

1. Assume that the coefficients of the quadratic equation $ax^2 + bx + c = 0$ are uniformly distributed over the interval $(0,1)$. Then the probability that the quadratic equation has real roots?

A) $\frac{2\log 2}{8}$ B) $\frac{3+2\log 4}{5}$ C) $\frac{2+3\log 2}{5}$ D) None of these
2. Consider a Markov chain with state space $\{1,2,3\}$ and transition matrix

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}.$$
 Then the stationary distribution is:

A) $\frac{2}{9}, \frac{4}{9}, \frac{1}{3}$ B) $\frac{1}{9}, \frac{3}{9}, \frac{5}{9}$ C) $\frac{4}{9}, \frac{1}{9}, \frac{4}{9}$ D) $\frac{3}{9}, \frac{3}{9}, \frac{1}{3}$
3. Let X_1, X_2, \dots, X_n ($n > 1$) be a random sample from uniform $U[2,10]$ population and define

$$Y = \log \left(\prod_{i=1}^n \frac{1}{Z_i^2} \right),$$
 where $Z_i = \frac{10-X_i}{8}$. Then $E(Y) = ?$

A) $n^2 + 2n$ B) n^2 C) $4n$ D) $2n$
4. Under which of the following conditions negative binomial $NB(r, p)$ converges to Poisson(θ)?

A) $r \rightarrow \infty, p \rightarrow 1$ and $r(1-p) = \theta$
 B) $r \rightarrow \infty, rp = \theta$
 C) $p \rightarrow 1$ and $r(1-p) = \theta$
 D) None of these
5. Let X be a non-negative integer valued random variable with pgf $P(s)$. Then the pgf of the random variable $(X < n)$ is:

A) $\frac{P(s)}{1-s}$ B) $\frac{sP(s)}{1-s}$
 C) $\frac{P\left(s^{\frac{1}{2}}\right) + P\left(s^{-\frac{1}{2}}\right)}{2}$ D) $\min \left\{ P\left(s^{\frac{1}{2}}\right), P\left(s^{-\frac{1}{2}}\right) \right\}$

6. Let X be a non-negative integer valued random variable with pgf $P(s)$. Then $\int_0^1 P(s)ds = ?$
- A) $E\left(\frac{1}{X+1}\right)$ B) $E\left(\frac{X}{X+1}\right)$ C) $E\left(\frac{1}{X}\right)$ D) $E\left(\frac{X}{X+2}\right)$
7. The correlation coefficient between the number of success and the number of failures in a binomial distribution is:
- A) -1 B) 0 C) 1 D) ± 1
8. Let X be a random variable with pdf $f(x) = \begin{cases} \frac{2x}{\pi^2}, & 0 < x < \pi \\ 0, & \text{otherwise} \end{cases}$. Then the pdf of $Y = \sin X$ is :
- A) $\frac{2}{\pi\sqrt{1-y^2}}, 0 < y < 1$ B) $\frac{1}{\pi\sqrt{1-2y^2}}, 0 < y < \frac{1}{\sqrt{2}}$
- C) $\frac{\pi}{2\sqrt{1-y^2}}, 0 < y < 1$ D) $\frac{\pi}{\sqrt{1-2y^2}}, 0 < y < \frac{1}{\sqrt{2}}$
9. Let X be a random variable with $E(X) = 0, E(X^2) = 1$ and fourth central moment $\mu_4 = 16$. Then the upper bound of $P\{|X| > 2\}$ is :
- A) $\frac{5}{8}$ B) $\frac{1}{4}$ C) $\frac{3}{4}$ D) $\frac{3}{8}$
10. Let (X, Y) be a random vector with distribution function $F(x, y) = (1 - e^{-x})(1 - e^{-y}), 0 < x < \infty, 0 < y < \infty$. Then $P(X < 2Y)$ is:
- A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) $\frac{2}{3}$ D) $\frac{3}{4}$
11. Let X_1 and X_2 be independent random variables with pdf $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$. Then $P\{|X_1 - X_2| < 0.5\}$ is :
- A) $\frac{1}{4}$ B) $\frac{1}{8}$ C) $\frac{3}{4}$ D) $\frac{3}{8}$

12. Let X_1, X_2, \dots, X_n be a random sample of size n from uniform distribution with pdf $f(x) = \begin{cases} \frac{1}{6}, & -3 < x < 3 \\ 0, & \text{otherwise} \end{cases}$. The large sample distribution of $\sqrt{n}\bar{X}$ is:
- A) $N(0,3)$ B) $U(-1,1)$ C) $U\left(-\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)$ D) $U\left(0, \frac{2}{\sqrt{n}}\right)$
13. Characteristic function of the random variable X is given by $\varphi(t) = \left(\frac{\sin t}{t}\right)^2, t \in R$. Then $P(1 < X < 2)$ is :
- A) $\frac{1}{8}$ B) $\frac{2}{8}$ C) $\frac{3}{8}$ D) $\frac{1}{2}$
14. Let Y_1, Y_2, Y_3 and Y_4 be four uncorrelated random variables each with variance 9, then the value of the correlation coefficient between $Y_1 + Y_2 + Y_3$ and $Y_2 + Y_3 + Y_4$ is :
- A) $\frac{2}{3}$ B) $\frac{1}{3}$ C) 0 D) 1
15. A normal population has mean 10 and standard deviation 2. What is the probability that the mean of a sample of size 256 will be greater than 10?
- A) 0.4562 B) 0.4265 C) 0.4652 D) 0.5
16. Two friends decided to meet between 2 pm and 3 pm with the condition that one waits the other for at most 20 minutes. The probability of chance of their meeting is:
- A) $\frac{4}{9}$ B) $\frac{5}{9}$ C) $\frac{1}{3}$ D) $\frac{2}{3}$
17. Let X and Y be two independent Poisson random variables with mean values 2 and 3 respectively. Then $var(X|X + Y = 10)$ is:
- A) 4 B) 6 C) $\frac{12}{5}$ D) $\frac{6}{5}$
18. Which one of the following is an example of negatively skewed data ?
- A) age of death from natural causes
 B) distribution of income in a country
 C) distribution of scores on a difficult exam
 D) distribution of movie ticket sales

19. Which of the following estimators is not a consistent estimator for θ based on a random sample of size n taken from uniform $U[0, \theta]$ distribution?
- A) $X_{(n)}$ B) $X_{(1)} + X_{(n)}$ C) $(n+1)X_{(n)}$ D) $2\bar{X}$
20. A random sample of size n is taken from $N(\mu, 1)$ population. Which one of the following is an unbiased estimator of μ^2 ?
- A) $\bar{X}^2 + \frac{n+1}{n}$ B) $\bar{X}^2 + \frac{n-1}{n}$ C) $\bar{X}^2 - \frac{1}{n}$ D) $\bar{X}^2 + \frac{1}{n}$
21. Let X be a single observation drawn from normal population $N(0, \sigma^2)$. Which of the following is a sufficient estimator of σ^2 ?
- A) X B) $|X|$ C) X^2 D) All of these
22. Let X_1, X_2 and X_3 be three independent observations on a random variable X , which follows Poisson distribution with parameter θ . Then which of the following is a sufficient estimator of θ ?
- A) $\frac{X_1 + 2X_2 + 3X_3}{7}$ B) $X_1 + 2X_2 + 3X_3$
- C) $X_1 + X_2 + X_3$ D) None of these
23. Let $X \sim b(1, p), p \in [\frac{1}{4}, \frac{3}{4}]$. The maximum likelihood estimator of p is given by:
- A) $\frac{2X+1}{4}$ B) $\frac{X+2}{4}$ C) X D) $\frac{3-2X}{4}$
24. Let X be a random variable with pdf $f_\theta(x) = \begin{cases} \frac{1}{\theta}, & 0 \leq x \leq \theta \\ 0, & \text{otherwise} \end{cases}$.
- The UMVUE of θ is given by:
- A) X B) $2X$ C) $\frac{X}{2}$ D) $\frac{X+1}{2}$
25. Let T_1 be the most efficient estimator of θ and T is another estimator whose efficiency is e . Then the correlation between T_1 and T is:
- A) \sqrt{e} B) $\frac{1}{\sqrt{e}}$ C) $\frac{1}{e}$ D) e

26. The most powerful test given by Neyman-Pearson for testing a simple hypothesis against a simple alternative is strictly unbiased if:
- A) $\alpha = 1$ B) $0 < \alpha < 1$ C) $0 \leq \alpha < 1$ D) $0 \leq \alpha \leq 1$
27. A sample of size n is taken from uniform $U[0, \theta]$. Then which of the following hypothesis is/are UMP?
- A) $H_0: \theta = \theta_0$ vs $H_1: \theta > \theta_0$
 B) $H_0: \theta = \theta_0$ vs $H_1: \theta < \theta_0$
 C) $H_0: \theta = \theta_0$ vs $H_1: \theta \neq \theta_0$
 D) All of these
28. A test is said to be consistent if:
- A) Size of the test tends to unity as the sample size tends to infinity
 B) Power of the test tends to unity as the sample size tends to infinity
 C) Power and size tends to zero as sample size tends to infinity
 D) power and size tend to unity as sample size tends to infinity
29. Let X_1, X_2, \dots, X_n be iid random variables with finite expectation and let N be a random variable independent of $X_i, i = 1, 2, \dots, n$. Define $Y = \sum_{i=1}^N X_i$, then $\text{var}(Y)$ is:
- A) $E(N)\text{var}(X_i)$
 B) $E(N)\text{var}(X_i) + [E(X_i)]^2\text{var}(N)$
 C) $[E(X_i)]^2\text{var}(N) + E[N^2]\text{var}(X_i)$
 D) None of these
30. If X_1, X_2, \dots, X_n is a random sample from uniform $U[0, \theta]$, then which of the following statements are correct?
- MLE of θ is same as the estimate given by method of moments
 - MLE of θ is $X_{(n)}$, the largest observation in the sample
 - $\frac{n+1}{n}X_{(n)}$ is an unbiased estimator of θ
 - Method of moment estimator is unbiased for θ
- A) 1, 3 & 4 only B) 2, 3 & 4 only
 C) 1 & 3 only D) 2 & 4 only

31. Let X_1, X_2, \dots, X_n be a random sample from a Poisson(θ) population and let $T = \sum_{i=1}^n X_i$. Then the UMVUE of $\exp(-\theta)$ is :
- A) $\left(\frac{n-1}{n}\right)^T$ B) $\frac{T}{n}\left(1 - \frac{1}{n}\right)^{T-1}$ C) $\left(\frac{n+1}{n}\right)^T$ D) None of these
32. If $X \sim N(\mu, \sigma^2)$, then approximate variance of $\sin X$ is :
- A) $\sigma^2 \sin^2 \mu$ B) $\sigma^2 \cos^2(\mu + \sigma^2)$
 C) $\sin^2(\sigma^2 + \mu)$ D) $\sigma^2 \cos^2 \mu$
33. Kendall's tau statistic for the bivariate data (1,9), (2,10), (3,8) and (4,11) is :
- A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) $\frac{1}{2}$ D) $\frac{1}{4}$
34. If $(X, Y) \sim BN(0,0,1,1,0.5)$, then $P\{XY > 0\}$ is:
- A) $\frac{1}{2}$ B) 0 C) $\frac{1}{3}$ D) $\frac{1}{4}$
35. Let Y_1, Y_2, \dots, Y_n be iid random variables with pdf $g(x) = \begin{cases} \frac{1}{N}, & x = 1, 2, \dots, N \\ 0, & \text{otherwise} \end{cases}$.
- The MLE of N has which one of the following properties?
- A) Unbiased, sufficient and complete
 B) Consistent, sufficient and complete
 C) Unbiased and sufficient
 D) Consistent and sufficient
36. A random sample {5,1,3,0} be taken from Poisson distribution with parameter θ . Then 95% asymptotic confidence interval of θ is :
- A) (0.78,4.72) B) (0.78,3.72)
 C) (2.25,3.72) D) (2.25,4.72)
37. Let X be a Poisson random variable with parameter θ . Then distribution function of X can be expressed in terms of:
- A) Gamma function B) Incomplete gamma function
 C) Beta function D) Incomplete beta function

38. Which of the following distributions do not belong to the exponential family?
1. Uniform family of distributions $U[\alpha, \beta]$
 2. Cauchy family of distributions $\mathcal{C}[\alpha, \beta]$
 3. Gamma family of densities $G[\alpha, \beta]$
 4. Family of multinomial distributions
- A) 1, 2 & 3 only B) 1, 3 & 4 only
C) 2 & 4 only D) 1 & 2 only
39. Let X_1, X_2, \dots, X_{10} be independent normal random variables with mean 2 and variance 1 and let $Z = \sum_{i=1}^{10} X_i^2$. Then $E(Z) = ?$
- A) 50 B) 40 C) 45 D) None of these
40. For the SPRT with stopping bounds A and B with $B < A$ and strength (α, β) , then the stopping bounds hold:
- A) $A \leq \frac{1-\beta}{\alpha}, B \geq \frac{1-\alpha}{\beta}$ B) $A \leq \frac{\alpha}{1-\beta}, B \geq \frac{\beta}{1-\alpha}$
C) $A \leq \frac{1-\beta}{\alpha}, B \geq \frac{\beta}{1-\alpha}$ D) None of these
41. For a random sample of size 2 from $N(0, \sigma^2)$ population, the value of $E(X_{(1)})$ is----, where $X_{(1)}$ is the first order statistic.
- A) $\frac{2\sigma}{\sqrt{\pi}}$ B) $\frac{\sigma}{\pi}$ C) $\frac{-\sigma}{\sqrt{\pi}}$ D) $\frac{-\sigma}{\sqrt{2\pi}}$
42. Lebesgue measure of the set of irrational numbers in $[-2, 2]$ is :
- A) 0 B) 4 C) less than 4 D) 2
43. Let $E_n = \left\{ \omega \in \mathbb{R} : 0 < \omega < 5 + \frac{\cos n\pi}{n} \right\}$ be a sequence of sets. Then $\lim(\sup A_n)$ is :
- A) (0,5) B) [0,5] C) (0,5] D) Does not exist
44. If i is a complex number, then the value of i^{-i} is :
- A) $\frac{i\pi}{2}$ B) $e^{\frac{-\pi}{2}}$ C) $e^{\frac{\pi}{2}}$ D) $-\frac{i\pi}{2}$

45. For what value of α does the strong law of large numbers (SLLN) hold for the sequence $P\{X_n = \pm n^\alpha\} = \frac{1}{2}$?
- A) $0 < \alpha < 1$ B) $\alpha > 1$ C) $\alpha < \frac{1}{2}$ D) $\alpha > \frac{1}{2}$
46. If X_1, X_2, \dots, X_n are n iid negative binomial random variables $NB(1; p)$, then the distribution of $\min(X_1, X_2, \dots, X_n)$ is :
- A) Geometric with parameter $(1 - p)^n$
 B) Geometric with parameter $1 - p^n$
 C) Geometric with parameter $1 - (1 - p)^n$
 D) None of these
47. Baye's estimator of the parameter θ under the quadratic loss function is the----.
- A) Mean of posterior distribution
 B) Median of posterior distribution
 C) Variance of posterior distribution
 D) None of these
48. The distribution of consistent solution of the likelihood equation is :
- A) Truncated random variable
 B) Standard normal variable
 C) Chi-square variable
 D) Degenerte random variable
49. Given P is an ergodic chain with stationary distribution $\left\{\frac{3}{8}, \frac{1}{8}, \frac{4}{8}\right\}$. Then $\lim_{n \rightarrow \infty} P^n = ?$
- A) $\begin{bmatrix} \frac{3}{8} & \frac{1}{8} & \frac{4}{8} \\ \frac{1}{8} & \frac{3}{8} & \frac{4}{8} \\ \frac{4}{8} & \frac{3}{8} & \frac{1}{8} \end{bmatrix}$ B) $\begin{bmatrix} \frac{3}{8} & \frac{1}{8} & \frac{4}{8} \\ \frac{3}{8} & \frac{1}{8} & \frac{4}{8} \\ \frac{3}{8} & \frac{1}{8} & \frac{4}{8} \end{bmatrix}$ C) $\begin{bmatrix} \frac{3}{8} & \frac{1}{8} & \frac{4}{8} \\ \frac{3}{8} & \frac{4}{8} & \frac{1}{8} \\ \frac{3}{8} & \frac{1}{8} & \frac{4}{8} \end{bmatrix}$ D) None of these
50. Identify the correct statement/s:
1. Uniformly most accurate lower confidence bound has the smallest probability of false coverage.
 2. Confidence level has the smallest probability of true coverage.
- A) 1 only B) 2 only C) Both 1 & 2 D) Neither 1 nor 2

51. Consider the following statements:

1. $X_i \sim N(\theta, \sigma^2)$; σ^2 known and θ unknown, $i = 1, 2, \dots, n$.

2. $X_i \sim b(1, \theta)$, θ unknown, $i = 1, 2, \dots, n$

The variance of sample mean coincides with the Cramer-Rao lower bound for every θ , for:

A) 1 only B) 2 only C) Both 1 & 2 D) Neither 1 nor 2

52. Let X_1, X_2, \dots, X_n be a random sample from uniform $U(0, \theta)$, $\theta > 0$. To test

$H_0: \theta = \theta_0$ vs $H_1: \theta = \theta_1$ ($\theta_1 > \theta_0$), consider the test function

$\varphi(x) = \begin{cases} 1 & \text{if } X_{(n)} > \theta_0 \\ \alpha, & \text{if } 0 < X_{(n)} < \theta_0 \end{cases}$, then the size and power of the test are respectively:

A) $\alpha, 1 - \alpha$ B) $\frac{\alpha}{2}, 1 - \alpha$

C) $\alpha, 1 - \left(\frac{\theta_0}{\theta_1}\right)^n (1 - \alpha)$ D) $1 - \alpha, 1 - \left(\frac{\theta_0}{\theta_1}\right)^n (1 - \alpha)$

53. Let $\Omega = \{0, 1, 2, \dots\}$ and \mathcal{A} is the power set of Ω . For any $A \subset \Omega$, define $\mu(A)$ = number of non-negative integers in A and if $X(\omega) = 2^{-\omega}$. Then $\int_0^\infty X d\mu$ is ?

A) 1 B) 2 C) 0 D) None of these

54. Let \bar{A} be any generalized inverse of A and $\bar{A}A = H$. Identify the correct statement/s:

1. $AH = A$ and $H\bar{A} = \bar{A}$

2. $\text{Trace } H = \text{Rank } H$

A) 1 only B) 2 only C) Both 1 & 2 D) Neither 1 nor 2

55. For the matrices $A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$.

Identify the correct statements:

1. $ABA = A$

2. BA is idempotent

3. $\text{Rank } A \neq \text{Rank } BA$

4. $\text{Rank } BA = \text{Trace } BA$

A) 2 & 3 only B) 1, 2 & 3 only C) 2, 3 & 4 only D) 1, 2 & 4 only

56. Consider a BIBD with parameters (b, k, v, r, λ) . Which of the following **cannot** possibly be the parameters of a BIBD?
1. $(b - 1, k - \lambda, b - k, k, \lambda)$
 2. $(b, v - k, v, b - r, b - 2r + \lambda)$
 3. $\left(\frac{v(v-1)}{2}, 2, v, v - 1, 1\right)$
 4. $(k, b, r, v, \lambda - 1)$
- A) 1, 2 & 3 only B) 1, 3 & 4 only
C) 2, 3 & 4 only D) 1, 2 & 4 only
57. Three different methods of analysis are used to determine the amount of a certain constituent in a sample. Five different analysts generate one observation each under each of these methods. Various sum of squares are obtained as TSS=97.6, SSA (Analysts)=4.27, SSB(Methods)=79.6. The value of MSE, mean sum of squares due to error is:
- A) 2.89 B) 1.96 C) 1.72 D) 1.53
58. The following data represents the yield from sugar cane plants under 5 treatments tried in 4 randomized blocks. The estimate of missing value is:
- | Treatment | Blocks | | | |
|-----------|--------|----|----|----|
| | 1 | 2 | 3 | 4 |
| A | 20 | 13 | 12 | 35 |
| B | 24 | 28 | 30 | 25 |
| C | 42 | 22 | 21 | 46 |
| D | 28 | 48 | 27 | ? |
| E | 67 | 33 | 39 | 50 |
- A) 40 B) 43 C) 46 D) 44
59. If the stratum sizes are in the ratio 1: 2: 3 and stratum mean squares are in the ratio 2:3:4, then the samples drawn from the stratum will be in the ratio of:
- A) 1:4:6 B) 2: 3:6 C) 1:3:6 D) 1: 6:3
60. Which sampling is the best for estimating population having linear trend?
- A) Systematic sampling B) Stratified sampling
C) Simple random sampling D) None of these

61. Let $X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \sim N(\mu, \Sigma)$, where $\mu = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ and $\Sigma = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Then $E(X'\Sigma^{-1}X)$ is?

- A) 15 B) 18 C) 12 D) $\frac{19}{3}$

62. Let s_w^2 and s_b^2 denote respectively the variance within clusters and variance between clusters. Which of the following is true?

- A) $s_b^2 = s_w^2$ B) $s_b^2 \geq s_w^2$ C) $s_b^2 \leq s_w^2$ D) None of these

63. In an experiment with 4 treatments and 5 blocks, the sum of the ranks for treatments were 12, 15, 6 and 17. Friedman's F based on above data is:

- A) 23.28 B) -5.6 C) -68.06 D) 8.28

64. A sample of size 2 from a population of size 4 with units U_1, U_2, U_3, U_4 is to be selected. Match List I (possible samples) with List II (sampling techniques):

List I

List II

a. All samples (U_i, U_j)
($i, j = 1, 2, 3, 4$)

1. Systematic sampling

b. All samples (U_i, U_j)
($i < j; i, j = 1, 2, 3, 4$)

2. SRSWR

c. All samples (U_i, U_{i+k})
($i = 1, 2; k = 2$)

3. SRSWOR

A) a-2, b-3, c-1

B) a-1, b-2, c-3

C) a-2, b-1, c-3

D) a-1, b-3, c-2

65. A simple random sample of 5 households was drawn from a village containing 150 households. The number of persons per household in the sample were 5, 6, 4, 7 and 3. The estimate of the total number of people in the village is:

- A) 3750 B) 750 C) 1500 D) 100

66. If a sample of size 2 is drawn from a population of size 3 by the method of SRSWR, then the probability of selecting a specified unit in any sample would be:
- A) $\frac{2}{3}$ B) $\frac{4}{9}$ C) $\frac{5}{9}$ D) $\frac{1}{9}$
67. With usual notations, the ratio estimator of the population mean is more precise than the sample mean whenever $\rho > \frac{3}{8}$ if :
- A) $C_y = C_x$ B) $C_y = 2C_x$
 C) $3C_y = 4C_x$ D) $4C_y = 3C_x$
68. Which of the following sets is/are neither open nor closed in $R = (-\infty, \infty)$?
- A) Set of rational numbers B) Set of irrational numbers
 C) Both A and B D) None of these
69. Let $\{a_n\}$ be a sequence of real numbers defined by $a_n = \begin{cases} \left(1 + \frac{1}{n}\right)^n, & \text{if } n \text{ is odd} \\ \left(1 - \frac{1}{n}\right)^n, & \text{if } n \text{ is even} \end{cases}$.
 Then $\limsup a_n$ is:
- A) e B) $\frac{1}{e}$ C) e^2 D) None of these
70. Let f be defined on $[0,1]$ by $f(0) = 0, f(x) = x \sin\left(\frac{1}{x}\right), x > 0$. The Lebesgue measure of the set $\{x: f(x) > 0\}$ is:
- A) $\frac{1-\log 2}{\pi}$ B) $\frac{\log 2}{\pi}$ C) $\frac{\pi}{1-\log 2}$ D) $\frac{\pi}{1+\log 2}$
71. Consider the multiple linear regression model $Y = X\beta + \epsilon$, where ϵ 's are uncorrelated with zero mean and variance σ^2 . The regression matrix
- $$X = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & 2 \end{bmatrix} \text{ and } \beta' = (\beta_0, \beta_1). \text{ Then } \text{var}(\hat{\beta}_1) ?$$
- A) $\frac{\sigma^2}{3}$ B) $\frac{\sigma^2}{5}$ C) $\frac{\sigma^2}{6}$ D) $\frac{\sigma^2}{2}$
72. If X is $N_m(\mu, \Sigma)$, where A is an $m \times m$ symmetric matrix and B is an $r \times m$ matrix, then $X'AX$ and BX are independent if and only if :
- A) $B\Sigma A = A$ B) $B\Sigma A = B$ C) $B\Sigma A = I$ D) $B\Sigma A = 0$

73. Let $h(x) = \frac{d}{dx} \int_x^{x^2} \log t dt$. Then $h(e)$ equals:
- A) $4e - 1$ B) $4 \log 2$ C) $4 \log 2 - 1$ D) $\frac{e}{\log 2}$
74. Which one of the following options is/are correct?
1. purchasing power of money = $\frac{1}{\text{cost of living index}}$
2. Real wages = $\frac{\text{money wages} \times 100}{\text{cost of living index}}$
- A) 1 only B) 2 only C) Both 1 & 2 D) None of these
75. The maximum value of the function $h(x_1, x_2, x_3) = x_1 x_2 x_3$, $0 \leq x_i \leq 3$, $i = 1, 2, 3$; $x_1 + x_2 + x_3 = 3$ is:
- A) 27 B) 9 C) 3 D) 1
76. Probability distribution of the offspring distribution is given as follows:

$X = x$	0	1	2	3
$f(x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

The probability of ultimate extinction is:

- A) 0.25 B) 0.5 C) 0.6 D) 1
77. Which one of the following functions is **not** convex in $(1, \infty)$?
- A) x^2 B) $x + \frac{1}{x}$ C) $\log\left(\frac{1}{x}\right)$ D) $\log x$
78. Let $g(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1, & \text{if } x \geq 0 \end{cases}$. Then the Lebesgue Stieltjes measure function generated by $g(x)$ on $(-1, 0)$ is :
- A) 1 B) -1 C) 0 D) None of these

79. Identify the correct statements:

1. R square value decreases or retains the same value when additional variables are included in the model.
2. R square has advantages over adjusted R square in regression modelling.
3. Adjusted R square value decreases when insignificant variables included in the model.
4. Adjusted R square value increases when significant variables included in the model.

- A) 1, 3 & 4 only B) 2 & 4 only
C) 1 & 3 only D) 3 & 4 only

80. The minimum value of $\text{var}(Y - cX)$, for all the values of 'c' is given by:

- A) $\frac{\rho^2 \text{var}(Y)}{\text{var}(X)}$ B) $\frac{\rho^2 \text{var}(X)}{\text{var}(Y)}$ C) $\rho^2 \text{var}(Y)$ D) $(1 - \rho^2) \text{var}(Y)$
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