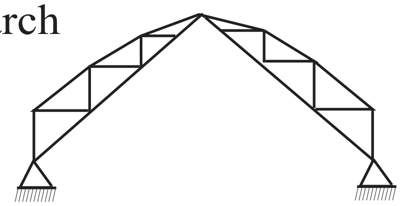
(f) Sawtooth



(g) Bowstring



(h) Three-hinged arch



Special Point: Pratt truss is better than Howe truss because the diagonal member in Pratt truss carries tension while in Howe truss, diagonal member carries compression. Thus if longer member carries compression, there is likely change of buckling of truss member.

Assumptions for design of truss members and connection

1. The members are joined together by smooth pins.
2. Self weight of the members is neglected.
3. All loading are applied to joints.

Zero-force Members:

1. If three members join at a point & out of them, two are collinear & also no external load acts at joint, the third member is a zero force member.
2. If only two non-collinear members exist at a truss joint & no external force or support reaction is applied to the joint, the members must be zero force member.

Methods of Analysis of statically determinate truss

Method of joint

$\Sigma F_x = 0, \Sigma F_y = 0$

Analysis should start at joint having atleast one known force & at most two unknown forces.

Method of section

Process use to solved for the unknown force acting on the members of a truss the method involved breaking the truss down into individual sections and analyzing each section as a separate rigid body.

Deflection of Truss Joint

1. Castigliano's Method

$$\Delta = \frac{\partial U}{\partial F} \quad ; \qquad U = \sum_{i=1}^{n} \frac{P_i^2 dx}{2AE} \qquad \text{so } \Delta = \frac{\partial}{\partial F}\left(\sum_{i=1}^{n} \frac{P_i^2 dx}{2AE}\right)$$

Δ = Deflection at the point of application of F & in the direction of F

U = Strain energy in the system

P_1 = Force in the i th member of truss due to combined action of external load & applied load (F) at the point at which deflection is to be found out.

2. Maxwell's unit load method

External virtual work = Internal virtual work

$$1 \times \Delta = \sum u_i \left(\frac{P_i dx}{A_i E_i} + l_i \alpha_i \Delta t_i + \lambda_i \right)$$

Where,

u_i = Internal member forces in the ith member due to unit load

p_i = Force the i th member due to combined external & applied load

Δ_i = Change in temperature of ith member

L_i = Length of ith member

α_i = Coefficient of thermal expansion for i th member

λ_i = Lack of fit or fabrication error

Special Point: Joint displacement equation method, Angle weights methods & Willot-Mohr (Graphical) method are also used in calculating truss deflection.

Methods of Analysis of Statically Indeterminate Truss

When no. of external reactions is exactly three or more than three, then only member forces can be taken as redundant & redundants should be selected in such a way that its removal does not make the truss unstable.

1. Castigliano's Method:

(i) Remove redundant member such that the truss becomes determinate & remains stable.

(ii) Apply equal & opposite force F at joints connecting the removed members.

(iii) Then calculate the member forces P_i due to F & external loading.

(iv) Find the strain energy of the system

$$U = \sum_{i=1}^{n} \frac{P_i^2 dx}{2A_iE_i}, \qquad n = \text{no. of member in the truss}$$

(v) Put $\frac{\partial U}{\partial F} = 0$ (minimum strain energy condition) & then calculate F.

(vi) Once F is known, then other member forces can be found out.

2. Maxwell's unit load Method

(i) Choose redundant such that its removal does not makes structure unstable.

(ii) Then remove redundant & find out member forces due to external loading

(iii) Remove external loading & apply the unit force (equal & opposite) at joints connecting the removed member. Calculate the member force u_i .

(iv) Net member force $F_i = P_i + u_i R$

(v) Minimise the strain energy by $\frac{\partial U}{\partial R} = 0$, where $U = \sum \frac{F_i^2 dx}{2A_iE_i}$

(vi) So, $R = \dfrac{-\sum_{i=1}^{n-1} u_i \frac{P_i dx}{A_iE_i}}{\sum_{i=1}^{n} \frac{u_i^2 dx}{A_iE_i}}$

Special point: If all factors are taken into account then

$$R = \dfrac{-\sum_{i=1}^{n-1} u_i \left(\frac{P_i dx}{A_i\, l_i}\, l\alpha\Delta t + \lambda_i \right)}{\sum_{i=1}^{n} \frac{u_i^2 dx}{A_iE_i}}$$

Development to force due to change in temperature or lack of fit

1. Externally and internally determinate truss: No force is developed in members.
2. Externally Indeterminate but internally determinate: No force developed in all members joining support.
3. Externally determinate but internally indeterminate: Rigid body motion will occur.

Truss	Frames
In truss forces act only along the axis of the members. Members are having tension or compression.	In frames forces are acting along the axis of the member, in addition to transverse forces.
Each member is acted upon by two equal and opposite forces having line of action along the centre of members. (every member of truss is a two force member.)	One or more than one member of frame is subjected to more than two forces (multiple force members).
Forces are applied at the joints only.	Forces may act anywhere on the member.
Member does not bend.	Members may be bend
Used for large loads.	Used for small & medium loads

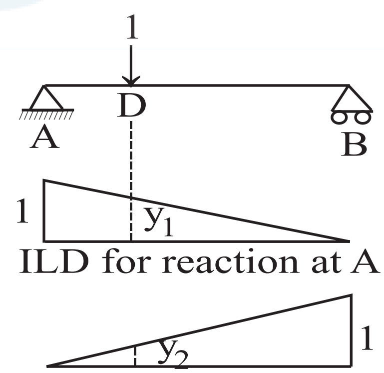
Influence-Line Digram

Influence line for bending moment indicates bending moment at a given section for any position of a point load.

* Influence lines represent the effect of a moving load only at a specified point on a member whereas shear & moment diagram represents the effects of fixed load (or a given position of load) at all points along the member.
* An influence line represents the variation of either the reaction, shear, moments & deflection at a specified point in a member as a concentrated unit force moves over the member.
* Influence line helps in deciding at a glance, where should the moving load (point load, movin load several point load) be placed on the structure so that it crestes greatest influence at the specified point.
* Influence line for stress functions for statically determinate structure consists of straight line segment. ILD for deflection may even be curved for determinate structure.

1. Simply supported beam

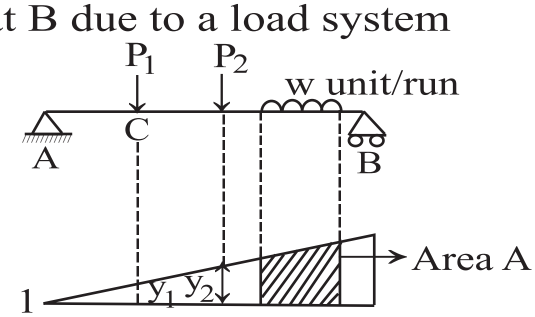
(a) End reactions due to unit load at D.



y_2 = Magnitude of reaction at B when unit load at D

y_1 = Magnitude of reaction at A when unit load at D

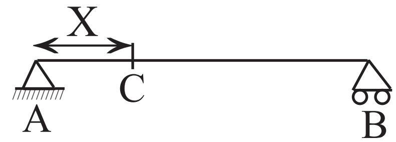
(b) End reaction at B due to a load system

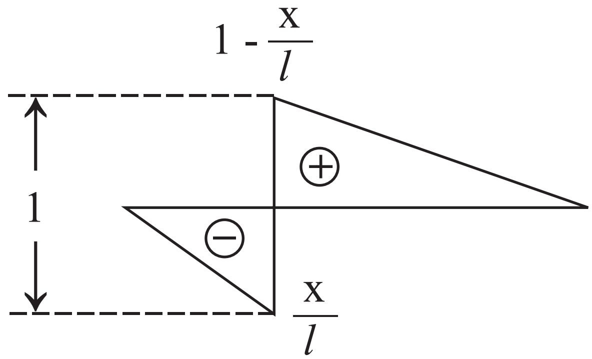


ILD for Reaction at B

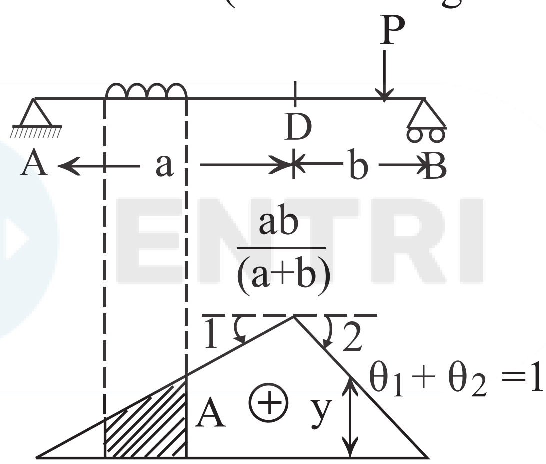
$R_B = p_1 y_1 + p_2 y_2 + \omega A$

(c) Shear force at C (assume slider support at C point)



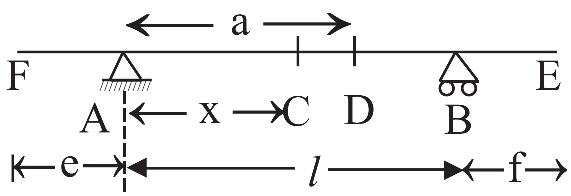


(d) Bending moment at D (Provide hinge at D point)

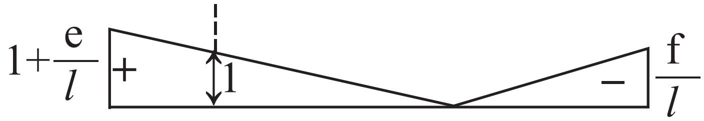


BM at D = wA + Py

2. Simply supported beam with overhang

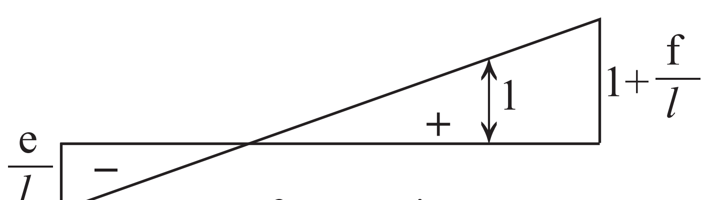


(a)



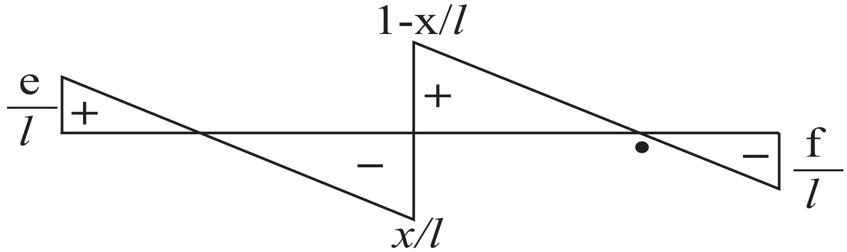
ILD for reaction at A

(b)



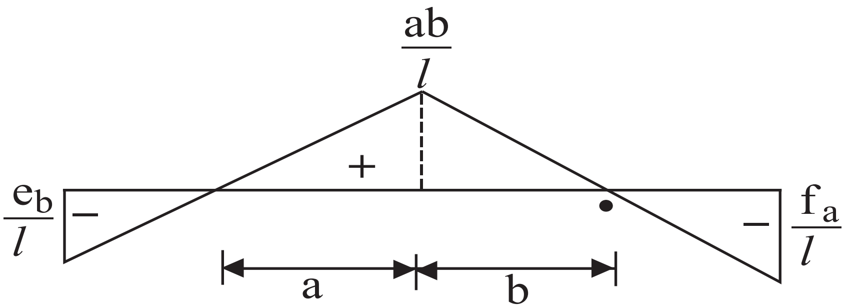
ILD for reaction at B

(c)



ILD for shear force reaction at C

(d)



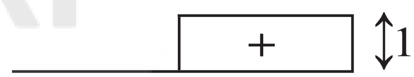
ILD for reaction at C

3. Cantilever Beam

(a)

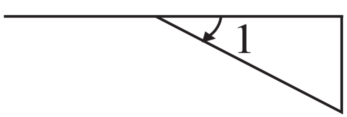
ILD for Reaction at A

(b)



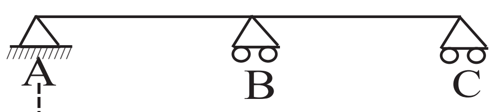
ILD for shear force at B

(c)

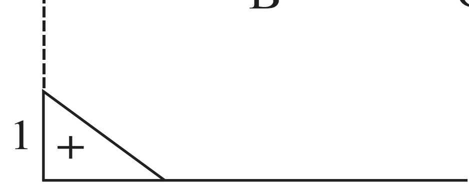


ILD for moment reaction at B

4. Simply supported beam with internal hinge

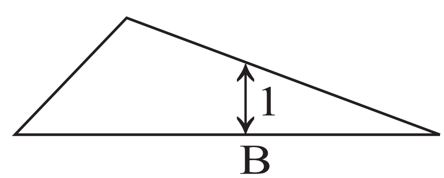


(a)



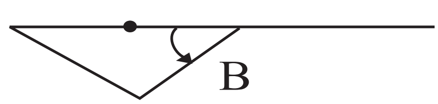
ILD for Reaction at A

(b)



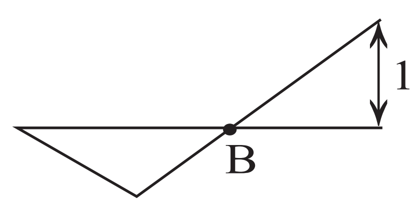
ILD for Reaction at B

(c)



ILD for Bending moment at B

(d)

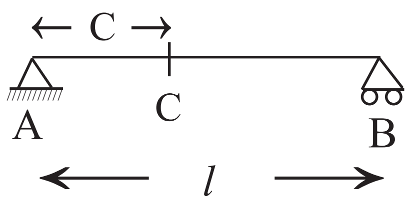


ILD for Reaction at C

Special point: The ordinate of ILD for reaction & shear force is dimensionless but ordinate of BM in ILD has dimensions of length.

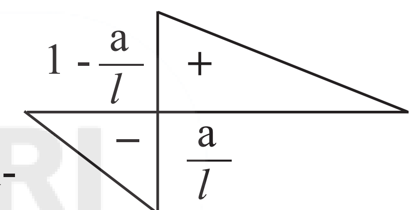
Muller-Breslau Principle: Influence line for any stress function may be obtained by removing the restrain offered by the function & introducing a directly related unit displacement at the location & in the direction of the function.

Effect of rolling Load



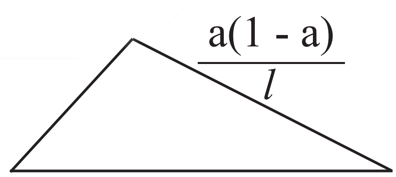
1. **Single point Rolling load**

   (i) For max. SF at section C, the point load should be just to the left or just to the right of the section where ordinate of ILD for SF is maximum.



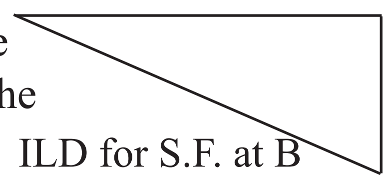
ILD for S.F. at C

   (ii) For maximum Bending moment at section C, the point load should be be on the section C itself.



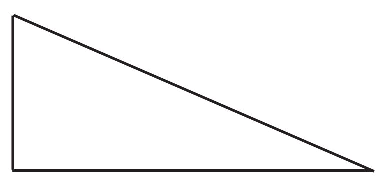
ILD for B.M. at C

   (iii) For maximum -ve shear force, anywhere in the span the point load should be on the right had support.



ILD for S.F. at B

   (iv) For maximum +ve shear force, anywhere in the span the point load should be on the left hand support.



ILD for S.F. at A

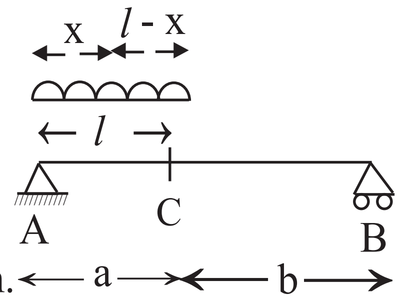
   (v) For absolute maximum BM, point load should be at mid span.

2. **UDL shorter than the span**

   (i) For maximum +ve SF at section C, the tail of the udl should reach section C.

(ii) For maximum -ve SF at Section C, span should be loaded such that head of the udl reaches at C.

(iii) For maximum bending moment at section C, the load should be placed in such a way that the section divides te load in the same ratio as in divides the span.



$$\frac{a}{b} = \frac{x}{l - x}$$

(iv) For absolute maximum BM, the CG of the load should be at the mid span.

(v) For absolute maximum +ve SF, the tail of the load should be at left support.

(vi) For absolute maximum -ve SF, the head of the load should be at right support.

3. UDL longer than span

(i) For maximum +ve SF at section C, the tail of the udl should reach section C.

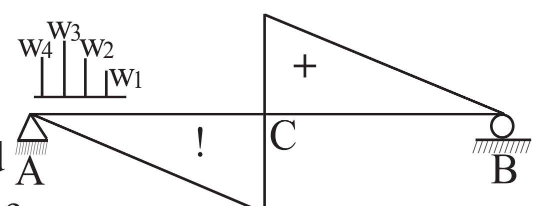
(ii) For maximum -ve SF at Section C, span should be loaded such that head of the udl reaches at C.

(iii) For absolute maximum +ve or -ve shear force complete span should be loaded.

(iv) For maximum Bending moment at Section C, complete span should loaded.

(v) For absolute maximum BM complete span should be loaded & absolute maximum BM will occur at mid span.

4. Train of concentrated loads.

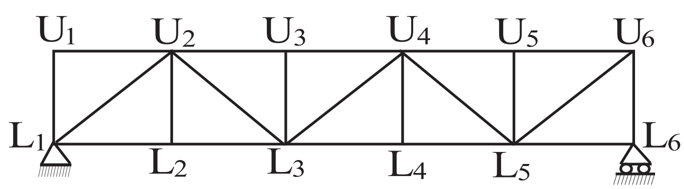


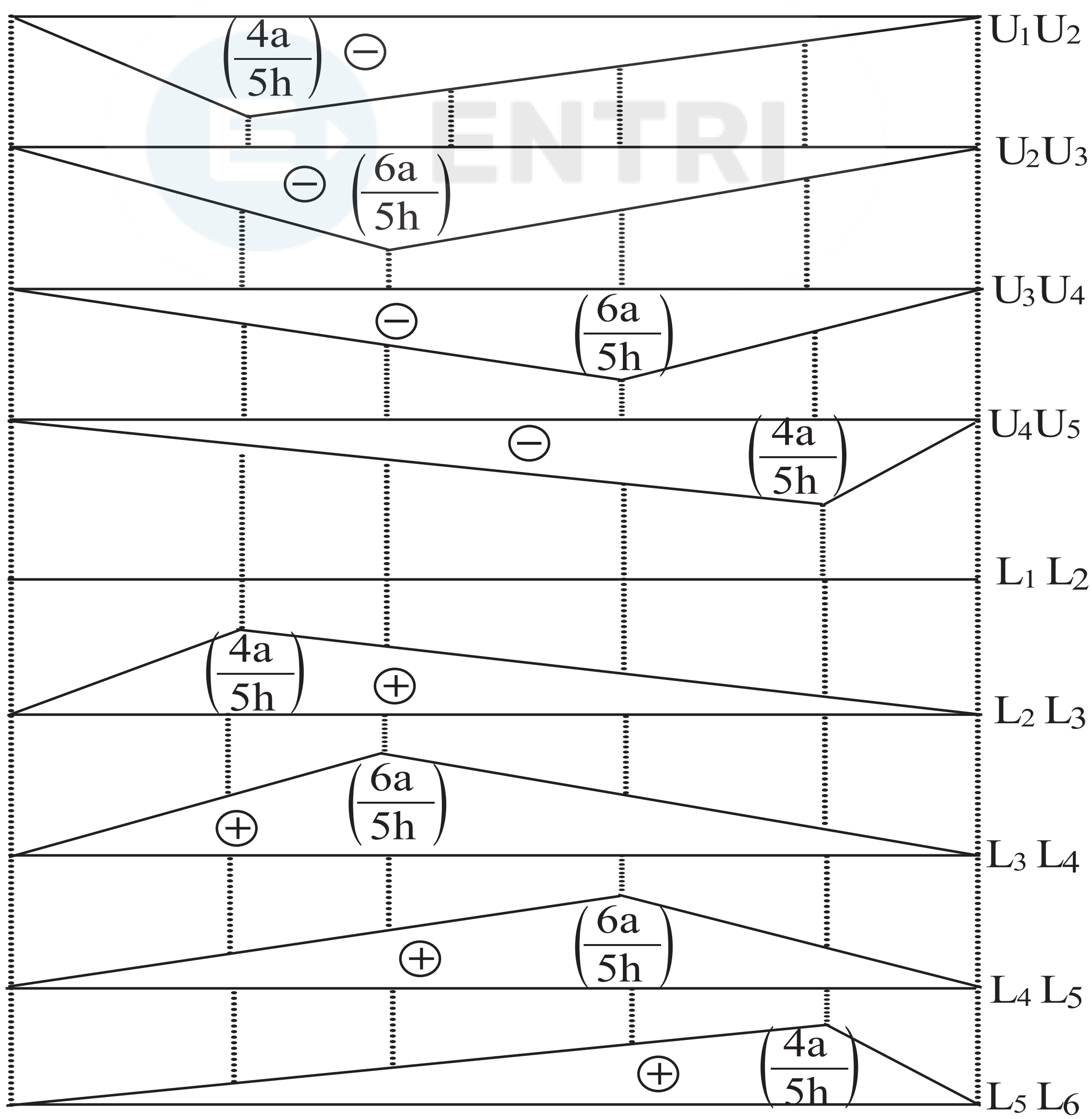
(i) For maximum +ve/-ve SF at section C, various loads will be placed on the section & values of SF at the section will be calculated. Maximum of these +ve/-ve values will be the maximum SF at the Section C.

(ii) For absolute maximum BM, loads are placed in such a way that resultant & the heavier loads are at equal distance from the mid-span. Under this condition bending moment will be calculated below the heavier load only.

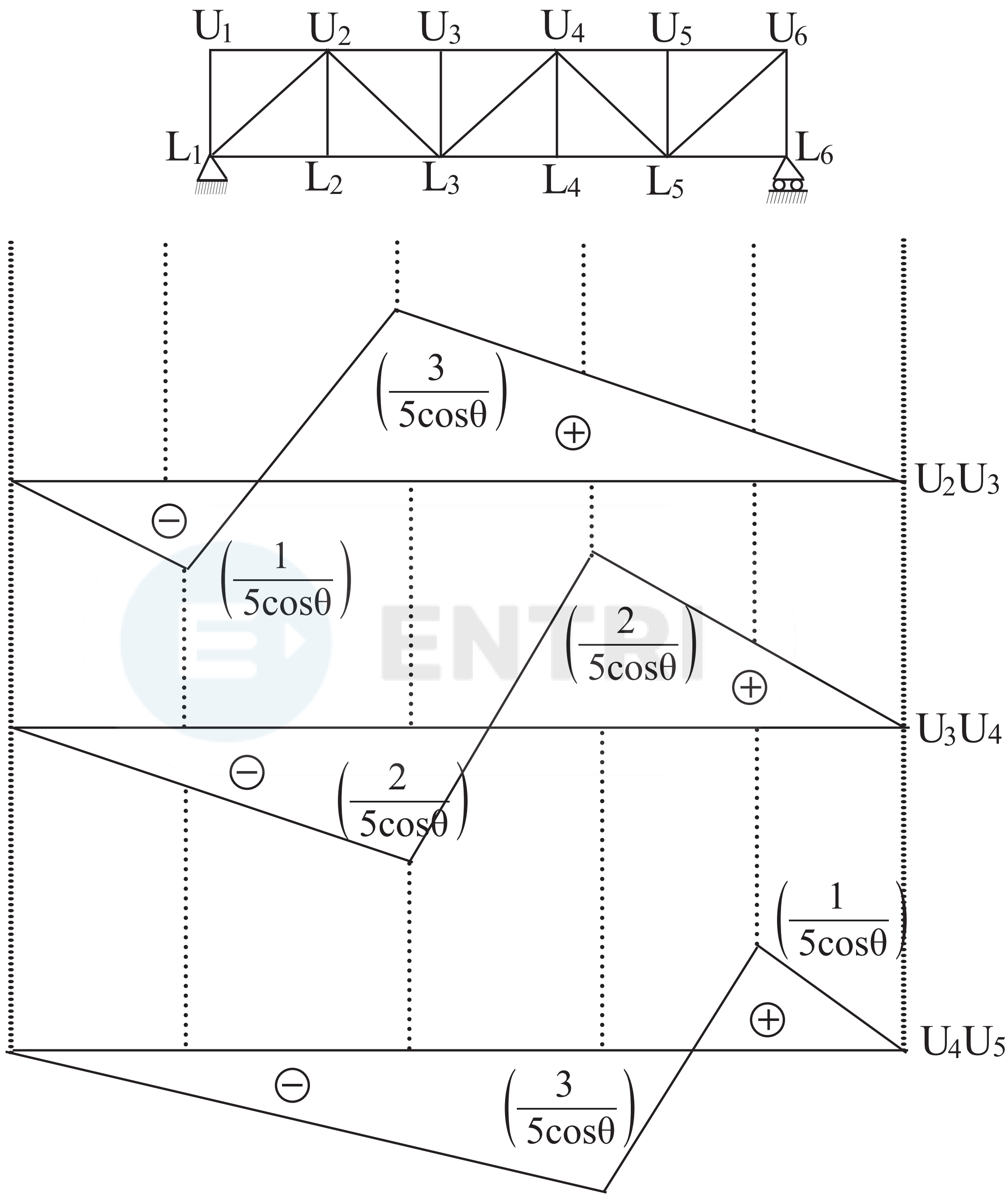
(iii) Maximum B.M. at the section C, will develop when one of the load is itself of the section, & load on the section should be such that when it rolls over the section & comes to the other side average loading (load/length of one section) on the left side changes from heavier loading to lighter loading (or vice versa).

ILD influence line diagram for truss members.





Influence line diagram for inclined member for truss



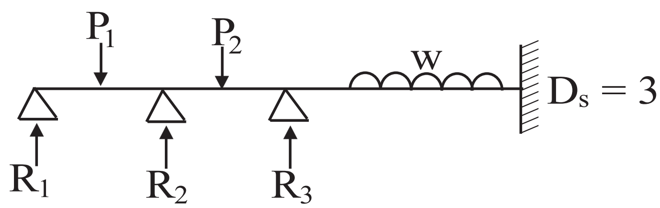
MATRIX-METHOD OF ANALYSIS

Matrix methods

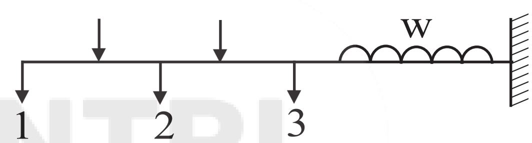
- Flexibility method (Force method of Analysis)
- Stiffness method (Displacement method of Analysis)

Flexibility Matrix-Method : Displacement caused by unit force.

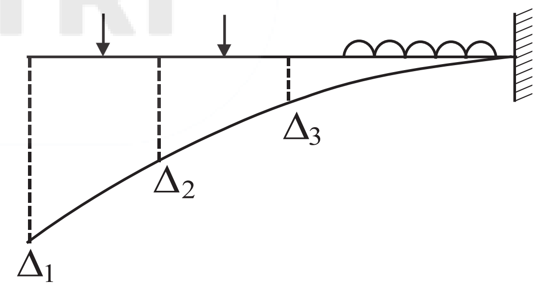
1. Identify the static indeterminacy of the structure & choose redundants.



2. Then assign coordinates to each redundant direction.



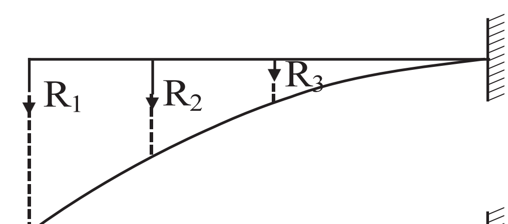
3. After it, release the redundants to obtain primary structure & then find out deflections in the co-ordinate at the locations of co-ordinate.

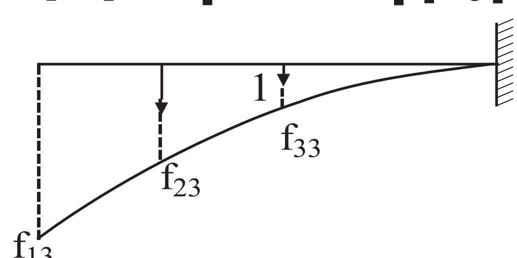


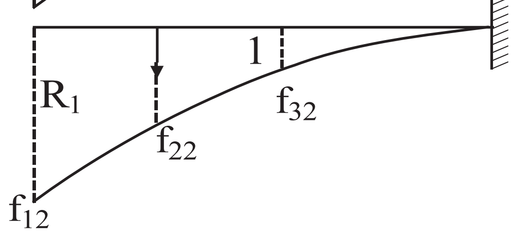
4. Apply redundants (after removing external loading) & find out deflections in the co-ordinate directions.

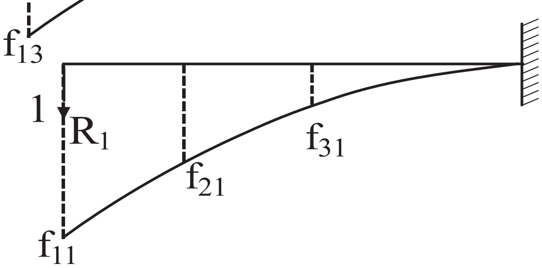
$$\delta_1 = f_{11}R_1 + f_{12}R_2 + f_{13}R_3$$
$$\delta_2 = f_{12}R_1 + f_{22}R_2 + f_{23}R_3$$
$$\delta_3 = f_{13}R_1 + f_{32}R_2 + f_{33}R_3$$

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}$$









Condition when D_1, D_2, D_3 are final deflections at co-ordinate locations 1, 2, 3, then $D_1 = \Delta_1 + \delta_1, D_2 = \Delta_2 + \delta_2, D_3 = \Delta_3 + \delta_3$

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} + \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}$$

$$[D] = [\Delta] + [f][R] \quad [R] = [F]^{-1}([D] - [\Delta])$$

Special point : f_{21} = deflection in direction 2 due to unit force in direction 1, f_{21} = element of second row, first column.

Properties of flexibility matrix

(i) Order of flexibility matrix = No. of co-ordinates choose in the problem (static indeterminacy).

(ii) All elements of flexibility matrix are displacements (δ or θ)

(iii) Flexibility matrix is a square symmetrical matrix.

(iv) Principle diagonal elements of flexibility matrix are always +ve.

(v) Flexibility matrix is calculated only for stable structures. There is no rigid body displacement.

(vi) Symmetry of flexibility matrix is owing to maxwell's reciprocal theorem.

(vii) Dimensions of all elements may not be same

$$f = \frac{\theta}{M} = \frac{1}{KNm} \quad \text{or} \quad f = \frac{\delta}{P} = \frac{1}{KN}$$

Stiffness Matrix-method : Force acquired to produce unit displacement.

1. Identify the kinematic indeterminacy of the structure & choose unknown displacement.
2. Then assign co-ordinates to each of the known displacements (θ,s).
3. The joints are restrained & forces developed due to applied load in the co-ordinate directions are found out. Let that be P_1', P_2', P_3'.
4. After it, the joints are released & deflections are permitted in the coordinate directions & forces required in the co-ordinate directions for these displacements are found out.

$P_1\Delta = K_{11}\Delta_1 + K_{12}\Delta_2 + K_{13}\Delta_3$, $P_2 = K_{21}\Delta_1 + K_{22}\Delta_2 + K_{23}\Delta_3$

$P_3\Delta = K_{31}\Delta_1 + K_{32}\Delta_2 + K_{33}\Delta_3$

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix}$$

Condition when P_1, P_2, P_3 are final developed at various co-ordinates axis

$P_1 = P_1' + P_{1\Delta}, \quad P_2 = P_2' + P_{2\Delta}, \quad P_3 = P_3' + P_{3\Delta}$

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} P_1' \\ P_2' \\ P_3' \end{bmatrix} \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix}$$

$[P] = [P'] + [K][\Delta] \qquad [\Delta] = [K]^{-1}([P] - [P'])$

Special point: K_{21} = force in the direction of 2 due to unit displacement in direction 1, K_{21} = element of 2nd row, 1st column

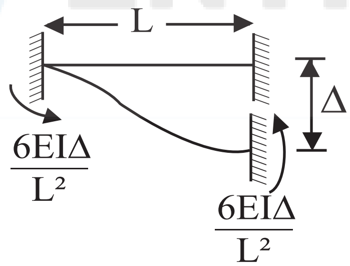
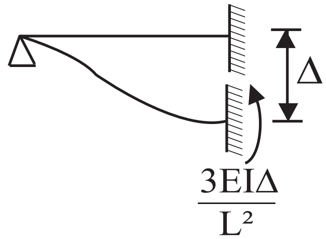
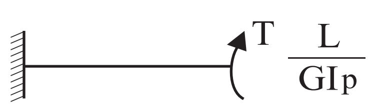
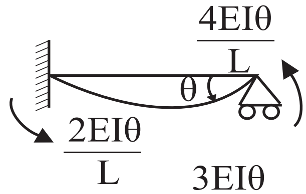
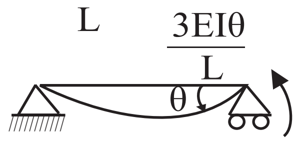
Properties of Stiffness Matrix

All the properties of flexibility matrix defined are also the properties of stiffness matrix except the following:

(i) Elements of stiffness matrix will be forces.

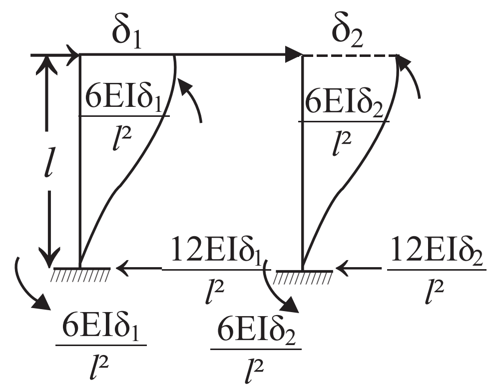
(ii) Order of stiffness matrix will be degree of kinematic indeterminacy

Simple Results

Type of Displacement	Diagram	Flexibility	Stiffness
(i) Axial displacement		$\frac{L}{AE}$	$\frac{AE}{L}$
(ii) Transverse displacement (a) With fixed end		$\frac{L^3}{12EI}$	$\frac{12EI}{L^3}$
(b) With far end hinged		$\frac{L^3}{3EI}$	$\frac{3EI\Delta}{L^3}$
(iii) Torsional displacement		$\frac{GI_p}{L}$	
(iv) Flexural displacement (a) With far end fixed		$\frac{L}{4EI}$	$\frac{4EI}{L}$
(b) With far end hinged		$\frac{L}{3EI}$	$\frac{3EI}{L}$

Special Case:

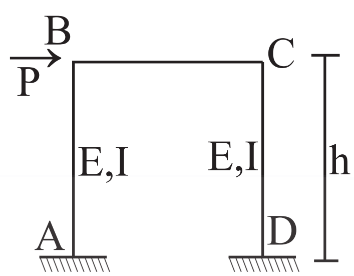
(i) When EI = ∞ for beam BC then

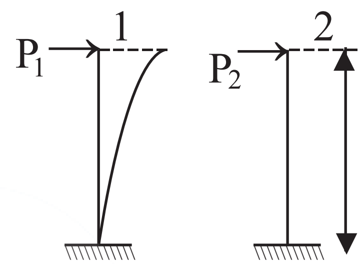


$$P = \frac{12EI\delta_1}{l^3} + \frac{12EI\delta_2}{l^3} \qquad (\delta_1 = \delta_2 = \delta)$$

$$P = \frac{12EI\delta_1}{l^3} \qquad \delta = \frac{Pl^3}{24EI}$$

(ii) When EI = 0 for beam BC, then





Deflection diagram

$$\delta_1 = \frac{P_1 h^3}{3EI}$$

$$\delta_2 = \frac{P_2 h^3}{3EI}$$

$$\delta_1 = \delta_2 \Rightarrow P_1 = P_2 = P/2$$

Hence, $\delta_1 = \delta_2 \Rightarrow \delta = \frac{Ph^3}{6EI}$

Stiffness matrix method for Truss: There are four co-ordinate direction for one member

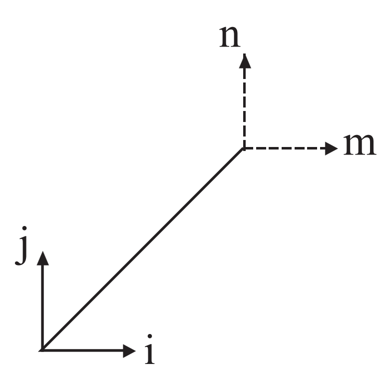
1. Displacement in i-direction without displacement in any direction

$$K_{ji} = \frac{\sum AE\cos\theta\sin\theta}{L}$$

$$K_{ij} = \frac{\sum AE}{L}\cos^2\theta$$

$$K_{mi} = \frac{\sum - AE}{L}\cos^2\theta$$

$$K_{ni} = \frac{\sum - AE\cos\theta\sin\theta}{L}$$



2. Displacement in j-direction without displacement in any direction

$$K_{ij} = \sum \frac{AE}{L}\sin\theta\cos\theta \qquad K_{ji} = \sum \frac{AE}{L}\sin^2\theta$$

$$K_{mj} = \sum - \frac{AE}{L}\sin\theta\cos\theta \qquad K_{nj} = \sum - \frac{AE}{L}\sin^2\theta$$

MOMENT-DISTRIBUTION METHOD

It is an interactive procedure of indirectly solving the equations of equilibrium as formulated in slope deflection method. Presented y **Prof. Hardy cross in 1930.** (Also called as Relaxation Method).

Basic Concept : Initially assuming each joint in the analysis to be fixed. Then by unlocking & locking each joint in succession, the internal moments at the joints are distributed & balanced until the joints have rotated to the final or nearly final position.

Special Point: Last cycle must end in balancing of hinged or continuous joints with carry over to fixed joint.

Stiffness: It is the force or moment required to be applied at a joint so as to produce unit deflection or rotation at that joint.

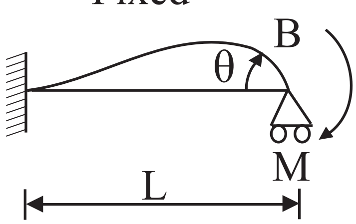
$$\boxed{\begin{array}{c} F = K\Delta \\ \hline M = K\theta \end{array}}$$

,Where K = stiffness, M = Moment required for rotation θ

F = Force required for deflection Δ

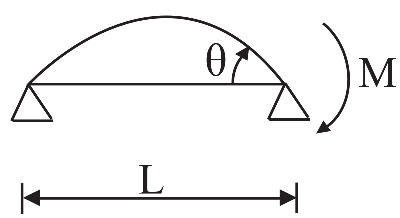
Member Stiffness: Stiffness of a member AB when farther end is

Fixed



$$K = \frac{M}{\theta} = \frac{4EI}{L}$$

Hinged



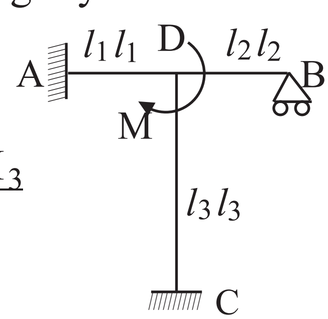
$$K = \frac{M}{\theta} = \frac{3EI}{L}$$

Joint Stiffness Factor: Considering joint 'D' to be rigidly connected

Applying moment 'M' at D

$M = M_1 + M_2 + M_3$; $M = K_1\theta + K_2\theta + K_3\theta$

$$K = \frac{M}{\theta} = K_1 + K_2 + K_3 + = \frac{4EI_1}{l_1} + \frac{3EI_2}{l_2} + \frac{4EI_3}{l_3}$$



Distribution Factor (D.F.):

Sum of DF for all members at a joint is always one

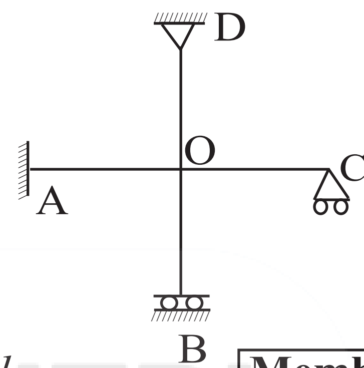
$$DF = \frac{\text{Stiffness of a member}}{\text{Sum of stiffness of all members at that joint}}$$

$$DF = \frac{\text{Relative stiffness of a member}}{\text{Sum of Relative stiffness of all members at that joint}}$$

Relative stiffness

(a) when far end is fixed $= I/l$

(b) when far end is hinged $= \frac{3I}{4l}$



Length of OA $= OB = OC = OD = l$

Stiffness $= \frac{EI}{L}$

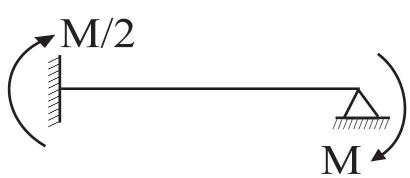
Member	Stiffness
OA	$\frac{4EI}{l}$
OD	$\frac{3EI}{l}$
OC	$\frac{3EI}{l}$
OB	$\frac{EI}{l}$

Carry Over Factor = (COF)

$$COF = \frac{\text{Carry over moment}}{\text{Applied moment}}$$

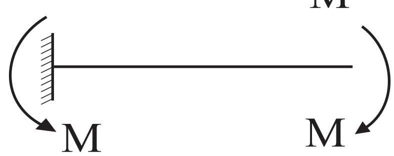
Standard Cases

(i)



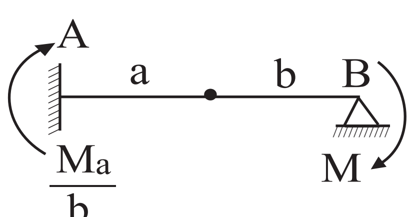
$COF = \frac{1}{2}$

(ii)



$COF = -1$

(iii)

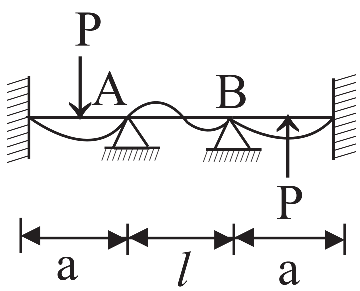


$COF = \frac{a}{b}$

Stiffness at B - $\frac{M}{\theta_B} = \frac{3EIb^2}{a^3 + b^3}$

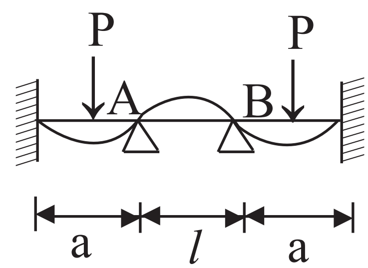
Modified stiffness factor for member AB

Symmetrical beam with unsymmetrical loading



$$K = \frac{6EI}{l}$$

Symmetrical beam with symmetrical loading



$$K = \frac{2EI}{l}$$