entri

OSCILLATIONS

VINU SREEDHAR

entri

Periodic Motion

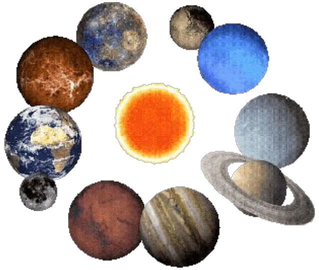
- A motion which repeats at regular intervals of time is called a periodic motion.

  Examples:-

i. The motion of any planet around the sun.

ii. The motion of moon around earth.

iii. The motion of the hands of a clock.



Oscillation or Harmonic motion

- To and fro motion of a body about a mean position is called oscillation or harmonic motion.
- Oscillations with high frequency are usually called vibrations.

Examples of Oscillation or Vibration:-

i. Oscillation of a simple pendulum.
ii. To and fro motion of the piston of an automobile engine.
iii. Vibrations of an excited turning fork.

entri

Difference between Periodic and Oscillatory motions:

- Every oscillatory motion is necessarily periodic. But every periodic motion need not be oscillatory. For example, The motion of earth around the sun is periodic but it is not oscillatory.

entri

Period (T)

- The smallest time interval after which a periodic motion is repeated is called the time period (T).
- If a particle oscillates N times in a time 't' seconds, its time period.

$$T = t / N$$

Frequency(V)

- The number of repetitions per second is called the frequency.
- If a particle oscillates N times in a time ‘t’ seconds, its frequency.

$$V = \frac{N}{t} = \frac{1}{T}$$

SI unit is S^{-1} or Hz (hertz).

entri

Angular Frequency(ω)

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

SI Unit:- radian per second (rad s^{-1})

entri

Problem 1:-

On an average a human heart is found to beat 72 times in a minute. Calculate period and frequency of the of heart beat.

Solution:

N = 72, t = 1 min. = 60 sec., T = ?, v = ?

T = t /N = 60 /72 = 0.83s

V = 1 /T = 1 /0.83 = 1.2 Hz

entri

Displacement Variable

- The physical quantity which changes with time in a Periodic motion is called displacement Variable or displacement.
- Displacement variable can be physical quantities such as position, angle, voltage, pressure, electric field, magnetic field etc.

entri

Examples:-

- When a body attached at the end of a spring vibrates, the displacement variable is the position vector from its equilibrium position.
- In the case of the oscillation of a simple pendulum, the displacement variable is the angle from the vertical.
- In the study of a.c, the displacement variable is the voltage or current.

entri

- For sound waves travelling through air, the displacement variable is the pressure.
- For the propagation of electromagnetic waves the displacement variable is the electric and magnetic field vectors.

Mathematical Representation of Periodic motion

- A periodic motion can be represented using a sine function, a cosine function or a linear combination of sine and cosine function.

  $f(t) = A \cos \omega t$

  Or

  $f(t) = A \sin \omega t$

  Or

  $f(t) = A \sin \omega t + B \cos \omega t$

- A periodic function has the property $f(t+T) = f(t)$, where T is the time period of the function.

Simple Harmonic Motion (SHM)

- Definition:- An oscillating particle is said to execute SHM if the restoring force on the particle at any instant of time is directly proportional to its displacement from the mean position and is always directed towards the mean position.

  Restoring force α Displacement

  F α x

entri

or **F = -kx**

k ⟶ force constant

By Newton's 2nd Law

F = ma

⇒ ma = -kx

⇒

$$a = \frac{-k}{m}x$$

entri

Examples of SHM:

i. Oscillations of a loaded spring
ii. Vibrations of a tuning fork
iii. Vibrations of balance wheel of a watch.
iv. Oscillations of a simple pendulum
v. Oscillations of a freely suspended magnet in a uniform magnetic field.

Differential equation of SHM

We have F = -kx for SHM.

By Newton's 2nd law F = ma,

$$a=\frac{d(v)}{dt}=\frac{d}{dt}\left(\frac{dx}{dt}\right)=\frac{d^2x}{dt^2}$$

$$\therefore m\frac{d^2x}{dt^2}=-kx$$

$$\text{or } \frac{d^2x}{dt^2}=\frac{-k}{m}x$$

Put $\frac{k}{m} = \omega^2$, then $\frac{d^2x}{dt^2} = -\omega^2 x$

I.e., $\frac{d^2x}{dt^2} + \omega^2 x = 0$,

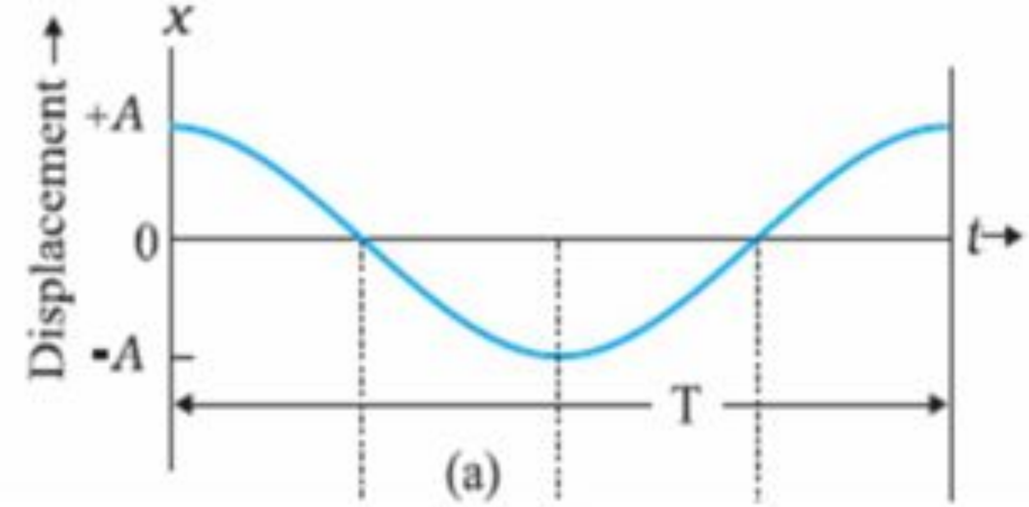
which is the differential equation of SHM.

The solution of this Differential equation is of the form

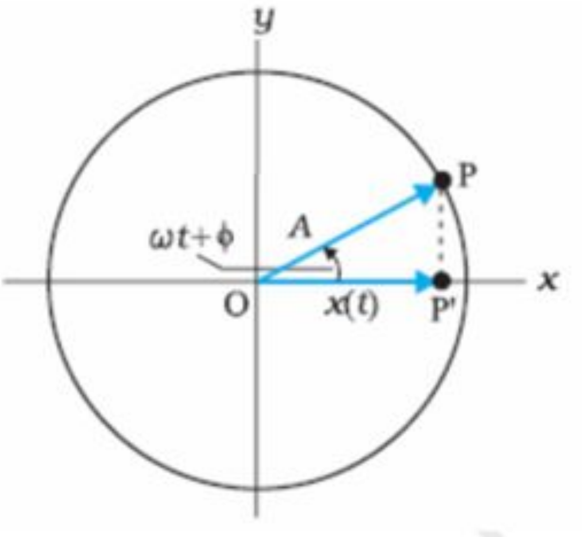
$x = A \cos(\omega t + \varnothing)$

Displacement in SHM

- Consider a particle vibrating back and forth about the origin of an x-axis between the limits –A and +A as shown in figure.
  The displacement x(t) of the particle is given by phase
  $x(t) = A\cos(\omega t + \varnothing)$
  $x(t) \longrightarrow$ Displacement
  $A \longrightarrow$ Amplitude
  $\varnothing \longrightarrow$ Initial phase or phase constant


x
Displacement →
+A
0
-A
t→
T
(a)

Simple Harmonic Motion and Uniform Circular Motion



- Consider the motion of a reference particle P executing uniform circular motion on a reference circle of radius A. At any time the angular position of the particle is $(\omega t+\varnothing)$, where $\varnothing$ is its angular position at $t = 0$.
  The projection of the point P on the x -axis is the point P′.
  The projection of position vector of the reference particle P on the x-axis gives the location x (t) of P′.

Thus we have,

$x(t) = A\cos(\omega t+\varnothing)$

- This shows that if the reference particle P moves in a uniform circular motion; its projection particle P′ executes a simple harmonic motion along a diameter of the circle.

- Thus a simple harmonic motion can be defined as the projection of uniform circular motion on a diameter of the circle.

entri

Question 1:

- A ball is fixed on the edge of a turn table. A light source and a screen are arranged such that the shadow of the ball at anytime during the rotation can be seen on the screen. What is the nature of motion of the shadow of the ball on the screen?

  Ans: Simple Harmonic Motion.

Velocity and Acceleration in SHM

- Displacement of a particle executing SHM is given by x(t) = A cos(ωt+∅)
  ∴ Velocity,

$$v(t) = \frac{d}{dt}\left(x(t)\right)$$

$$= \frac{d}{dt}\left(A\cos(\omega t + \phi\right)$$

$$= A \times {}^{-}\sin(\omega t + \phi) \times \omega$$

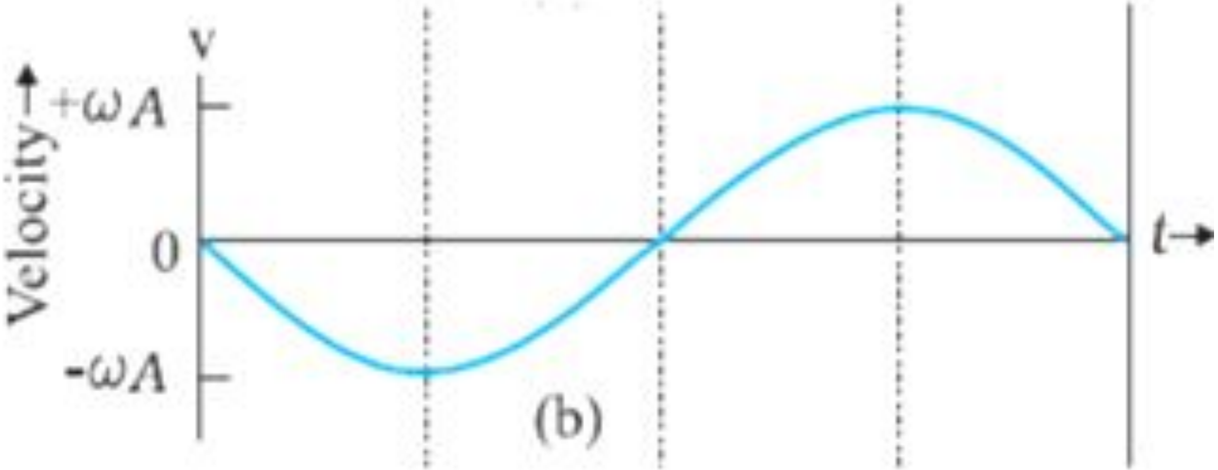
$$v(t) = -\omega A \sin(\omega t + \phi)$$

$$= -\omega A \sqrt{1 - \cos^2(\omega t + \phi)}$$

$$= -\omega \sqrt{A^2 - A^2 \cos^2(\omega t + \phi)}$$

$$= -\omega \sqrt{A^2 - x^2}$$

$$\boxed{v(t) = -\omega \sqrt{A^2 - x^2}}$$



v
Velocity
+ωA
0
-ωA
t
(b)

Acceleration,

$$a(t) = \frac{dv(t)}{dt}$$

$$= \frac{d}{dt}\left(-\omega A \sin(\omega t + \phi)\right)$$

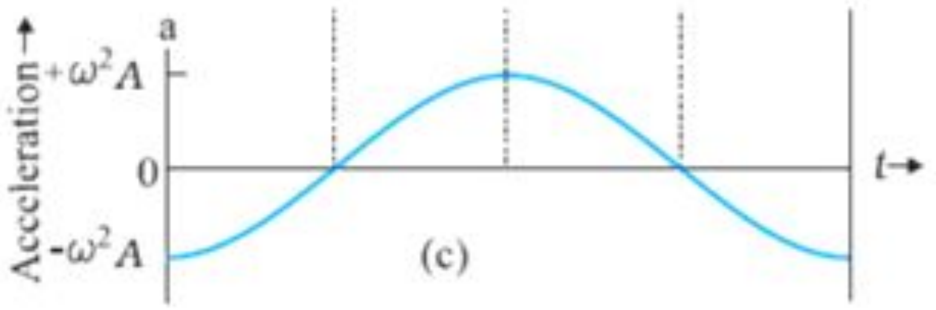
$$= -\omega A \cos(\omega t + \phi) \times \omega$$

$$a(t) = -\omega^2 A \cos(\omega t + \phi)$$

$$\boxed{a(t) = -\omega^2 x(t)}$$

entri

- The above equation shows that in SHM, the acceleration is proportional to displacement and is always directed to the mean position.



Force in SHM

$F = ma$

$= m(-\omega^2 x)$, but $m\omega^2 = k$

$\therefore F = -kx$

$k \rightarrow$ force constant

SI unit of k is N/m

Time Period in SHM

We have, $\omega^2 = \frac{k}{m}$

$$\omega = \sqrt{\frac{k}{m}}$$

We have $\Rightarrow \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$

$$\Rightarrow T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

$$\Rightarrow \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Frequency in SHM

Frequency v = 1/T

$$= \frac{1}{2\pi\sqrt{\frac{k}{m}}}$$

Energy in SHM

- A particle executing SHM has kinetic and potential energies, both varying between the limits, zero and maximum.

Kinetic energy,

$$K = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m[-\omega A\sin(\omega t + \phi)]^2$$

$$= \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$

$$K = \frac{1}{2}kA^2 \sin^2(\omega t + \phi)$$

entri

Potential Energy,

$$U = \frac{1}{2}kx^2$$

$$= \frac{1}{2}k\left(A\cos(\omega t + \phi)\right)^2$$

$$U = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$$

Total energy,

$$E = K + U$$

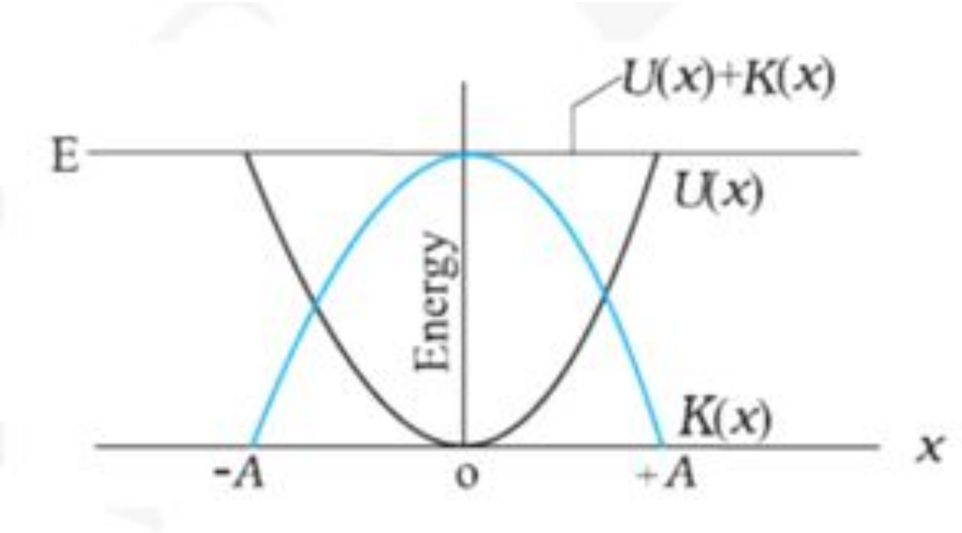
$$= \frac{1}{2}kA^2\sin^2(\omega t + \phi) + \frac{1}{2}kA^2\cos^2(\omega t + \phi)$$

$$=\frac{1}{2}kA^2\left[\sin^2(\omega t+\phi)+\cos^2(\omega t+\phi)\right]$$

$$=\frac{1}{2}kA^2$$

$$\therefore \boxed{E=\frac{1}{2}kA^2}$$

- Total energy of a harmonic oscillation is independent of time, for any conservative force.



THANK YOU