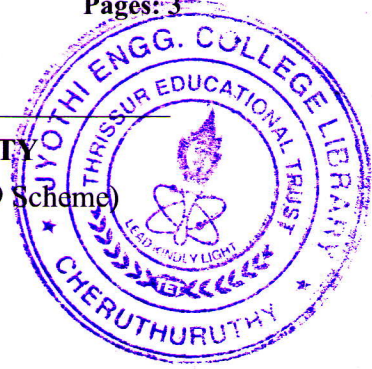


Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
Third Semester B.Tech Degree Examination December 2020 (2019 Scheme)



**Course Code: MAT203**

**Course Name: Discrete Mathematical Structures**

Max. Marks: 100

Duration: 3 Hours

**PART A**

*Answer all questions. Each question carries 3 marks*

- |    |   | Marks |
|----|---|-------|
| 1  | Without using truth tables, show that<br>$p \rightarrow (q \rightarrow r) \equiv p \rightarrow (\sim q \vee r) \equiv (p \wedge q) \rightarrow r$ | (3)   |
| 2  | Define the terms: Converse, Inverse and Contrapositive.   | (3)   |
| 3  | What is Pigeonhole Principle? Given a group of 100 people, at minimum, how many people were born in the same month?                               | (3)   |
| 4  | In how many ways can the letters of the word 'MATHEMATICS' be arranged such that vowels must always come together?                                | (3)   |
| 5  | If $A = \{1,2,3,4\}$ , give an example of a relation on $A$ which is reflexive and transitive, but not symmetric.                                 | (3)   |
| 6  | Define a complete lattice. Give an example.   | (3)   |
| 7  | Define a recurrence relation. Give an example.  | (3)   |
| 8  | Determine the coefficient of $x^{15}$ in $f(x) = (x^2 + x^3 + x^4 + \dots)^4$   | (3)   |
| 9  | Define semi-group. Give an example.   | (3)   |
| 10 | Show that the set of idempotent elements of any commutative monoid forms a submonoid.   | (3)   |

**PART B**

*Answer any one full question from each module. Each question carries 14 marks*

**Module 1**

- 11(a) Check whether the propositions  $p \wedge (\sim q \vee r)$  and  $p \vee (q \wedge \sim r)$  are logically equivalent or not. (6)
- (b) Check the validity of the statement (8)

$$p \rightarrow q$$

$$q \rightarrow (r \wedge s)$$

$$\sim r \vee (\sim t \vee u)$$

$$p \wedge t$$

---


$$\therefore u$$

12(a) Show that  $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$  is a tautology. (6)

(b) Let  $p, q, r$  be the statements given as (8)

$p$ : Arjun studies.  $q$ : He plays cricket.  $r$ : He passes Data Structures.

Let  $p_1, p_2, p_3$  denote the premises

$p_1$ : If Arjun studies, then he will pass Data Structures.

$p_2$ : If he doesn't play cricket, then he will study.

$p_3$ : He failed Data Structures.

Determine whether the argument  $(p_1 \wedge p_2 \wedge p_3) \rightarrow q$  is valid.

### Module 2

13(a) State Binomial theorem. Find the coefficient of  $xyz^2$  in  $(2x - y - z)^4$  (6)

(b) Determine the number of positive integers  $n$  such that  $1 \leq n \leq 100$  and  $n$  is not divisible by 2, 3 or 5. (8)

14(a) Prove that if 7 distinct numbers are selected from  $\{1, 2, 3, \dots, 11\}$ , then sum of two among them is 12. (6)

(b) An urn contains 15 balls, 8 of which are red and 7 are black. In how many ways can 5 balls be chosen so that (i) all the five are red (ii) all the five are black (iii) 2 are red and 3 are black (iv) 3 are red and 2 are black. (8)

### Module 3

15(a) If  $f, g$  and  $h$  are functions on integers,  $f(n) = n^2, g(n) = n + 1,$  (6)

$h(n) = n - 1,$  then find (i)  $f \circ g \circ h$  (ii)  $g \circ f \circ h$  (iii)  $h \circ f \circ g$

(b) If  $A = \{a, b, c\}$  and  $P(A)$  be its power set. The relation  $\leq$  be the subset relation defined on the power set. Draw the Hasse diagram of  $(P(A), \leq).$  (8)

16(a) Let  $R$  be a relation on  $Z$  by  $xRy$  if  $4|(x - y)$ . Then find all equivalence classes. (6)

(b) Find the complement of each element in  $D_{42}$ . (8)

### Module 4

17(a) Solve the recurrence relation  $a_{n+1} = 2a_n + 1, n \geq 0, a_0 = 0.$  (6)

(b) Solve the recurrence relation  $a_{n+2} = a_{n+1} + a_n, n \geq 0, a_0 = 0, a_1 = 1$  (8)

18(a) Solve the recurrence relation  $a_{n+2} - 4a_{n+1} + 3a_n = -200, n \geq 0,$  (6)  
 $a_0 = 3000, a_1 = 3300$

(b) Solve the recurrence relation  $a_n = 2a_{n-1} - 4a_{n-2}, n \geq 3, a_1 = 2, a_2 = 0$  (8)

**Module 5**

19(a) If  $f: (R^+, \cdot) \rightarrow (R, +)$  as  $f(x) = \ln x$ , where  $R^+$  is the set of positive real (6)  
numbers. Show that  $f$  is a monoid isomorphism from  $R^+$  onto  $R$ .

(b) Show that every subgroup of a cyclic group is cyclic. (8)

20(a) State and prove Lagrange's Theorem. (6)

(b) If  $A = \{1,2,3\}$ . List all permutations on  $A$  and prove that it is a group. (8)

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