ENTRI

MATHEMATICAL PHYSICS
COMPLEX ANALYSIS

NICEMON THOMAS

17th century

$$z^2 - 4z + 5 = 0$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{4 \pm \sqrt{-4}}{2}$$

$$= \frac{4 \pm 2i}{2}$$

$$= 2 \pm 1i$$

Real part — imaginary part

Properties of complex numbers

① complex numbers obey closure property in addition, multiplication and exponents

ie, $z_1 + z_2 = z_3 \in \mathbb{C}$

$z_1 \cdot z_2 \in \mathbb{C}$

$z_1^{z_2} \in \mathbb{C}$

② Existance of identity

$z + 0 = z$ → additive identity

$z \cdot 1 = z$ → multiplicative identity

ENTRI

(3) Existance of inverse

$z + (-z) = 0$

↳ additive inverse

$z \cdot \frac{1}{z} = 1$

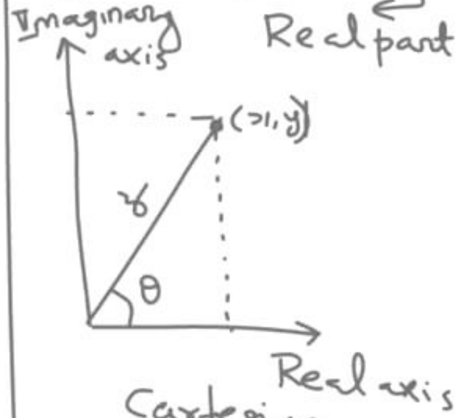
↳ multiplicative inverse

Representation of a complex number

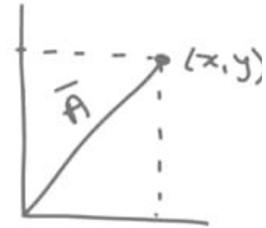
$z = (x + iy) = (x, y)$

Real part

↳ imaginary part



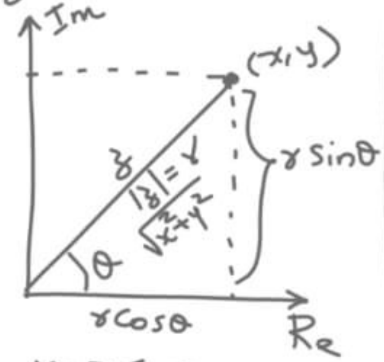
Cartesian Representation



$\bar{A} = x\hat{i} + y\hat{j}$

Polar Form

$z = (x+iy) = (x,y)$



$x = r\cos\theta$

$y = r\sin\theta$

$\frac{y}{x} = \tan\theta$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$z = x+iy$

$z = r\cos\theta + i(r\sin\theta)$

$= r(\underbrace{\cos\theta + i\sin\theta})$

$z = \quad re^{i\theta}$

$\boxed{z = re^{i\theta}}$

$z = re^{i\theta}$

$r = |z|$ magnitude

θ → angle b/w z and +x axis (argument)

ENTRI

Properties

Let $z_1 = r_1 e^{i\theta_1}$

$z_2 = r_2 e^{i\theta_2}$

① $z_1 z_2 = r_1 r_2 e^{i(\theta_1+\theta_2)}$

$|z_1 z_2| = |r_1||r_2|$

$= |z_1||z_2|$

Argument $(z_1 z_2)$

$= \text{Arg}.\, z_1 + \text{Arg}.\, z_2$

② $\dfrac{z_1}{z_2} = \dfrac{r_1 e^{i\theta_1}}{r_2 e^{i\theta_2}} = \dfrac{r_1}{r_2} e^{i(\theta_1-\theta_2)}$

$\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$

$\text{Arg}\left(\dfrac{z_1}{z_2}\right) = \text{Arg}.\, z_1 - \text{Arg}.\,(z_2)$

THANK YOU