# A

**25121** **120 MINUTES**

1. Let $R$ be a relation on the set $\mathbb{Z}$ defined by $R$ ={(a+b) | $a^2 + b^2$ is even}.Which of the following is true about $R$ ?

A) $R\ is\ reflexive\ but\ not\ symmetric$

B) $R\ is\ symmetric\ but\ not\ transitive$

C) $R\ is\ reflexive\ and\ transitive\ but\ not\ symmetric$

D) $R\ is\ an\ equivalence\ relation$

2. Let $f:\ \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = e^x + x^2$. Which of the following is true about $f$ ?

A) $f\ is\ strictly\ increasing$

B) $f\ is\ strictly\ decreasing$

C) $f\ is\ both\ increasing\ and\ decreasing$

D) $f\ is\ neither\ increasing\ nor\ decreasing$

3. If the diagonal of a square is the line joining the points (1, 3, 2) and (2, 1, 3), then area of the square is:

A) 3 square units B) 4 square units

C) 2 square units D) 1 square unit

4. Maximum value of $y$ in the equation $9x^2 + 4y^2 - 72 = 0\ is$:

A) Zero B) Infinity C) $2\sqrt{3}$ D) $3\sqrt{2}$

5. Area enclosed by the curve $y^2 = 2 - x^2$ between $x = -\sqrt{2}$ and $x = \sqrt{2}$ is equal to:

A) $2\pi$ B) $\sqrt{2}\pi$ C) $3\pi$ D) $\sqrt{3}\pi$

6. A box contains 5 red balls and 3 blue balls. If two balls are drawn randomly without replacement, what is the probability that both balls are red?

A) $\frac{5}{8}$ B) $\frac{5}{14}$ C) $\frac{3}{8}$ D) $\frac{25}{56}$

7. If A = {x : x> 0 and $x^2 > 2$}, then which of the following is true ?

A) A is an empty set
B) infimum of A is $\sqrt{2}$
C) supremum of A is $\sqrt{2}$
D) infimum of A is zero

8. If a function $f$:[a,b] → $\mathbb{R}$ is of bounded variation, then which of the following statement is correct ?

A) $f$ must be continuous
B) $f$ must be differentiable
C) $f$ can be written as the difference of two monotone increasing functions
D) None of these

9. Let A be the set of all sequences whose elements are the digits 0 and 1, then the set A is:

A) Finite
B) Countable
C) Uncountable
D) None of these

10. If $S_n = n^2$, then the sequence $\{S_n\}$ is:

A) bounded and convergent
B) bounded and divergent
C) unbounded and convergent
D) unbounded and divergent

11. Taylor series of $\frac{1}{z}$ about $z$ =1is:

A) $1 + (z-1) + (z-1)^2 + (z-1)^3 + \cdots$
B) $1 - (z-1) - (z-1)^2 - (z-1)^3 - \cdots$
C) $1 - (z-1) + (z-1)^2 - (z-1)^3 + \cdots$
D) $1 - (z+1) + (z+1)^2 - (z+1)^3 + \cdots$

12. Up to isomorphism, the number of abelian groups of order 108 is:

A) 12 B) 9 C) 6 D) 5

13. The number of generators of the cyclic group of order 8 is:

A) 1 B) 2 C) 3 D) 4

14. In the ring $\mathbb{Z}_n$, the divisors of zero are precisely those elements that are:

A) 1 and $n$ B) relatively prime to $n$

C) not relatively prime to $n$ D) none of these

15. Which among the following is a **wrong** statement?

A) Every field is an integral domain

B) For any positive integer $p$, $\mathbb{Z}_p$is a field

C) Every finite integral domain is a field

D) None of these

16. The characteristic of the ring $\mathbb{Z}_3 \times \mathbb{Z}_4$ is:

A) 3 B) 4 C) 7 D) 12

17. Which among the following is a gcd of 22, 471 and 3, 266?

A) 17 B) 23 C) 31 D) 29

18. If $A$ is a square matrix such that $A^2 = A$, then $(I - A)^3 + A$ is equal to:

A) zero matrix B) unit matrix C) $I - A$ D) $I + A$

19. Rank of the matrix $A = \begin{pmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{pmatrix}$

A) 0 B) 1 C) 2 D) 3

20. If $V_1$ and $V_2$ are 3-dimensional subspaces of a 4-dimensional space $V$, then the smallest possible dimension of $V_1 \cap V_2$ is:

A) 1 B) 2 C) 3 D) 4

21. Let $T_1$ and $T_2$ be two maps from $\mathbb{R}^2$ into $\mathbb{R}^2$ defined by $T_1$ (a,b) = (1+a,b) and $T_2$ (a,b) = (b,a). Then which of the following is correct ?

A) $T_1$ is not a linear transformation and $T_2$ is a linear transformation

B) $T_1$ is a linear transformation and $T_2$ is not a linear transformation

C) $T_1$ is a linear transformation and $T_2$ is also a linear transformation

D) $T_1$ is not a linear transformation and $T_2$ is also not a linear transformation

22. If $\phi$ is the Euler's totient function, then $\phi$ ($\phi$ (1001)) is equal to:

A) 144 B) 192 C) 298 D) 96

23. If $y = Ae^{2x} + Be^{-x}$, the differential equation satisfied by $y$ is:

A) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$ B) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - y = 0$

C) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$ D) $\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$

24. General solution of the differential equation $\frac{dy}{dx} = \frac{2x}{y}$ is:

A) $y^2 = x^2 + C$ B) $y^2 = 2x^2 + C$

C) $y^2 = 4x^2 + C$ D) $y^2 = x^2 - C$

25. The one dimensional wave equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$ is:

A) parabolic B) elliptic C) hyperbolic D) none of these

26. Which of the following statements about compact sets is **false**?

A) Every compact set in a Hausdorff space is closed

B) A compact subset of a topological space is bounded

C) Every compact set in a metric space is sequentially compact

D) A continuous image of a compact set is compact

27. Let $(X, d)$ be a metric space. Which of the following statements is **false**?

A) Every convergent sequence in $X$ is a Cauchy sequence

B) A subset of $X$ is compact if and only if it is closed and bounded

C) If $(X, d)$ is complete, then every Cauchy sequence in $X$ converges

D) Every bounded sequence in $X$ has a convergent subsequence

28. Which of the following topological spaces is **not** a Hausdorff space?

A) The Euclidean space $\mathbb{R}^n$ with the standard topology

B) The discrete topology on any set

C) The metric space $(X, d)$, where $d$ is a metric

D) None of these

29. If θ is the angle between the vectors (2,3,5) and $(1, -4, 3)$ in the real inner product space $\mathcal{R}^3$over $\mathcal{R}$, then which of the following is correct?

A) $sin^2\theta = \frac{963}{988}$ B) $sin^2\theta = 0$ C) $sin^2\theta = \frac{25}{988}$ D) $sin^2\theta = 1$

30. Which of the following is a necessary condition for a normed space to be a Banach space?

A) Every bounded sequence has a convergent subsequence

B) The space is finite-dimensional

C) The space is separable

D) Every Cauchy sequence converges

31. Let $X$ and $Y$ be Banach spaces and $F : X \to Y$ linear. If the graph of $F$ is closed in $X \times Y$ and $F$ is one to one and onto, then which of the following is true?

A) $F$ is a continuous map from $X$ into $Y$

B) $F$ is a non continuous map from$X$ into $Y$

C) $F$ is a linear homomorphism from $X$ onto $Y$

D) $F$ is a linear homomorphism from $X$ into $Y$

32. For $1 \leq p < \infty$ consider the sequence space $l^p$with p norm. If $x, y \in l^p$such that $x = (1, 0, 0, \dots)$ and $y = (0, 1, 0, \dots)$, then $\|x - y\|_p$is equal to:

A) 1 B) 2 C) $2^p$ D) $2^{\frac{1}{p}}$

33. If E is an orthonormal subset of an inner product space $X$, then for $u \neq v \in E$, the value of $\| u - v \|$ is:

A) 2 B) $\sqrt{2}$ C) $2^p$ D) $2^{\frac{1}{p}}$

34. The domain of the function $f(x) = \sqrt{16 - x^2}$ is:

A) $[0, \infty)$ B) $(-4,4)$ C) $[-4,4]$ D) $[4, \infty)$

35. Let $f: \mathbb{Z} \rightarrow \mathbb{N}$ be defined by $(x) = |x|$ . Then which of the following is TRUE?

A) $f$ is both one-one and onto

B) $f$ is one-one but not onto

C) $f$ is onto but not one-one

D) $f$ is neither one-one nor onto

36. If $x - axis$ is the perpendicular bisector of the line segment joining $A(3,5)$ and $B(x, y)$. Then the point $B$ is:

A) $(-3,5)$ B) $(3,-5)$ C) $(-3,-5)$ D) $(5,3)$

37. If the line $x + y = 2$ intersects the circle $x^2 + y^2 = 4$ at two points $A$ and $B$ then the length of $AB$ is:

A) 2 B) 4 C) $2\sqrt{2}$ D) 8

38. $\lim_{x \to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x}$ is:

A) $\sqrt{2}$ B) 2 C) $\frac{1}{\sqrt{2}}$ D) $\frac{1}{2\sqrt{2}}$

39. The least value of $a$ so that the function $f(x) = x^2 + ax + 5$ is increasing on [1,2] is:

A) $-2$ B) 0 C) 2 D) 4

40. An unfair die is such that the faces 1 to 5 are equally likely while face 6 is twice as likely as any other face. Then the probability of getting an even number is:

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{7}$ D) $\frac{4}{7}$

41. In how many ways can 3 red and 5 blue chairs be kept around a circular table if no two red chairs come together ?

A) 2880 B) 1440 C) 720 D) 240

42. Which of the following series is **not** convergent?

A) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \ldots$ B) $\frac{1}{2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^4} + \ldots$

C) $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots$ D) $1 + \frac{1}{4^{2/3}} + \frac{1}{9^{2/3}} + \frac{1}{16^{2/3}} + \ldots$

43. Which of the following sequence of functions converges uniformly on the given domain?

A) $f_n(x) = \frac{x}{n}, x\epsilon\mathbb{R}$  B) $f_n(x) = x^n, x\epsilon\mathbb{R}$

C) $f_n(x) = x^{\frac{1}{n}}, x > 0$  D) $f_n(x) = \frac{\sin nx}{n}, x\epsilon\mathbb{R}$

44. The value of $\int_0^2 |2x - 1|\, dx$ is:

A) 1  B) 2  C) $\frac{3}{2}$  D) $\frac{5}{2}$

45. The loci of the points $z$ satisfying the equation $|z - 1| + |z + 1| = 4$ is:

A) A circle centered at $(1, -1)$ and radius 2

B) A straight line passing through $(1, -1)$ and at a perpendicular distance of 4 from the origin

C) An ellipse with major axis 4 and minor axis $2\sqrt{3}$

D) A circle with $(-1,0)$ and $(1,0)$ as diameter

46. Which of the following is a single valued function of $z$

A) $w = z^2$  B) $w = \sqrt{z}$  C) $w = e^z$  D) $w = arg\ z$

47. Which of the following is **not** true?

A) If $f(z)$ is analytic then $\frac{df}{d\bar{z}} = 0$ at $z$

B) $f(z) = |z|^2$ is analytic everywhere

C) If $f(z) = u + iv$ is analytic then the curves $u = a\ constant$ and $v = a\ constant$ form an orthogonal system

D) If $f(x) = u + iv$ is analytic then $u$ and $v$ are both harmonic functions

48. The function $f(z) = sin\left(\frac{1}{1-z}\right)$ has:

A) a simple pole at $z = 1$

B) a removable singularity at $z = 1$

C) a non isolated essential singularity at $z = 1$

D) an isolated essential singularity at $z = 1$

49. The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n!}{n^n} z^n$ is:

A) 0 B) $e$ C) $\frac{1}{e}$ D) $\infty$

50. The remainder when $3^{256}$ is divided by 14 is:

A) 3 B) 7 C) 9 D) 11

51. The number of homomorphism from $\mathbb{Z}_8$ to $\mathbb{Z}_{12}$ is:

A) 1 B) 2 C) 4 D) 8

52. Let $\varphi: \mathbb{Z}[x] \to \mathbb{Z}$ given by $\varphi(f(x)) = f(0)$ be an onto mapping. Then which of the following is **not** true?

A) $\varphi$ is a ring homomorphism with $ker(\varphi) = \langle x \rangle$

B) $\frac{\mathbb{Z}[x]}{\langle x \rangle} \approx \mathbb{Z}$

C) The ideal $\langle x \rangle$ is prime

D) The ideal $\langle x \rangle$ is maximal

53. Which of the following is **not** true about the polynomial $f(x) = 2x^2 + 4$ ?

A) $f(x)$ is reducible over $\mathbb{Z}$

B) $f(x)$ is irreducible over $\mathbb{Q}$

C) $f(x)$ is reducible over $\mathbb{R}$

D) $f(x)$ is reducible over $\mathbb{C}$

54. The number of homomorphism from the group $\mathbb{Z}_6$ to $\mathbb{Z}_6$x$\mathbb{Z}_4$ is:

A) 4 B) 6 C) 8 D) 12

55. The dimension of the field $\mathbb{Q}(\sqrt{2}, \sqrt[3]{2})$ over the field $\mathbb{Q}(\sqrt{2})$ is:

A) 2 B) 3 C) 4 D) 6

56. The splitting field of the polynomial $x^4 + 1$ over the field $\mathbb{Q}$ is:

A) $\mathbb{Q}$ B) $\mathbb{Q}(\sqrt{2})$ C) $\mathbb{Q}(i)$ D) $\mathbb{Q}(i, \sqrt{2})$

57. The gcd of the polynomials $f(x) = x^3 + x^2 + x + 1$ and $g(x) = x^3 + 1$ is:

A) $x + 1$ B) $x^2 + 1$ C) $x^2 + x + 1$ D) $x^2 - x + 1$

58. Let $G$ be a non-abelian group of order 21. Then the number of sylow-3 subgroups is:

A) 1 B) 3 C) 7 D) 21

59. If the nullity of the matrix $\begin{pmatrix} k & 1 & 2 \\ 1 & -1 & -2 \\ 1 & 1 & 4 \end{pmatrix}$ is 1, then the value of $k$ is:

A) $-1$ B) 0 C) 1 D) 2

60. The rank of the matrix $\begin{bmatrix} 1 & 2 & 0 & 2 \\ -1 & -2 & 1 & 1 \\ 1 & 2 & -3 & -7 \\ 1 & 2 & -2 & -4 \end{bmatrix}$ is:

A) 1 B) 2 C) 3 D )4

61. Which of the following matrices cannot be obtained by elementary operation on the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{pmatrix}$?

A) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ B) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{pmatrix}$ C) $\begin{pmatrix} 3 & 0 & 1 \\ 2 & 2 & 2 \\ 1 & 3 & 1 \end{pmatrix}$ D) $\begin{pmatrix} 1 & 3 & 1 \\ 2 & 2 & 2 \\ 3 & 1 & 3 \end{pmatrix}$

62. Let $A = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ be a matrix with integer entries such that $q \neq 0$.
If $A^2 + A + I_2 = 0$, then:
A) $p^2 - p - qr = 1$ B) $p^2 - p - qs = 1$
C) $p^2 + p + qr = -1$ D) $p^2 + p - qr = -1$

63. Let $\{u, v\}$ be linearly independent subset of a real vector space $V$. Then which of the following is **not** linearly independent?
A) $\{u, u - v\}$ B) $\{u + \sqrt{2}v,\ u - \sqrt{2}v\}$
C) $\left\{v, 2v - \frac{u}{2}\right\}$ D) $\{2u + v, -4u - 2v\}$

64. The dimension of the subspace $W = \{(x, y, z) \in \mathbb{R}^3 | 2x + 2y + z = 0,$ $3x + 3y - 2z = 0, x + y - 3z = 0\}$ of $\mathbb{R}^3$ is:

A) 0 B) 1 C) 2 D) 3

65. Let V be the vectorspace of all polynomials of degree less than or equal to 2. Let $S = \{x^2 + x + 1,\ x^2 + 2x + 2,\ x^2 + 3\}$. Then:

A) $S$ is linearly independent and does not span $V$

B) $S$ is linearly independent and spans $V$

C) $S$ is linearly dependent and does not span $V$

D) $S$ is linearly dependent and spans $V$

66. Which of the following is a linear transformation?

A) $T: \mathbb{R}^2 \to \mathbb{R}^3$ given by $T(x, y) = (x, y, xy)$

B) $T: \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (2x + y, x)$

C) $T: \mathbb{R} \to \mathbb{R}$ given by $T_a(x) = x + a$ for some non zero $a \in \mathbb{R}$

D) $T: \mathbb{R} \to \mathbb{R}$ given by $T(x) = x^2$

67. Let $P(\mathbb{R})$ denote the vectorspace of all polynomials over real field $\mathbb{R}$. Then the linear transformations $T: P(\mathbb{R}) \to P(\mathbb{R})$ given by $f(x) \to f'(x)$ is:

A) Both one-one and onto B) One-one but not onto

C) Not one-one but onto D) Neither one-one nor onto

68. If $\lambda$ is an eigenvalue of a matrix $A$ then the eigenvalue of $adj\ A$ is:

A) $\lambda$ B) $\frac{1}{\lambda}$ C) $|A|\lambda$ D) $\frac{|A|}{\lambda}$

69. The last digit of the number $2023^{2024^{2025}}$ is:

A) 1 B) 2 C) 3 D) 4

70. If $p$ is a prime number then by Fermat's Theorem:

A) $2^{p-1} - 2$ is divisible by $p$

B) $2^p - 1$ is divisible by $p$

C) $2^p - 2$ is divisible by $p$

D) $2^{p-1} - 1$ is divisible by $p$

71. The differential equation of all circles of radius $r$ whose centres lie on the

x-axis:

A) $y\left(1+\frac{dy}{dx}\right)^2=r^2$ B) $y^2\left(1+\left(\frac{dy}{dx}\right)^2\right)=r^2$

C) $y^2\left(1+\frac{dy}{dx}\right)=r^2$ D) $y^2\left(1+\frac{dy}{dx}\right)^2=r^2$

72. The solution of the differential equation $(x+y)(dx-dy)=dx+dy$ is:

A) $x^2+y^2=c$ B) $e^{x+y}=c(x-y)$

C) $x+y=ce^{x-y}$ D) $e^{x+y}=e^{x-y}+c$

73. The orthogonal trajectory of the family of parabolas $y=ax^2$ is:

A) $x^2+y^2=c$ B) $2x^2+y^2=c$

C) $x^2-y^2=c$ D) $x^2+2y^2=c$

74. The value of the Bessel function $J_{\frac{1}{2}}(x)$ is:

A) $\sqrt{2\pi}\sin x$ B) $\sqrt{\frac{2}{\pi}}\sin x$ C) $\sqrt{\frac{2}{\pi x}}\sin x$ D) $\sqrt{\frac{\pi}{2x}}\sin x$

75. The solution of the partial differential equation $\frac{\partial^2 z}{\partial x^2}+z=0$ given when $x=0, z=e^y$ and $\frac{\partial z}{\partial x}=1$

A) $z=e^y\cos x+\sin x$ B) $z=e^y\sin x+\cos x$

C) $z=e^y(\sin x+\cos x)$ D) $z=e^y(\sin x-\cos x)$

76. Let $(X,d)$ be a metric space and $A(\neq \emptyset)$ be a subset of $X$. Consider the following statements:

(I) Every convergent sequence in $A$ is a Cauchy sequence in $A$

(II) Every Cauchy sequence in $A$ is a convergent sequence in $A$

Then which of the following is correct?

A) (I) is True but (II) is False
B) (I) is False but (II) is True
C) Both (I) and (II) are True
D) Both (I) and (II) are False

77. Which of the following is **not** true about compact sets:

A) $\mathbb{R}$ with usual metric is not compact
B) Infinite discrete metric space is compact
C) Continuous image of a compact set is compact
D) Let $\mathbb{R}$ be with usual metric then $A = [0,1] \subseteq \mathbb{R}$ is compact

78. Let $\tau$ be the smallest topology on the set $\mathbb{R}$ containing :

$\beta = \{[a, b) | a < b, a, b \in \mathbb{R}\}$

(I) $\mathbb{R}$ is compact in the topology $\tau$
(II) $\tau$ is Hausdorff
Which of the following is correct?

A) Both (I) and (II) are True
B) (I) is True but (II) is False
C) (II) is True but (I) is False
D) Both (I) and (II) are False

79. Dual space of $l^p$ $(p > 1)$ is:
A) $l^\infty$
B) $l^q$ where $q = 1 - p$
C) $l^q$ where $\frac{1}{p} + \frac{1}{q} = 1$
D) $l^q$ where $\frac{1}{p} - \frac{1}{q} = 1$

80. If $W$ is a subspace of an inner product space $V$ and $W^\perp$ is orthogonal complement of $W$. Then which of the following is **not** true?
A) $W^\perp$ is a subspace of $V$
B) $(W^\perp)^\perp = W$
C) $W \cap W^\perp \neq \{0\}$
D) $V = W \oplus W^\perp$