APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2019 Course Code: MAT101 | Course Name: LINEAR ALGEBRA AND CALCULUS Max. Marks: 100 | Duration: 3 Hours

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PART A

Answer all questions; each carries 3 marks.

1. **Determine the rank** of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 5 \end{bmatrix}$.

2. If 2 is an eigenvalue of $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$, find the other eigenvalues **without using its characteristic equation**.

3. **Show that** $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$, where $z = e^x \sin y + e^y \cos x$.

4. If $f(x,y) = xe^{-y} + 5y$, find the **slope** of $f(x,y)$ in the x-direction at (4,0).

5. Find the **mass** of the square lamina with vertices (0,0), (1,0), (1,1), and (0,1) and density function $x^2 y$.

6. **Evaluate** $\int_{0}^{\infty} \int_{0}^{\infty} e^{-(x^2 + y^2)} \, dx \, dy$ by changing to polar coordinates.

7. **Test the convergence** of the series $\sum_{k=1}^{\infty} \frac{k}{2k+1}$.

8. **Check the convergence** of $\sum_{k=1}^{\infty} \frac{1}{k^{k/2}}$.

9. Find the **Taylor series** for $f(x) = \cos x$ about $x = \frac{\pi}{2}$ up to third-degree terms.

10. Find the **Fourier half-range sine series** of $f(x) = e^x$ in $0 < x < 1$.

PART B

Answer one full question from each module; each carries 14 marks.

Module I

11. a) Solve the system of equations by the G**auss elimination method**:

- $x + 2y + 3z = 1$
- $2x + 3y + 2z = 2$
- $3x + 3y + 4z = 1$

b) Find the **eigenvalues and eigenvectors** of $\begin{bmatrix} 4 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 3 \end{bmatrix}$.

OR

12. a) Find the values of λ and μ for which the system of equations:

- $2x + 3y + 5z = 9$
- $7x + 3y - 2z = 8$
- $2x + 3y + \lambda z = \mu$ has (i) no solution (ii) a unique solution, and (iii) infinite solution.

b) Find the matrix of transformation that **diagonalizes** the matrix $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$. Also write the diagonal matrix.

Module II

13. a) Let f be a differentiable function of three variables and suppose that $w = f(x-y, y-z, z-x)$, show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$.

b) Locate all **relative extrema** of $f(x,y) = 4xy - y^4 - x^4$.

OR

14. a) Find the **local linear approximation** L to $f(x,y) = \sqrt{x^2 + y^2}$ at the point $P(3,4)$. Compare the error at $Q(3.04, 3.98)$ with the distance PQ.

b) The radius and height of a right circular cone are measured with errors of at most 1% and 4%, respectively. Use differentials to approximate the maximum percentage error in the calculated volume.

Module III

15. a) Evaluate $\iint_R y \, dx \, dy$ where R is the region bounded by $y^2 = 4x$ and $x^2 = 4y$.

b) Use a double integral to find the **area** of the region enclosed between $y = \frac{x^2}{2}$ and the line $y = 2x$