

SCIENTIFIC OFFICER: PHYSICS


CALCULUS

UNNI R

Limits of Trigonometric Functions

$$\lim_{x \rightarrow 0} \sin x = 0$$

$$\lim_{x \rightarrow 0} \cos x = 1$$

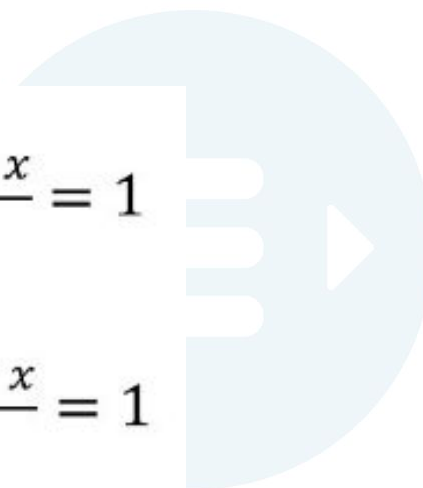
$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x} = 1$$



Limits of Log & Exponential Fns.

$$\lim_{x \rightarrow 0} e^x = 1$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a$$

$$\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^x = e^a$$

Limits of the form 1^∞

$$\lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}} = e$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^x = e^a$$



Limits of the form x^n

$$\lim_{x \rightarrow a} \frac{(x^n - a^n)}{x - a} = n(a)^{n-1}$$



Existence of limit

- To check if limit exist for $f(x)$ at $x = a$

$$\text{i.e. } \lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$$

➤ **Differentiability** $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

L-Hospital's Rule

If $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ gives $\frac{0}{0}$ form

where

$$f(a) = 0$$

$$g(a) = 0$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$$

Basic Derivatives

$$\frac{dk}{dx} = 0$$

where $k = \text{constant}$

$$\frac{d(x)}{dx} = 1$$

$$\frac{d(kx)}{dx} = k$$

$$\frac{d(x^n)}{dx} = nx^{n-1}$$



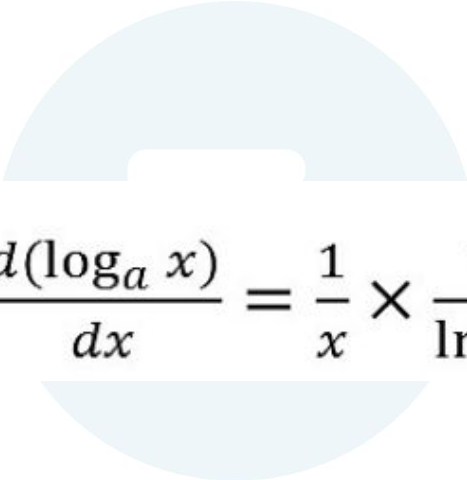
Differential of Log & Exponential Fns.

$$\frac{d(e^x)}{dx} = e^x$$

$$\frac{d(\ln(x))}{dx} = \frac{1}{x}$$

$$\frac{d(a^x)}{dx} = a^x \log a$$

$$\frac{d(x^x)}{dx} = x^x (1 + \ln x)$$


$$\frac{d(\log_a x)}{dx} = \frac{1}{x} \times \frac{1}{\ln a}$$


Differential of Trigonometric Fns.


$$\frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d(\cos x)}{dx} = -\sin x$$

$$\frac{d(\tan x)}{dx} = \sec^2 x$$

$$\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$$


$$\frac{d(\sec x)}{dx} = \sec x \tan x$$


$$\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x \cot x$$

Differential Rules

Product Rule $\frac{d}{dx} (f(x) g(x)) = f'(x) g(x) + f(x) g'(x)$

Quotient Rule $\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x) g(x) - f(x) g'(x)}{(g(x))^2}$

Chain Rule $\frac{d(f(g(x)))}{dx} = f'(g(x)) g'(x)$

Hyperbolic Functions.

$\sinh(x) = \frac{e^x - e^{-x}}{2}$	$\operatorname{csch}(x) = \frac{1}{\sinh(x)} = \frac{2}{e^x - e^{-x}}, x \neq 0$
$\cosh(x) = \frac{e^x + e^{-x}}{2}$	$\operatorname{sech}(x) = \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}}$
$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$	$\operatorname{coth}(x) = \frac{1}{\tanh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}, x \neq 0$

Differential of Hyperbolic Fns.

$$\frac{d}{dx} \sinh(x) = \cosh(x)$$

$$\frac{d}{dx} \cosh(x) = \sinh(x)$$

$$\frac{d}{dx} \tanh(x) = \operatorname{sech}^2(x)$$

$$\frac{d}{dx} \operatorname{coth}(x) = -\operatorname{csch}^2(x)$$

$$\frac{d}{dx} \operatorname{sech}(x) = -\tanh(x) \operatorname{sech}(x)$$

$$\frac{d}{dx} \operatorname{csch}(x) = -\operatorname{coth}(x) \operatorname{csch}(x)$$

$$\lim_{x \rightarrow 0} \frac{\sin x^\circ}{x}$$



- $\lim_{x \rightarrow 0} \frac{\log x}{\cot x}$



- $\lim_{x \rightarrow \frac{\pi}{2}} (1 - \sin x) \tan x$



- $\lim_{x \rightarrow 0} x^x$



- $\lim_{x \rightarrow 0} [3\cos x + 2\sin 3x]^{\frac{1}{x}}$

$x \rightarrow 0$



- Find the maximum & minimum value of the function $f(x) = 2x^3 - 24x + 107$ in $[1,3]$



THANK YOU

